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Brief paper

Robust sliding mode regulation of nonlinear systems[☆]

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ABSTRACT

In this paper, the problem of nonlinear Sliding Mode (SM) output regulation is addressed. In particular, a state feedback SM regulator problem is formulated, taking the concepts related to the zero output tracking submanifold as a starting point, and a solution is proposed for general nonlinear affine control systems subject to unmodeled time-varying disturbance. Then, the problem is studied for the particular class of nonlinear systems presented in the so-called Regular form. The effectiveness of the proposed method is demonstrated by the application to the Pendubot system.

1. Introduction

One of the most important problems in nonlinear control theory is the design of a feedback law having the output of a controlled plant asymptotically tracking a prescribed smooth reference signal while rejecting a disturbance generated by some external autonomous system or exosystem. The first solutions to this problem were presented in Francis and Wonhem (1976) for linear systems, and then extended to the nonlinear setting in Huang (2004) and Isidori (1995). In these approaches, the feedback schemes were based on the "internal model principle", incorporating a solution of the *regulator equation*: the Francis equation in the linear case, and the Francis–Isidori–Byrnes (FIB) equation for nonlinear systems.

In recent years, the robustness issue has become important. The design of robust regulators for nonlinear systems with uncertain parameters has been investigated in several works (see Chen & Huang, 2005; Huang, 1995; Huang & Chen, 2004; Huang & Lin, 1994; Marconi, Praly, & Isidori, 2012; Memon & Khalil, 2010; Seshagiri & Khalil, 2005; Zheng & Zhong, 2013). However, in these works, the robustness issue has been considered for *plant parameters variations* only, while in the real situation, model uncertainty can be presented due to both parameters variations and *external unmodeled disturbance*. It is worth noting that the presence of external unknown time-varying disturbance leads to a time-varying regulator equation (Yang & Huang, 2012) which cannot be solved due to the presence of unknown disturbance term.

An alternative approach for dealing with this problem is to combine the output regulation theory with the SM control technique (Utkin, Guldner, & Shi, 1999), which allows decomposing and simplifying the regulator design procedure and imposing robustness properties with respect to unmodeled disturbance, at least to a matched one (Drazenović, 1969). The output regulation problem solution via SM technique has been broadly studied in the last two decades by several authors (see, among others Elmali & Olgac, 1992; Govindaswamy, Floquet, & Spurgeon, 2014; Lai, Edwards, & Spurgeon, 2005; Zheng & Zhong, 2013) mainly for *minimum phase* systems. Few works were addressed to *non-minimum phase* systems, however, just for the case of linear systems (Jeong & Utkin, 1999; Utkin & Utkin, 2014), and for the particular case of nonlinear systems with unitary relative degree (Bonivento, Marconi, and Zanasi, 2001).

In this paper, the SM output regulation problem is studied in the general setup for nonlinear affine control systems and class of systems presented in Regular form subject to unmodeled time-varying disturbance, that included both minimum phase and non-minimum phase systems with arbitrary relative degree. Using the results presented in Yang and Huang (2012), it is shown that the corresponding time-varying regulator equation obtained for the complete system cannot be solved because of the presence of an unknown disturbance term. Based on the equivalent control method (Utkin et al., 1999), the internal model approach is applied to the reduced order SM equation which is invariant w.r.t. the time-varying matched disturbance resulting in the corresponding time-invariant regulator equation. Once solving this reduced order

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regulator equation, getting the center manifold and defining the control errors, a sliding manifold is proposed on which the dynamics of the closed-loop nonlinear system are constrained to evolve by means of a discontinuous control law, instead of designing a linear stabilizing feedback, as in the case of the classical output regulation problem. The designed sliding manifold contains the particular center manifold, and the dynamics of the closed-loop system tends asymptotically along the sliding manifold, to the steady-state behavior, achieving the asymptotic output tracking and thus making unnecessary to obtain the analytic solution of the time-varying regulator equation.

The main contributions of the paper are:

- the deriving of the solution existence conditions for nonlinear affine control systems in the general setup and for a class of those systems presented in Regular form with unmodeled time-varying matched disturbance,
- the designing of a SM regulator with a sliding manifold on which the tracking error tend asymptotically to zero in the presence of unmodeled time-varying matched disturbance.

It is worth noting that the presented results are applicable to nonlinear non-minimum phase systems. Therefore, to show the effectiveness of the proposed approach, the SM regulator approach is applied to the sub-actuated Pendubot, which is a non-minimum phase system.

The rest of this work is organized as follows. In Section 2, the state feedback SM output regulation problem formulation is presented, and, in Section 3, the problem is solved for the general setup of nonlinear affine control systems. The solvability conditions for the presented problem are derived for a class of systems having the Regular Form in Section 4. Section 5 deals with the application of the proposed method to the Pendubot system. Final comments conclude the work in Section 6.

2. Problem statement and assumptions

Consider a nonlinear affine system subject to perturbations

$$\dot{x} = f(x) + B(x)(u + \gamma(x,t)) + D(x)w \tag{1}$$

$$y = h(x) \tag{2}$$

with state x, defined in a neighborhood X of the origin of $\Re^n$, and $u \in \Re^m$, $y \in \Re^p$; $\gamma(x,t)$ is an unmodeled disturbance vector that includes plant parameter variations and external disturbance, $\text{rank}B(x) = m \; \forall \, x \in X$.

The control objective is to design a state feedback controller which enables bringing the tracking error

$$e = y - q(w), \;\; q(0) = 0 \tag{3}$$

to zero. Here, the reference signal $q(w)$ is generated by the exosystem

$$\dot{w} = \phi(w), \;\; w \in W \subset R^s, \;\; \phi(0) = 0. \tag{4}$$

Denoting $A_0 = \left[\frac{\partial f(x)}{\partial x}\right]_{(0)}$, $C_0 = \left[\frac{\partial h}{\partial x}\right]_{(0)}$, $B_0 = B(0)$, the following assumptions are introduced:

H1. *The pair $\{A_0, B_0\}$ is stabilizable.*

H2. *The Jacobian matrix $S = \left[\frac{\partial \phi}{\partial w}\right]_{(0)}$ at the equilibrium point $w = 0$ has all eigenvalues on the imaginary axis.*

H3. *The unknown disturbance vector $\gamma(x,t)$, $\gamma \in \Re^m$ is bounded by*

$$\|\gamma(x,t)\| \le \gamma_0, \;\; \gamma_0 > 0, \tag{5}$$

in an admissible region Ω.

H4. *The vectors x and w are available for measurement.*

Assumption **H1** is clearly needed to locally stabilize the SM dynamics. Assumption **H2** is standard for the output regulation problem while Assumption **H3** specifies a class of unknown disturbances which are bounded and satisfy the matching condition (Drazenović, 1969), as it is usually considered in a robust SM control system design.

Finally, due to lack of space, the attention is focused on the solvability of the State Feedback Regulator (SFR) problem with known x and w (Assumption **H4**), and a robust error feedback problem is left for future work.

In the *classical* setup, in absence of the disturbance, that is, $g(x,t) = 0$, it has been shown that the solvability of the SFR problem can be stated in terms of the existence of a pair of mappings $x = \pi(w)$ and $u = c(w)$ with $\pi(0) = 0$ and $c(0) = 0$ which solves the following regulator equation:

$$\frac{\partial \pi(w)}{\partial w}\phi(w) = f(w) + B(w)c(w) + D(w)w \tag{6}$$

$$0 = h(\pi(w)) - q(w) \tag{7}$$

and the classical control provided by a linear state feedback $u = \lambda(w) + K_0(x - \pi(w))$ with a Hurwitz matrix $(A_0 + B_0K_0)$ can locally stabilize the system (1) ensuring the tracking error (3) tends asymptotically to zero. In presence of $\gamma(x,t)$, it can be assumed that there exist smooth functions $\pi_s(w,t)$ and $c_s(w,t)$ with $\pi_s(0,t) = 0$ and $c_s(0,t) = 0$ such that the following expression holds (Yang & Huang, 2012):

$$\frac{d\pi_s(w,t)}{dt} = f(w,t) + B(w,t)(c_s + \gamma(w,t)) + D(t,w)w \tag{8}$$

$$0 = h(\pi_s(w,t)) - q(w). \tag{9}$$

Obviously these equations are impossible to solve with unknown $\gamma(w,t)$. To overcome this problem, in this paper, the SM technique will be implemented to design a regulator with a sliding manifold which will use the solution $\pi(w)$ of Eqs. (6)–(7) instead of $\pi_s(w,t)$.

Following the regulation theory, the local center manifold is introduced based on (6)–(7) as

$$\varepsilon(x,w) = 0, \; \varepsilon = x - \pi(w) \text{ with } \pi(0) = 0. \tag{10}$$

Then, the local change of variables (10) transforms (1) and (2) into

$$\dot{\varepsilon} = f(\varepsilon,w) + B(\varepsilon,w)[u + \gamma(\varepsilon,w,t)] + D(\varepsilon,w)w - \frac{\partial \pi(w)}{\partial w}\phi(w) \tag{11}$$

$$e = h(\varepsilon,w) - q(w) \tag{12}$$

where $f(\varepsilon,w) = f(x)_{x=\varepsilon+\pi(w)}$, $B(\varepsilon,w) = B(x)_{x=\varepsilon+\pi(w)}$, $D(\varepsilon,w) = D(x)_{x=\varepsilon+\pi(w)}$, $\gamma(\varepsilon,w,t) = \gamma(x,t)_{x=\varepsilon+\pi(w)}$ and $h(\varepsilon,w) = h(x)_{x=\varepsilon+\pi(w)}$.

Now, the *state feedback* SM output regulation problem is defined as the problem of finding a smooth sliding function $s(\varepsilon)$, $s \in \Re^m$ such that the following conditions hold:

- (SMS_{ef}) (Sliding Mode Stability). The state of the system (1) (or (11)) with a quasi continuous (or discontinuous) state feedback converges in finite time to the sliding manifold

$$s(\varepsilon) = 0, \;\; s = (s_1, \ldots, s_m)^T \tag{13}$$

which contains the steady-state (central) manifold (10), and the closed-loop system dynamics tend asymptotically along the sliding manifold (13) to the steady-state behavior.