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Brief paper

Leader-following control of high-order multi-agent systems under directed graphs: Pre-specified finite time approach[☆]

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ABSTRACT

In this work we address the full state finite-time distributed consensus control problem for high-order multi-agent systems (MAS) under directed communication topology. Existing protocols for finite time consensus of MAS are normally based on the signum function or fractional power state feedback, and the finite convergence time is contingent upon the initial conditions and other design parameters. In this paper, by using regular local state feedback only, we present a distributed and smooth finite time control scheme to achieve leader–follower consensus under the communication topology containing a directed spanning tree. The proposed control consists of a finite time observer and a finite time compensator. The salient feature of the proposed method is that both the finite time intervals for observing leader states and for reaching consensus are independent of initial conditions and any other design parameters, thus can be explicitly pre-specified. Leader-following problem of MAS with both single and multiple leaders are studied.

1. Introduction

Background and motivation. Distributed consensus is known as one of the most important issues in cooperative control of MAS (Qu, 2009). The common important goal in this aspect is to achieve consensus with the least possible topological requirement and in a timely and distributive manner. In this regard, finite time distributed consensus has been an emerging popular topic in the control community in recent years as it offers numerous benefits including faster convergence rate, better disturbance rejection and robustness against uncertainties (Bhat and Bernstein, 2000) leading to fruitful results in literature, see Cao and Ren (2014), Chen, Lewis, and Xie (2011), Du, Wen, Yu, Li, and Chen (2015), Huang, Wen, Wang, and Song (2015), Hui, Haddad, and Bhat (2008), Li, Du, and Lin (2011), Li and Qu (2014), Liu, Lam, Yu, and Chen (2016), Wang and Xiao (2010), Wang, Song, and Krstic (2017), Wang, Song, Krstic, and Wen (2016a, b), Wang, Song, and Ren (2017) and Zuo (2015), to just name a few. However, among the existing results, there is no reported work on finite time consensus that is with smooth control action, further, there is no reported effort in achieving consensus with an explicitly pre-specified finite convergence time, not even for single integrator MAS, to the authors' best knowledge. As for non-networked systems, although prescribed regulation is achievable with the existing fractional approaches (Polyakov, Efimov, & Perruquetti, 2015; Polyakov & Fridman, 2014) by properly choosing the design parameters, it is nontrivial to make such choice as initial conditions and other constraints are involved, this is particularly true for high-order systems.

Note that in engineering, many systems are modeled by higher-order dynamics. For instance, a single link flexible joint manipulator is well modeled by a fourth-order nonlinear system (Khalil, 2002). In this paper we present a smooth control method for high-order MAS under directed communication constraint to achieve leader-following consensus within a finite time that is independent of initial conditions and any other design parameters, thus can be explicitly pre-specified. We achieve this by employing two different time-varying scaling functions to construct a finite-time observer and a finite time compensator, respectively, where the concept of time-varying scaling function, originated from the recent work on finite-time regulation of SISO system by Song, Wang, Holloway, and Krstic (2017), is used.

Contributions of the work. The novelty and contributions of the proposed solution can be summarized as follows: (1) A pre-specified finite time observer for each follower is constructed, and

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we have developed a lemma (Lemma 2) as a tool to analyze the stability and performance of the pre-specified finite time observer. Note that for leader–follower tracking (containment) control of MAS with high-order dynamics, observer is needed to estimate the states information of each order of the leader(s), and this has to be done within a pre-specified finite time, otherwise it would be uncertain when the control for leader-following tracking (containment) should be initiated. This is difficult to achieve with existing finite time control methods, for instance, Li et al. (2011) and Wang, Li, and Shi (2014), because the finite time T therein cannot be preset; (2) the proposed leader-following consensus protocol for the high-order MAS, consisting of a finite time observer and a finite time compensator, is able to achieve consensus within a finite time T that can be uniformity pre-specified, such T is fully independent of initial conditions and any other design parameters. Technical difficulty occurs in analyzing the boundedness of the high-order residue term θ, and we have introduced a lemma (Lemma 3) to deal with such situation; (3) the proposed finite-time control is built upon regular feedback of local system states, and the control action is continuous everywhere and smooth almost everywhere except for one single switching time instant; and (4) the proposed controller can be extended to solve the case with multiple dynamic leaders, where the containment problem involving multiple dynamic leaders arises.

Notation: $\otimes$ represents the Kronecker product; 0 is a vector/matrix with each entry being 0 with appropriate dimension; I_n represents the identity matrix of dimension n; $A > 0$ represents that A is positive definite; we call $A = [a_{ij}] \in R^{N\times N}$ a nonsingular M-matrix, if $a_{ij} < 0, i \neq j$, and all eigenvalues of A have positive real parts.

2. System dynamical model

We consider a multi-agent system consisting of N follower agent(s) and M leader agent(s) $(N, M \geq 1)$. Let $\mathcal{F} = \{1, 2, \ldots, N\}$ and $\mathcal{L} = \{N+1, \ldots, N+M\}$ be the follower set and leader set, respectively. The dynamics of the ith $(i \in \mathcal{F})$ follower agent is of the form,

$$\begin{aligned} \dot{x}_{i,q} &= x_{i,q+1}, \quad q = 1, \ldots, n-1, \\ \dot{x}_{i,n} &= u_i, \end{aligned} \tag{1}$$

where $x_{i,q} \in R^m$ $(q = 1, \ldots, n)$ and $u_i \in R^m$ are the system state and control input, respectively. For convenience, we take $m = 1$ (the case of $m > 1$ can be established similarly). The dynamics of the ith $(i \in \mathcal{L})$ leader agent is,

$$\begin{aligned} \dot{x}_{i,q} &= x_{i,q+1}, \quad q = 1, \ldots, n-1, \\ \dot{x}_{i,n} &= 0. \end{aligned} \tag{2}$$

Suppose that the communication topology among the follower(s) and the leader(s) is described by a directed graph $\mathcal{G} = (\iota, \varepsilon)$, where $\iota = \{\iota_1, \ldots, \iota_{N+M}\}$ is the set of vertices representing $N+M$ agents and $\varepsilon \subseteq \iota \times \iota$ is the set of edges of the graph (Ren & Cao, 2010). The directed edge $\varepsilon_{ij} = (\iota_i, \iota_j)$ denotes that vertex ι_j can obtain information from ι_i. The set of in-neighbors of vertex ι_i is denoted by $\mathcal{N}_i = \{\iota_j \in \iota | (\iota_j, \iota_i) \in \varepsilon\}$. We denote by $\mathcal{A} = [a_{ij}] \in R^{(N+M)\times(N+M)}$ the weighted adjacency matrix of $\mathcal{G}$, where $\varepsilon_{ji} \in \varepsilon \Leftrightarrow a_{ij} > 0$, otherwise, $a_{ij} = 0$. In addition, $a_{ii} = 0$. $\mathcal{D} = \mathrm{diag}(\mathcal{D}_1, \ldots, \mathcal{D}_N) \in R^{(N+M)\times(N+M)}$, with $\mathcal{D}_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the weighted in-degree of node i, denotes the in-degree matrix such that the Laplacian matrix is defined as $L = [l_{ij}] = \mathcal{D} - \mathcal{A}$.

3. Coordinated tracking control with one leader

In this section, we consider the pre-specified finite time coordinated tracking problem with 1 leader, i.e., $M = 1$. In such case, a leader is an agent without in-neighbors, a follower is an agent that has at least one in-neighbor, and the Laplacian L is represented as $L = \begin{bmatrix} L_1 & L_2 \\ 0_{1\times N} & 0 \end{bmatrix}$ with $L_1 \in R^{N\times N}$ and $L_2 \in R^{N\times 1}$.

Assumption 1. The topology $\mathcal{G}$ contains a directed spanning tree, where the leader acts as the root.

Lemma 1 (*Li, Wen, Duan, & Ren, 2015*). *Under Assumption 1, L_1 is a nonsingular M-matrix and is diagonally dominant, then there exists a matrix $Q = diag\{q_1, \ldots, q_N\} > 0$, in which $q_1, \ldots, q_N$ are determined by $[q_1, \ldots, q_N]^T = (L_1^T)^{-1} 1_N$, such that*

$$QL_1 + L_1^T Q > 0. \tag{3}$$

Definition 1. The pre-specified finite time full state tracking consensus of leader–follower MAS (1)–(2) with the pre-specified finite time T^* is said to be solved if, for any given initial state, it holds that

$$x_{i,q} \to x_{N+1,q} \quad \text{as} \quad t \to t_0 + T^* \tag{4}$$

$$x_{i,q} = x_{N+1,q} \quad \text{when} \quad t \geq t_0 + T^*, \tag{5}$$

for all $i \in \mathcal{F}$ and $q = 1, \ldots, n$.

In the following, we begin the controller design and stability analysis. We first introduce the local neighborhood error for the ith $(i \in \mathcal{F})$ follower as,

$$\epsilon_{i,q} = \sum_{j\in\mathcal{F}\cup\mathcal{L}} a_{ij}(x_{i,q} - x_{j,q}), \quad q = 1, \ldots, n. \tag{6}$$

We denote the state error between the ith $(i \in \mathcal{F})$ follower and the leader by

$$\delta_{i,q} = x_{i,q} - x_{N+1,q}, \quad q = 1, \ldots, n. \tag{7}$$

Denote by $\epsilon_i = [\epsilon_{i,1}, \ldots, \epsilon_{i,n}]^T \in R^n$, $x_i = [x_{i,1}, \ldots, x_{i,n}]^T \in R^n$, $x_{N+1} = [x_{N+1,1}, \ldots, x_{N+1,n}]^T \in R^n$, $\delta_i = x_i - x_{N+1}$, for $i \in \mathcal{F}$, and $E = [\epsilon_1^T, \ldots, \epsilon_N^T]^T \in R^{nN}$, $X = [x_1^T, \ldots, x_N^T]^T \in R^{nN}$, $X_{N+1} = 1_N \otimes x_{N+1} \in R^{nN}$, $\delta = X - X_{N+1}$, such that

$$E = [L_1 \otimes I_n](X - X_{N+1}) = [L_1 \otimes I_n]\delta. \tag{8}$$

To achieve finite-time control with uniformly pre-specified finite time, we need to introduce two time-varying scaling functions,

$$\varrho(t) = \frac{T_1^{1+h}}{(T_1 + t_l - t)^{1+h}}, \quad t \in [t_l, t_l + T_1), \tag{9}$$

$$\eta(t) = \begin{cases} 1, & t \in [t_0, t_1), \\ \dfrac{T^{n+h}}{(T + \tau_l - t)^{n+h}}, & t \in [\tau_l, \tau_l + T), \end{cases} \tag{10}$$

where $h > 1$ $(h \in Z_+)$ is a user chosen constant, $t_{l+1} = t_l + T_1$ $(l \in Z_+ \cup \{0\})$, and $\tau_{l+1} = \tau_l + T$ $(l \in Z_+)$ with $\tau_1 = t_0 + T_1 = t_1$. Here $T_1 > 0$ and $T > 0$ denote the pre-specified finite convergence time, respectively, both are designer-specified real number satisfying $T_1 \geq T_r$ and $T \geq T_r$, where T_r denotes the time period needed for signal processing/computing and information transmission/communication.

Properties of $\varrho(t)$: for $l \in Z_+ \cup \{0\}$ and $p > 0$, it holds that
(i) $\varrho(t)^{-p}$ is monotonically decreasing on $[t_l, t_l + T_1)$;
(ii) $\varrho(t_l)^{-p} = 1$ and $\lim_{t\to(t_l+T_1)^-} \varrho(t)^{-p} = 0$.

Properties of $\eta(t)$: for $l \in Z_+$ and $p > 0$, it holds that
(i) $\eta(t)^{-p}$ is monotonically decreasing on $[\tau_l, \tau_l + T)$;
(ii) $\eta(\tau_l)^{-p} = 1$ and $\lim_{t\to(\tau_l+T)^-} \eta(t)^{-p} = 0$.

Hereafter, we denote by $\bullet^{(q)}$ $(q = 0, \ldots, n)$ the qth derivative of $\bullet$ with $\bullet^{(0)} = \bullet$, and denote by $\bullet^k = \underbrace{\bullet \cdots \bullet}_{k}$ $(k \in N_+)$ the kth power