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Brief paper

Stochastic thresholds in event-triggered control: A consistent policy for quadratic control[☆]

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ABSTRACT

We propose an event-triggered control scheme for discrete-time linear systems subject to Gaussian white noise disturbances. The event-conditions are given in terms of the deviation between the actual system state and the state of a nominal undisturbed system whose state is identical to the real system state at the event times. In order to ensure that the conditional distribution of the deviation between the two systems, under the condition that no event occurs, remains a normal distribution, we employ thresholds that are themselves random variables. This allows us to: (i) provide expressions for the probability mass function of the times between events and, in turn, arbitrarily select this function; (ii) synthesize controllers associated with the proposed transmissions scheduler that are optimal in terms of an average quadratic cost. In particular, these two properties allow us to show that our event-triggered scheme is consistent in the sense that it outperforms (in a quadratic cost sense) traditional periodic control for the same average transmission rate and does not generate transmissions in the absence of disturbances. We demonstrate the effectiveness of our scheme in a numerical example and describe a way to solve the non-convex optimization problem arising in the approach.

1. Introduction

Event-triggered control has been proposed in recent years to reduce the communication burden of traditional periodic control in networked control systems. A standard networked control system for a single control loop is depicted in Fig. 1. An event-triggered control scheme is composed of: (i) a scheduler which based on sensor measurements of the process determines when transmissions to a remote controller occur; (ii) a controller which based on the *received* sensor measurements decides the control input. In most of the previous works (e.g. Anderson, Milutinović, & Dimarogonas, 2015; Årzén, 1999; Åström & Bernhardsson, 2002; Cassandras, 2013; Demirel, Gupta, & Johansson, 2013; Garcia & Antsaklis, 2013; Heemels, Johansson, & Tabuada, 2012; Lunze & Lehmann, 2010; Rabi & Johansson, 2009; Tabuada, 2007; Tallapragada, Franceschetti, & Cortés, 2016; Yook, Tilbury, & Soparkar, 2002), the scheduler decides when transmissions should occur based on deterministic criteria, such as the deviation between process state and a state estimation on the controller side (e.g., the last transmitted state value or a model-based estimation) exceeding a given value or, more generally, a function of the information known to the trigger mechanism at a given time crossing a given threshold. While designing the controller and the scheduler in an optimal way, according to given performance specifications, is typically hard, it is still possible to design these to meet desired specifications. In particular a desired property, one of the consistency properties defined in Antunes and Khashooei (2016), is that the performance of such event-triggered control is better than that of periodic control for the same average transmission rate.

In this paper, we propose consistent event-triggered controllers that rely on stochastic thresholds. We consider a process disturbed by Gaussian white noise. As has been recognized recently, if deterministic threshold policies are employed in event-triggered control, the state does not preserve the Gaussian nature of the probability distribution of the disturbances (Wu, Ren, Han, Shi, & Shi, 2016). This severely complicates, or even totally prevents, the exact stochastic characterization of the state and, therefore, the closed-loop design and analysis. In the context of event-triggered estimation, the authors of Han et al. (2015), Shi, Chen, and Darouach (2016) and Wu et al. (2016) have therefore proposed *stochastic* threshold policies for which the state preserves the

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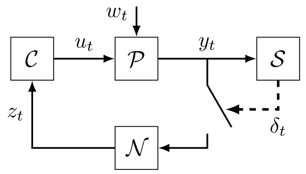

Fig. 1. Event-triggered control loop consisting of plant $\mathcal{P}$, controller $\mathcal{C}$, scheduler $\mathcal{S}$, and network $\mathcal{N}$.

Gaussian nature of the probability distribution of the disturbances. The main idea is to define the trigger condition in terms of a threshold on the value of a quadratic form of the deviation between the actual system state and a nominally predicted system state. The value of the threshold is itself a random variable, drawn (independently from all other variables) from a certain exponential[1] distribution. In Han et al. (2015) and Shi et al. (2016), the shaping matrix of the quadratic form is a fixed tuning parameter in the estimation scheme. Here, we choose this matrix equal to the covariance matrix of the current distribution of the deviation between the actual system state and the nominally predicted state, similarly to Wu et al. (2016). This allows us to obtain explicit expressions of the trigger probabilities in terms of the intensity of the exponential distributions defining the thresholds, and, in turn, enables us to assign these probabilities arbitrarily. The use of stochastic thresholds is then exploited in the context of closed-loop control to design an event-triggered controller that minimizes a quadratic cost, as in standard discrete-time (periodic) linear quadratic control, with communication constraints captured by constraints on the (average) transmission rate. This is in general a hard problem, see for example Antunes and Heemels (2014), Molin and Hirche (2013) and Ramesh, Sandberg, Bao, and Johansson (2011), which consider similar problems where the transmission rate is also penalized in the cost function. While several results have been established on the structural properties of optimal designs, the optimal – and combined – design of the state estimator, scheduler, and controller remains challenging. We propose to fix the scheduler to be a stochastic scheduler as discussed above, and show that in the state-feedback case the optimal controller can then be found and is, in fact, given by certainty equivalence feedback, consistent with the results in Molin and Hirche (2013). By optimizing the parameters of the proposed scheduler subject to a desired transmission rate constraint, we show that the proposed event-triggered controller is never outperformed by an optimal periodic control with the same average transmission rate, which is the first of two properties of a consistent periodic controller as defined in Antunes and Khashooei (2016). We also show that the second property is satisfied, that is, that no transmissions are generated in the absence of disturbances. While the resulting optimization problem needed to obtain the scheduling law is non-convex, we show how it can be solved such that the performance of the resulting closed loop is not worse than if periodic control is employed.

The remainder of the paper is organized as follows. This introductory section concludes with some remarks on notation. The stochastic scheduler and its main properties are presented in Section 2. In Section 3, consistency and optimality of the control scheme are discussed. A numerical example illustrating the results is presented in Section 4, certain aspects of the proposed scheme are discussed in Section 5, and Section 6 concludes the paper.

Notation: $\mathbb{N}$ denotes the set of natural numbers, $\mathbb{N}_0$ denotes the set of non-negative integers. For $x \in \mathbb{R}^n$ and for a positive definite $Q \in \mathbb{R}^{n\times n}$ we define $\|x\|_Q := \sqrt{x^\top Qx}$. The rank of a matrix $M \in \mathbb{R}^{n\times n}$ is denoted by $\mathrm{rk}(M)$, its Moore–Penrose pseudo-inverse by $M^\dagger$, and the product of all of its non-zero eigenvalues (each according to its algebraic multiplicity) by $\det^\dagger(M)$. Moreover, $z \sim \mathcal{N}(\bar{z}, \Xi)$ $(z|I \sim \mathcal{N}(\bar{z}, \Xi))$ indicates that the (conditioned) probability distribution of $z \in \mathbb{R}^n$ (on the information I) is Gaussian with mean $\bar{z}$ and covariance $\Xi \succeq 0$. Finally, $r \sim \exp(\lambda)$ indicates that $r \in \mathbb{R}$ is an exponentially distributed random variable with intensity $\lambda \geq 0$.

2. Schedulers with stochastic thresholds

We detail the event-triggered setting in Section 2.1 and define the proposed class of stochastic schedulers in Section 2.2. We then state a first key result regarding the Gaussian nature of the state distribution in Section 2.3. In Section 2.4 we consider the special case of identically distributed disturbances. In Section 2.5, we investigate the stability of the closed-loop system.

2.1. Event-triggered setting

Consider the networked control configuration presented in Fig. 1. The process $\mathcal{P}$ is assumed to be described by

$$x_{t+1} = Ax_t + Bu_t + w_t \tag{1}$$

with state $x_t \in \mathbb{R}^n$, input $u_t \in \mathbb{R}^m$, and disturbance $w_t \in \mathbb{R}^n$ at time $t \in \mathbb{N}_0$, respectively, where w_t, $t \in \mathbb{N}_0$, is generated by a Gaussian white noise process with zero mean and covariance $0 \neq W_t \in \mathbb{R}^{n\times n}$ (that is, $w_t \sim \mathcal{N}(0, W_t)$ are independent but not necessarily identically distributed at every $t \in \mathbb{N}_0$). We assume that there exists a uniform bound $\mathrm{tr}(W_t) \leq \bar{W}$ on the disturbance covariances with $\bar{W} \in (0, \infty)$. Further, the pair (A, B) is assumed to be controllable and the initial state x_0 to be given deterministically. Finally, we assume the network $\mathcal{N}$ to be ideal, that is, free from delays or packet drop-outs.

The scheduler must decide at each time t whether to transmit the state information to the controller based on the information available up to time t, being

$$I_t^{\mathrm{s}} := (y_0, y_1, \ldots, y_t),$$

where we assume perfect state information, that is,

$$y_t = x_t.$$

We define $\delta_t = 1$ if the scheduler decides to transmit at time t and $\delta_t = 0$ otherwise and let μ_t denote the scheduler policy

$$\delta_t = \mu_t(I_t^{\mathrm{s}}).$$

The controller must decide at each time t the control input u_t to apply to the plant based on the information available to the controller

$$I_t^{\mathrm{c}} = (z_0, z_1, \ldots, z_t),$$

where

$$z_t = \begin{cases} y_t & \text{if } \delta_t = 1 \\ \emptyset & \text{otherwise.} \end{cases}$$

We denote the controller policy by γ_t, hence

$$u_t = \gamma_t(I_t^{\mathrm{c}}). \tag{2}$$

Note that since u_t is a function of I_t^{c}, the controller can use previously computed control inputs to infer the next control input, i.e., we can assume that the information available to the controller is

$$I_t^{\mathrm{c}} = ((z_0, z_1, \ldots, z_t), (u_0, u_1, \ldots, u_{t-1})).$$

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[1] The exact realization of the stochastic trigger rules is slightly different in Han et al. (2015), Shi et al. (2016) and Wu et al. (2016), but can be shown to be equivalent to the realization in the present manuscript. It is also equivalent to the trigger rule proposed in Demirel, Leong, and Quevedo (2017), where a similar approach to event-triggered control as presented here is considered for scalar systems based on dead-beat control.