Automatica 89 (2018) 235–240

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Practical second order sliding modes in single-loop networked control of nonlinear systems[☆]

Michele Cucuzzella[a,*], Antonella Ferrara[b]

[a] ENTEG, Faculty of Science and Engineering, University of Groningen, Nijenborgh 4, 9747 AG Groningen, The Netherlands
[b] Dipartimento di Ingegneria Industriale e dell'Informazione, University of Pavia, via Ferrata 5, 27100 Pavia, Italy

ARTICLE INFO

Article history:
Received 14 December 2015
Received in revised form 30 June 2017
Accepted 6 October 2017

Keywords:
Sliding mode control
Robust control of nonlinear systems
Networked control systems

ABSTRACT

This paper presents a novel Second Order Sliding Mode (SOSM) control algorithm for a class of nonlinear systems subject to matched uncertainties. By virtue of its Event-Triggered nature, it can be used as a basis to construct robust networked control schemes. The algorithm objective is to reduce as much as possible the number of data transmissions over the network, in order not to incur in problems typically due to the network congestion such as packet loss, jitter and delays, while guaranteeing satisfactory performance in terms of stability and robustness. The proposed Event-Triggered SOSM control strategy is theoretically analysed in the paper, showing its capability of enforcing the robust ultimately boundedness of the sliding variable and its first time derivative. As a consequence, it is also possible to prove the practical stability of the considered system, in spite of the reduction of transmissions with respect to a conventional SOSM control approach. Moreover, in order to guarantee the avoidance of the notorious Zeno behaviour, a lower bound for the time elapsed between two consecutive triggering events is provided.

1. Introduction

Networked Control Systems (NCSs) are the obvious solution to control problems in several field implementations because of their advantages in terms of flexibility and reduction of modification and update costs. In NCSs, the presence of the network in the control loop can determine a deterioration of the performance because of critical issues such as packet loss and transmission delays (Hespanha, Naghshtabrizi, & Xu, 2007). Usually, the network malfunctions tend to increase with the network congestion. Thus, the design of control schemes able to reduce the transmissions over the network can be beneficial. In the literature, the so-called Event-Triggered (ET) control (Donkers & Heemels, 2012; Tabuada, 2007; Tallapragada & Chopra, 2014; Wu, Gao, Liu, & Li, 2017) has been proposed as an effective solution for NCSs. In contrast to conventional time-triggered implementation, which features periodic transmissions of the state measurements, ET control approach enables the state transmission only when some triggering condition is satisfied (or violated). For this reason, ET control can reduce the transmissions over the network avoiding the possible network congestion.

On the other hand, Sliding Mode (SM) control is a well-known robust control approach, especially useful to control systems subject to matched uncertainties (Utkin, 1992). The same holds for higher order and, in particular, Second Order Sliding Mode (SOSM) control (Dinuzzo & Ferrara, 2009; Levant, 2003), in which not only the sliding variable but also its time derivatives are steered to zero in a finite time. This is confirmed by the numerous applications described in the literature (see, for instance, Cucuzzella, Incremona, & Ferrara, 2015, 2017; Cucuzzella, Rosti, Cavallo, & Ferrara, 2017; Cucuzzella, Trip, De Persis, & Ferrara, 2017).

In this paper, SOSM control and ET control are coupled to design a novel robust control scheme with a reduced transmission requirement that can be appropriate for NCSs (Cucuzzella & Ferrara, 2016; Ferrara & Cucuzzella, 2018). The proposed control approach is based on two triggering conditions and two control laws that depend on the sliding variable and its first time derivative. Moreover, the proposed control strategy is very easy to implement, it does not require to transmit the state at any time instant, and by virtue of its low implementation complexity, it can be adequate also in case of NCSs. Moreover, the proposed algorithm provides the reduction of the control amplitude when the origin of the auxiliary system state space is approached, with a consequent reduction of the total control energy. The considered system controlled via the proposed strategy is theoretically analysed in the paper, proving

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Zhihua Qu under the direction of Editor Daniel Liberzon.

* Corresponding author.
E-mail addresses: m.cucuzzella@rug.nl (M. Cucuzzella), antonella.ferrara@unipv.it (A. Ferrara).

https://doi.org/10.1016/j.automatica.2017.11.034
0005-1098/

the ultimately boundedness, in a suitable convergence set, of the sliding variable and its first time derivative, even in presence of the uncertainties. In the paper it is also proved that in the convergence set an approximability property analogous to that of classical SM control holds. As a consequence, it is also possible to prove the practical stability of the considered uncertain nonlinear system. Finally, in order to guarantee the avoidance of the notorious Zeno behaviour, a lower bound for the time elapsed between two consecutive triggering events is provided.

2. Problem formulation

Consider the uncertain nonlinear system

$$\dot{x} = a(x) + b(x)u + d_m(x), \tag{1}$$

where $x \in \Omega$ ($\Omega \subset \mathbb{R}^n$ bounded) is the state vector, the value of which at the initial time instant t_0 is $x(t_0) = x_0$, and $u \in [-U_{\max}, U_{\max}]$ is the input, while $a(x) : \Omega \to \mathbb{R}^n$ and $b(x) : \Omega \to \mathbb{R}^n$ are uncertain functions of class $C^1(\Omega)$. Moreover, the external disturbance d_m is assumed to be matched, i.e.,

$$d_m(x) = b(x)d, \quad d \in \mathcal{D} \subset \mathbb{R}, \tag{2}$$

$\mathcal{D}^{\sup} := \sup_{d\in\mathcal{D}}\{|d|\}$ being a known positive constant. Define a suitable output function (the so-called "sliding variable") $\sigma : \Omega \to \mathbb{R}$ of class $C^2(\Omega)$, it being defined as follows.

Definition 1 (*Sliding Variable*). σ is a sliding variable for system (1) provided that the pair (σ, u) has the following property: if u in (1) is designed so that, in a finite time $t_r^\star \geq t_0$, $\forall x_0 \in \Omega$, $\sigma = 0$ $\forall t \geq t_r^\star$, then $\forall t \geq t_r^\star$ the origin is an asymptotically stable equilibrium point of (1) constrained to $\sigma = 0$.

Now, regarding the sliding variable σ as the controlled variable associated with system (1), assume that system (1) is complete in Ω and has a uniform relative degree equal to 2. The following definitions are introduced.

Definition 2 (*Ideal SOSM*). Given $t_r^\star \geq t_0$ (ideal reaching time), if $\forall x_0 \in \Omega$, $\sigma = \dot{\sigma} = 0$ $\forall t \geq t_r^\star$, then an "ideal SOSM" of system (1) is enforced on the sliding manifold $\sigma = \dot{\sigma} = 0$.

Definition 3 (*Practical SOSM*). Given $t_r \geq t_0$ (practical reaching time), if $\forall x_0 \in \Omega$, $|\sigma| \leq \delta_1$, $|\dot{\sigma}| \leq \delta_2$ $\forall t \geq t_r$, then a "practical SOSM" of system (1) is enforced in a vicinity of the sliding manifold $\sigma = \dot{\sigma} = 0$.

Moreover, assume that system (1) admits a global normal form in Ω, i.e., there exists a global diffeomorphism of the form $\Phi = [\Psi, \sigma, a\cdot\nabla\sigma]^T = [x_r, \xi]^T$, with $\Phi : \Omega \to \Phi_\Omega$ ($\Phi_\Omega \subset \mathbb{R}^n$ bounded), and $\Psi : \Omega \to \mathbb{R}^{n-2}$, $\nabla\sigma = (\partial\sigma/\partial x_1, \ldots, \partial\sigma/\partial x_n)^T$, $x_r \in \mathbb{R}^{n-2}$, $\xi = [\sigma, \dot{\sigma}]^T \in \mathbb{R}^2$, such that

$$\begin{cases} \dot{x}_r = a_r(x_r, \xi) & \text{(a)} \\ \dot{\xi}_1 = \xi_2 & \text{(b)} \\ \dot{\xi}_2 = f(x_r, \xi) + g(x_r, \xi)(u + d), & \text{(c)} \end{cases} \tag{3}$$

with $a_r = \frac{\partial\Psi}{\partial x}a$, $f = a\cdot\nabla(a\cdot\nabla\sigma)$, and $g = b\cdot\nabla(a\cdot\nabla\sigma)$. Note that, since a, b are functions of class $C^1(\Omega)$, and σ is a function of class $C^2(\Omega)$, with $\Omega \subset \mathbb{R}^n$ bounded, then functions f, g exist for all $(x_r, \xi) \in \Phi_\Omega$. Moreover, as a consequence of the uniform relative degree assumption, one has that $g \neq 0$. In the literature, see for instance Dinuzzo and Ferrara (2009), subsystem (3)(b)–(3)(c) is called "auxiliary system". Since a_r, f, g are continuous functions and Φ_Ω is a bounded set, one has that

$$\exists\, F > 0 : |f(x_r, \xi)| \leq F, \quad \exists\, G_{\max} > 0 : g(x_r, \xi) \leq G_{\max}. \tag{4}$$

In this paper we assume that F and $G_{\max}$ are known. Moreover, we assume that

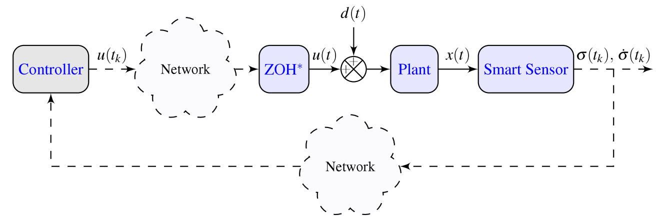

Fig. 1. The proposed single-loop networked control scheme.

$$\exists\, G_{\min} > 0 : g(x_r, \xi) \geq G_{\min}, \tag{5}$$

$G_{\min}$ being a priori known.

Relying on (3)–(5), a first control problem can be stated.

Problem 1. Design a feedback control law

$$u^\star = \kappa(\sigma, \dot{\sigma}), \tag{6}$$

with the following property: $\forall x_0 \in \Omega$, $\exists\, t_r^\star \geq t_0$ such that $\sigma = \dot{\sigma} = 0$, $\forall t \geq t_r^\star$, in spite of the uncertainties.

Note that the solution to Problem 1 is in fact a control law capable of robustly enforcing an "ideal SOSM" of system (1)–(5) in a finite time (see Definition 2). In other terms, any SOSM control law is an admissible solution to Problem 1. Note that, since σ is selected to be a sliding variable (see Definition 1), if Problem 1 is solved, one has that $\forall x_0 \in \Omega$, the origin of the state space is a robust asymptotically stable equilibrium point for (1)–(5).

Typically, the state is sampled at time instants t_k, $k \in \mathbb{N}$, and the control law is computed as $u(t) = u(t_k)$, $\forall t \in [t_k, t_{k+1}[$, the sequence $\{t_k\}_{k\in\mathbb{N}}$ being periodic, with $T = t_{k+1} - t_k$ a priori fixed ("time-triggered"). In this paper, instead of relying on time-triggered executions, we will introduce two triggering conditions, transmitting the values of $\sigma, \dot{\sigma}$ and u only when such conditions are verified ("event-triggered"). Moreover, we assume that the plant is equipped with a particular zero-order-hold, indicated in Fig. 1 with ZOH*, capable of holding constant u, $\forall t \in [t_k, t_{k+1}[$. Relying on (3)–(5), we can formulate the problem that will be solved in the paper.

Problem 2. Design a feedback control law

$$u = u(t_k) = \kappa(\sigma(t_k), \dot{\sigma}(t_k)) \quad \forall t \in [t_k, t_{k+1}[, \tag{7}$$

with the following property: $\forall x_0 \in \Omega$, $\exists\, t_r \geq t_0$ such that $|\sigma| \leq \delta_1$, and $|\dot{\sigma}| \leq \delta_2$, $\forall t \geq t_r$, in spite of the uncertainties, with δ_1 and δ_2 positive constants arbitrarily set.

Note that the solution to Problem 2 is an event-triggered control law capable of robustly enforcing a "practical SOSM" of system (1)–(5) in a finite time (see Definition 3) when a ZOH* is used to generate $u(t)$.

3. The proposed solution

The control scheme proposed to solve Problem 2 is reported in Fig. 1. The existence of a communication network is considered. Yet, we do not explicitly model the network, but we propose a control strategy such that the number of transmissions is reduced to avoid the network congestion. Under these considerations we assume that at the time instants when the triggering conditions are verified, the network is available (we refer to Ferrara and Cucuzzella, 2018 for the case with delayed transmissions due to the unavailability of the network). The proposed control scheme contains two key blocks: the "Smart Sensor" and the "Controller".