Automatica 89 (2018) 300–307

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

An internal model approach for multi-agent rendezvous and connectivity preservation with nonlinear dynamics☆

Yi Dong [a], Youfeng Su [b,*], Yue Liu [b], Shengyuan Xu [a]

[a] Department of Automation, Nanjing University of Science and Technology, Nanjing, China
[b] College of Mathematics and Computer Science, Fuzhou University, Fuzhou, China

ARTICLE INFO

Article history:
Received 13 November 2016
Received in revised form 20 May 2017
Accepted 2 October 2017

Keywords:
Rendezvous
Nonlinear systems
Multi-agent systems
Internal model
Connectivity preservation

ABSTRACT

In this paper, we present an internal model approach for the leader-following rendezvous and connectivity preservation problem for a class of multi-agent systems, where both the follower and leader systems can be assumed to contain the strong nonlinearity, as well as the external disturbances and the parametric uncertainties. With the suitable choice of the potential function, two types of internal model based distributed controllers are developed for handling both cases of the certain and uncertain leaders, respectively. By innovatively constructing the intrinsical nonlinear relationship between the rendezvous error and the virtual rendezvous error, we can obtain the corresponding gain functions in both controllers that are able to conquer not only the strong nonlinearity of control plant but also the nonlinearity caused by the potential function. Finally, the validity of our design is illustrated by an example.

1. Introduction

The rendezvous problem of multi-agent systems originated from the nature behaviors of the social animals, the study of which is well motivated by the real applications such as mobile sensing network, intelligent transportation system and so on, see the survey paper (Zavlanos, Egerstedt, & Pappas, 2011). Different from the other coordinated control problems, such as consensus, where the network is predefined and imposed with connectivity assumptions, the rendezvous problem requires the network among the agents to dynamically evolve and be determined by agents' states themselves. Some representative works concerning such state-dependent network can be found in Ando, Oasa, Suzuki, and Yamashita (1999) and Lin, Morse, and Anderson (2007). In order to solve the rendezvous problem, it is necessary to enable the distributed controller the property of achieving the global convergence of the rendezvous errors of all agents, and, more importantly, the capability of maintaining the connectivity of the network. The potential function has been shown to be a significant tool for connectivity preservation, which can generate sufficient force to keep initially connected agents close enough so that they can stay in the sensing range. So far, by combining the potential function and some other robust or adaptive control tools, the rendezvous problem has been intensively studied for linear multi-agent systems (Dimarogonas & Johansson, 2010; Ji & Egerstedt, 2007; Saboori, Nayyeri, & Khorasani, 2013; Su, Wang, & Chen, 2010; Zavlanos et al., 2011), some mechanical systems including under-actuated vehicles (Ajorlou & Aghdam, 2013) and Euler–Lagrange robots (Mao, Dou, Fang, & Chen, 2013), some simple nonlinear multi-agent systems, such as homogeneous networked Lipschitz nonlinear multi-agent system with unit relative degree (Cao, Ren, Casbeer, & Schumacher, 2016) and in the second order form (Su, Chen, Wang, & Lin, 2011), heterogeneous second-order nonlinear multi-agent systems with system dynamics satisfying certain bounded conditions (Feng, Sun, & Hu, 2015), just to name a few. With the aid of some bounded potential function, Saboori et al. (2013) studied such problem for linear multi-agent systems subject to actuator saturation, while Wen, Duan, Su, Chen, and Yu (2012) extended the technique to solve the flocking problem for the same system dynamics as Su et al. (2011).

Also by applying the potential function technique, the rendezvous problem has recently been solved under the output regulation framework by a distributed observer based feedforward control (Dong & Huang, 2014b) and a distributed internal model design (Su, 2015), respectively. However, both of them focused only on linear multi-agent systems. In contrast with the other control schemes, rendezvous control under the output regulation

☆ This work has been supported in part by National Natural Science Foundation of China under Grant Nos. 11571075, 61403082, 61603185, and 61673215, in part by Natural Science Foundation of Fujian Province of China under Grant No. 2016J06014, and in part by the 333 Project (BRA2017380). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Claudio De Persis under the direction of Editor Christos G. Cassandras.

* Corresponding author.
E-mail addresses: yidong09@hotmail.com (Y. Dong), yfsu@fzu.edu.cn (Y. Su), liuyue@fzu.edu.cn (Y. Liu), syxu@njust.edu.cn (S. Xu).

https://doi.org/10.1016/j.automatica.2017.12.018
0005-1098/

framework is able to achieve asymptotic tracking and disturbance rejection simultaneously for a multi-agent system in the leader-following scenario. In particular, the internal model based approach can further deal with the parametric uncertainty in both follower and leader systems.

To satisfy the requirement for the application of rendezvous problem in the real physical systems, the purpose of this paper is to further present an internal model approach for the leader-following rendezvous problem for a group of nonlinear multi-agent systems subject to both plant uncertainties and external disturbances. Both cases of the certain and uncertain leaders are taken into consideration, where in the former case, the controller can be obtained by the internal model based robust control design, while in the latter, we need to further combine adaptive control for estimating the uncertain parameter in the leader system. The contribution of this paper is mainly of two folds. First, in contrast with those concerning connectivity preservation algorithms intended for linear multi-agent systems and nonlinear multi-agent systems satisfying global Lipschitz condition or with bounded system dynamics (Cao et al., 2016; Feng et al., 2015; Su et al., 2011), this paper develops a novel internal model based connectivity preservation algorithm that has the ability of handling the more general strong nonlinearities as well as uncertainties and disturbances. Second, compared with the internal model approaches for the output regulation problem of nonlinear multi-agent systems in the fixed network (Dong & Huang, 2014a; Su & Huang, 2013; Xu, Hong, & Wang, 2014) and for the rendezvous problem of linear multi-agent systems (Su, 2015), our design is much more challenging in that the design of the gain function has to handle not only the strong nonlinearity of each plant but also the nonlinearity caused by the potential function so as to preserve the network connectivity. Technically, as will be shown in Lemma 3 and later, this challenge can be conquered by establishing the inequality of the nonlinear functions between the rendezvous error and the virtual rendezvous error. Notice that such an inequality is non-trivial here, since the relationship between the rendezvous error and the virtual rendezvous error is intrinsical nonlinear instead of the linear one in Dong and Huang (2014a), Su and Huang (2013) and Xu et al. (2014), and the functions in this inequality are all intrinsical nonlinear ones as opposed to the linear ones in Su (2015).

The reminder of this paper is organized as follows. In Section 2, we formulate our problem precisely and provide some preliminaries. In Section 3, we present our main result. An example is illustrated in Section 4. Finally, we close this paper in Section 5 with some concluding remarks.

2. Problem statement & preliminaries

In this section, we first describe our problem, and then provide some preliminaries for the solvability of our problem.

2.1. Problem statement

Consider a class of nonlinear multi-agent systems as follows:

$$\dot{y}_i = g_i(y_i, v, w) + u_i, \; i = 1, \dots, N, \tag{1}$$

where $y_i \in \mathbb{R}^n$ is the position vector of agent i, $u_i \in \mathbb{R}^n$ is the input, $w \in \mathbb{R}^{n_w}$ is an uncertain parameter vector, and $v(t) \in \mathbb{R}^{n_v}$ is an exogenous signal representing both reference input and disturbance. The signal $v(t)$ is assumed to be generated by the leader system that is of the following form:

$$\dot{v} = S(\sigma)v, \quad y_0 = q(v, w), \tag{2}$$

where $y_0 \in \mathbb{R}^n$ is the output of the leader system and represents the reference signal, and $S(\sigma) \in \mathbb{R}^{n_v \times n_v}$ with $\sigma \in \mathbb{R}^{n_\sigma}$ denoting the parameter in the leader system. Here we call the N subsystems of (1) as N followers of the leader (2). We call the leader certain if the parameter σ is known for the feedback design, and uncertain if otherwise. We suppose that all functions in (1) and (2) are globally defined, sufficiently smooth, and satisfy $g_i(0, 0, w) = 0$, $i = 1, \dots, N$, and $q(0, w) = 0$ for all $w \in \mathbb{R}^{n_w}$.

Inspired by Ji and Egerstedt (2007), with respect to the system composed of (1) and (2), we can define a state-dependent time-varying graph[1] $\bar{\mathcal{G}}(t) = (\bar{\mathcal{V}}, \bar{\mathcal{E}}(t))$ where $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$ with node 0 associated with the leader system (2) and node $i = 1, \dots, N$, associated with the ith subsystem of (1), and $\bar{\mathcal{E}}(t) \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The set $\bar{\mathcal{V}}$ is called the node set of $\bar{\mathcal{G}}(t)$ and the set $\bar{\mathcal{E}}(t)$ is called the edge set of $\bar{\mathcal{G}}(t)$. We use the notation $\bar{\mathcal{N}}_i(t) \triangleq \{j : (i, j) \in \bar{\mathcal{E}}(t)\}$ to denote the neighbor set of the node i for $i = 1, \dots, N$. The definition of $\bar{\mathcal{E}}(t)$ associated with the system composed of (1) and (2) is determined by the following rules: given any $r > 0$ and $\epsilon \in (0, r)$, for any $t \geq 0$, $\bar{\mathcal{E}}(t) = \{(i, j) \mid i, j \in \bar{\mathcal{V}}\}$ is defined such that

1. $\bar{\mathcal{E}}(0) = \{(i, j) : \|y_i(0) - y_j(0)\| < r - \epsilon, i, j = 1, \dots, N\} \cup \{(0, j) : \|y_0(0) - y_j(0)\| < r - \epsilon, j = 1, \dots, N\}$;
2. if $\|y_i(t) - y_j(t)\| \geq r$, then $(i, j) \notin \bar{\mathcal{E}}(t)$;
3. $(i, 0) \notin \bar{\mathcal{E}}(t)$, for $i = 0, 1, \dots, N$;
4. for those $(i, j) \notin \bar{\mathcal{E}}(t^-)$ with $i = 0, 1, \dots, N, j = 1, \dots, N$, if $\|y_i(t) - y_j(t)\| < r - \epsilon$, then $(i, j) \in \bar{\mathcal{E}}(t)$.

Here $(i, j) \in \bar{\mathcal{E}}(t)$ means that position difference between the ith and jth agents is smaller than the sensing range, i.e., $\|y_i - y_j\| < r$. In this case, the ith agent can sense the signal $y_i - y_j$ for the feedback at time t. We also define the rendezvous error between the leader and the ith follower as $e_i = y_i - y_0$, $i = 1, \dots, N$. Notice that e_i relies on the leader's signal y_0, and hence is not available for those that are not connected with the leader system. Alternatively, we will resort to the virtual rendezvous error that is $e_{vi} = \sum_{j \in \bar{\mathcal{N}}_i(t)} w_{ij}(t)(y_i - y_j)$, which depends on the position difference of its neighbors. Here $w_{ij}(t)$ are some nonlinear functions determined by the potential function that will be given later. Then the problem of leader-following rendezvous with connectivity preservation is described as follows:

Given the multi-agent system composed of (1) and (2), *and any arbitrarily prescribed compact set* $\mathbb{V}_0 \times \mathbb{W} \times \mathbb{\sigma} \subseteq \mathbb{R}^{n_v + n_w + n_\sigma}$, *find a control law of the form* $u_i = h_{1i}(\eta_i, e_{vi})$, $\dot{\eta}_i = h_{2i}(\eta_i, e_{vi})$, $i = 1, \dots, N$, *where* $\eta_i \in \mathbb{R}^{n_{\eta_i}}$, h_{1i} *and* h_{2i} *are functions vanishing at the origin, such that, for all* $v(0) \in \mathbb{V}_0$, $\text{col}(w, \sigma) \in \mathbb{W} \times \mathbb{\sigma}$ *and all initial conditions* $y_i(0)$, $i = 1, \dots, N$, *that make* $\bar{\mathcal{G}}(0)$ *connected, the closed-loop system has the properties that* $\bar{\mathcal{G}}(t)$ *is connected for all* $t \geq 0$ *and* $\lim_{t \to \infty} e_i(t) = 0$, $i = 1, \dots, N$.

Remark 1. Considering the real physical systems, the rendezvous problem we study here is addressed to the nonlinear multi-agent systems given by (1), containing strong nonlinearities, as well as uncertainties and disturbances, leading to the failure of the existing rendezvous algorithms. For simultaneously handling these issues, we will develop a novel internal model approach for this nonlinear case.

2.2. Preliminaries

In what follows, we will seek an internal model approach for solving this problem. The internal model for nonlinear systems has been well-defined, see Chen and Huang (2015, Sections 7.2 & 7.3). In the rest of this section, for making the paper self-contained, we briefly repeat its construction as well as the standard assumptions that guarantee its existence. It is well-known that the solvability of the regulator equations (Isidori & Byrnes, 1990) is necessary for

[1] See the monograph (Godsil & Royle, 2001) for the graph theory and notation.