Automatica 89 (2018) 125–134

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Cascade observer design for a class of uncertain nonlinear systems with delayed outputs[☆]

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ARTICLE INFO

Article history:
Received 22 July 2016
Received in revised form 4 October 2017
Accepted 11 November 2017

Keywords:
Nonlinear systems
Delayed output
Uncertain system
Cascade observer
High gain observer
Sampling process

ABSTRACT

This paper proposes a state observer with a cascade structure for a class of nonlinear systems in the presence of uncertainties in the state equations and an arbitrarily long delay in the outputs. The design of the observer is achieved under an appropriate set of assumptions allowing to establish the ultimate boundedness of the observation error. Indeed, a suitable expression of the asymptotic observation error, involving the delay, the bound of the uncertainties and the Lipschitz constant of the system nonlinearities, is derived. Besides, it is shown that this ultimate bound is a decreasing function of the cascade length and is equal to zero in the uncertainty-free case. The observer design is first carried out in the case where the output measurements are continuously available and subsequently extended to the case where the outputs are available only at (non equally spaced) sampling instants. The performance of the proposed observer and its main properties are highlighted through illustrative simulation results involving an academic example.

1. Introduction

During the last two decades, an intensive research activity has been devoted to investigate the stability, control and state estimation for linear and nonlinear systems with time delays (see for instance Gu, Kharitonov, and Chen (2003), Kharitonov and Hinrichsen (2004), Lei and Khalil (2016a, b), Obuz, Klotz, Kamalapurkar, and Dixon (2017) and Zheng and Bejarno (2017) with references therein for nonlinear ones). It is worth mentioning that in most works dealing with state estimation for delay systems, the output is assumed to be delay-free. In many real-time applications, some state variables may not be available instantaneously and corresponding measurements are systematically tainted with delay. This is the case of bioreactors where most of the component measurements are obtained with a more or less important time delay since they result from time consuming laboratory analyses. Another typical example is that of network connected systems where some output data are transmitted through low-rate communication systems. This generally introduces non negligible time-delays that have to be accounted for in order to ensure the viability of the control and monitoring system.

The problem of observer design with output delay has been comprehensively examined in Kristic (2009) for linear systems. For nonlinear systems, state observers assuming a cascade structure has been proposed in Besançon, Georges, and Benayache (2007), Germani, Manes, and Pepe (2002), Kazantzis and Wright (2005) and Vafaei and Yazdanpanah (2016). The cascade observer (predictor) is constituted by a chain of subsystems where each subsystem predicts the state of the preceding one in the chain. Such a prediction is made over a horizon whose length is equal to a fraction of the original delay in such a way that the state of the last subsystem provides an estimate of the system actual state. An integral predictor approach has also been proposed in Battilotti (2015) and Karafyllis, Krstic, Ahmed-Ali, and Lamnabhi-Lagarrigue (2014) to estimate the actual state from a delayed output. As noted in Cacace, Germani, and Manes (2014) and Farza et al. (2015), although the integral approach is conceptually simple, it involves the calculation of open-loop integral terms which may be computationally prohibited for real-time application. Moreover,

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Thomas Bo Schön under the direction of Editor Torsten Söderström.

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https://doi.org/10.1016/j.automatica.2017.12.012
0005-1098/

the open-loop structure related to this approach makes it sensitive to uncertainties and modelling error. In all the aforementioned works, the considered classes of systems do not involve uncertainties. Results dealing with state observation of nonlinear systems subject to unknown disturbances and delayed outputs are rather limited. In Fridman and Shaked (2011), a H_∞ filter was designed for a class of linear time delay systems with a non delayed output and involving a disturbance in the state equation. Using LMI tools, state and unknown input observers have been proposed for a class of nonlinear systems with bounded exogenous inputs and delay-free noisy outputs in Chakrabarty, Corless, Buzzard, Zak, and Rundell (2017). A tentative for extending this observer design to the case of delayed outputs is described in Chakrabarty, Buzzard, Fridman, and Zak (2016). A bound on the underlying state and unknown inputs estimation error was provided and this bound depends on some parameters issued from the resolution of a set of LMI's provided that the underlying LMI's are feasible.

In this paper, one shall focus on a particular (and known) class of nonlinear uncertain systems where the output is available with an arbitrarily long delay. Our objective is to extend the cascade observer design proposed in Farza et al. (2015) and Hernández-González, Farza et al. (2016) to this class of systems while providing a full convergence analysis which accounts for the uncertain terms. An expression of the asymptotic estimation error has been derived and according to this expression, the exponential convergence to zero of the observation error is established in the uncertainty-free case. In the presence of uncertainties, the asymptotic estimation error remains in a ball whose radius particularly depends on the delay and on the Lipschitz constant of the system nonlinearities as well as on the bound of the uncertainties. A deep analysis of the so derived expression leads to the fact that the ultimate bound of the asymptotic estimation error is a decreasing function of the cascade length and henceforth smallest values of this bound could be obtained by considering high values of the cascade length. The proposed cascade observer design is first designed by assuming that the output measurements are continuously available. Then, it is appropriately redesigned to deal with the (delayed) sampled outputs case. Moreover, it is shown that the redesigned version inherits the main properties of the original cascade observer. Preliminary results related to the observer design have already been given in Farza et al. (2017) and Hernández-González, Ménard et al. (2016).

This paper is organized as follows. In Section 2, the class of considered systems is introduced and some requisite preliminaries related to the observer design in the delay-free output case are briefly presented. In Section 3, the cascade observer equations are given and the main properties of the asymptotic estimation error are emphasized. The cascade observer is redesigned in Section 4 in order to account for sampled outputs and it is shown that the redesigned continuous–discrete time cascade observer inherits the main properties of the original one. In Section 5, the performance and the main properties of the proposed continuous–discrete time cascade observer are highlighted through an academic example. Some concluding remarks are given in Section 6. For clarity purposes, the proof of some technical results and the convergence analysis of the prediction error related to the cascade observer in the continuous outputs case are given in Appendix.

2. Problem formulation and preliminaries

Consider the class of multivariable nonlinear systems that are diffeomorphic to the following block triangular form:

$$\begin{cases}\dot{x}(t) = Ax(t) + \varphi(u(t), x(t)) + B\varepsilon(t) \\ y_d(t) = Cx(t-d) = x^{(1)}(t-d)\end{cases} \tag{1}$$

with

$$\begin{aligned} x &= \left(x^{(1)T} \ \dots \ x^{(q)T}\right)^T \in \mathbb{R}^n, \\ A &= \begin{pmatrix} 0_{(q-1)p,p} & I_{(q-1)p} \\ 0_p & 0_{p,(q-1)p}\end{pmatrix}, \ C = \left(I_p \ 0_p \ \dots \ 0_p\right), \\ \varphi(u,x) &= \left(\varphi^{(1)}(u,x^{(1)}) \ \varphi^{(2)}(u,x^{(1)},x^{(2)}) \ \dots \ \varphi^{(q)T}(u,x)\right)^T, \\ B &= \left(0_p \ \dots \ 0_p \ I_p\right)^T, \end{aligned} \tag{2}$$

where $x^{(i)} \in \mathbb{R}^p$, $i = 1, \dots, q$, are the state variables (block) components, the nonlinearities $\varphi(u,x)$ are triangular with respect to x, $u \in U$ a compact subset of $\mathbb{R}^m$ denotes the system input and $y_d \in \mathbb{R}^p$ denotes the delayed output of the system, $d > 0$ is the constant (known) measurement delay, $\varepsilon : [-d, +\infty[\ \mapsto \mathbb{R}^p$ is an unknown function describing the system uncertainties. It shall be treated as an unknown function which explicitly depends on the time t for $t \geq -d$.

As it is mentioned in the introduction, our main objective is to design a cascade observer providing an estimation of the actual state of system (1) by using the delayed output measurements. It should be emphasized that two main obstacles have to be handled simultaneously when designing the observer. The first is dealing with the presence of a time delay in the output measurements and the second one is related to the presence of the uncertainties in the state equations. As it shall be seen later, a third obstacle will also be considered when the outputs are no longer available in a continuous manner but only at (not equally spaced) sampling instants. To the authors' best knowledge, these obstacles have not been handled simultaneously in the context of observer design. The uncertain system (1)–(2) has been considered in Bouraoui et al. (2015) but the outputs were not delayed. A high gain observer has been first proposed when the outputs are available in a continuous manner. Then, an appropriate redesign of the so proposed observer has been carried out in order to account for the sampling process of the outputs. The main property of the proposed observer, with continuous as well as sampled outputs, lies in the fact that the asymptotic observation error lies in a ball whose radius can be made as small as desired by appropriately choosing the observer design parameter. In Farza et al. (2015), system (1)–(2) has also been considered but with no uncertainty and a cascade observer has been proposed. The design proposed in Farza et al. (2015) has been extended to a more general class of uncertainty-free systems in Hernández-González, Farza et al. (2016) in the case of continuous-time as well as sampled outputs. It has been shown in Farza et al. (2015) and Hernández-González, Farza et al. (2016) that the underlying observer estimation error converges exponentially to zero for an appropriate choice of the cascade length. The observer which shall be proposed is designed in the spirit of works proposed in Bouraoui et al. (2015), Farza et al. (2015), Hernández-González, Farza et al. (2016) and in particular its equations are similar to those of the observer given in Hernández-González, Farza et al. (2016). Nevertheless, the determination of the main properties of the observer, i.e. the behaviour of the underlying estimation error is not trivial. These properties are indeed different from those related to the observers proposed in the aforementioned references and cannot be deduced from the results given in these references. Indeed, the characterization of the asymptotic estimation error behaviour will be carried out through a deep analysis of an appropriate expression derived for this error. Moreover thanks to the approach adopted when dealing with sampled outputs, the conclusions derived through this analysis are shown to hold in continuous-time and sampled outputs cases.

The observer design will be carried out under the following assumptions.