Automatica 87 (2018) 166–175

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Output feedback stabilization of stochastic feedforward systems with unknown control coefficients and unknown output function ☆

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ARTICLE INFO

Article history:
Received 22 February 2017
Received in revised form 4 September 2017
Accepted 10 September 2017
Available online 7 November 2017

Keywords:
Output feedback control
Unknown control coefficient
Unknown output function
Stochastic feedforward system
Time-varying
Backstepping

ABSTRACT

This paper investigates the problem of output feedback control for a class of stochastic feedforward systems with unknown control coefficients and unknown output function. Since many unknowns occur into the systems, all the states of the systems are unmeasurable or unknown which means that they are not available in the control scheme. To compensate these unmeasurable/unknown states, a new form of K-filters with time-varying low-gain is introduced in our design. As long as output function belongs to any close sector included in the maximal sector region of systems' output function, we can design a time-varying output feedback control law by integrating the well-known backstepping framework with the time-varying technique. Later, based on the improved LaSalle-type theorem, we analyze the regulation of the closed-loop systems. Finally, an induction heater circuit system with unknown inductance/capacitance is given to show the effectiveness of our control scheme.

1. Introduction

Recently, based on the stochastic stability theory (Khasminski, 1980; Mao, 2007), the problem of output feedback stabilization for stochastic nonlinear systems has been paid more attention by many researchers. Deng and Krstić (1999) proposed a powerful tool–*backstepping design method* to study the globally asymptotic stabilization of stochastic nonlinear systems via output feedback. By employing the homogeneous system theory, Li, Xie, and Zhang (2011) considered the output feedback stabilization of stochastic nonlinear systems in lower-triangular form under weaker condition. More efforts have been achieved in Deng and Krstić (2000), Krstić and Deng (1998), Liu and Zhang (2006) and references therein. However, these works only focused on stochastic lower-triangular systems. As we all know, many practical physical models can be described by upper-triangular (feedforward) systems, such as cart-pendulum systems (Mazenc & Bowong, 2003), ball-beam systems with a friction term (Sepulchre, Jankovic, & Kokotovic, 1997) and an induction heater circuit system (Lander, 1987). Hence, the output feedback control of such systems is an important topic in the control field. By designing homogeneous reduced-order observers with a static low-gain, Liu, Yu, Yu, and Zhou (2014) investigated the output feedback stabilization of stochastic feedforward systems with state time-delay. Zhang, Zhao, and Xie (2015) extended the results to stochastic high-order case. Besides, Zha, Zhai, Fei, and Wang (2014) considered the global finite-time stabilization by output feedback of stochastic feedforward systems.

Note that the above works mainly relied on the precise information on the control coefficients and the growth rate of nonlinearity during the control scheme. However, these information may always be unknown in practical applications, therefore how to cope with the global stabilization of nonlinear systems with these unknowns is a hot issue. To overcome this obstacle, Liu (2013) developed *the time-varying technique* to design output feedback controllers for uncertain lower-triangular systems with unknown control coefficients and unknown growth rate. For the stochastic case, Li and Liu (2015) introduced a form of K-filters with time-varying high-gain to study the output feedback regulation of stochastic lower-triangular systems. Jiao, Zheng, and Xu (2016) studied the state feedback stabilization of stochastic feedforward systems with unknown growth rate via the time-varying technique and the improved LaSalle-type theorem. As unknown output function is admitted into systems, the authors in Jiang, Zhang, and Xie (2017) considered the global stabilization of stochastic nonlinear systems in lower-triangular form. More related results can be found in Jia, Xu, Cui, Zhang, and Ma (2016), Liu, Ge, and Zhang (2008), Li and Liu (2017), Li, Xie, and Zhang (2017), Liu, Zhang, and Jiang (2007), Wang and Wei (2015), Wu, Chen, and Li (2016),

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Changyun Wen under the direction of Editor Miroslav Krstic.
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https://doi.org/10.1016/j.automatica.2017.10.004
0005-1098/

Wu, Li, Zong, and Chen (2017), Zhang, Mu, and Liu (2017) and references therein. Naturally, an important and unsolved problem is proposed: *How can we achieve the output feedback stabilization of stochastic feedforward systems with unknown control coefficients and unknown output function?*

In this paper, we will answer this question. The main contributions of this paper are concluded as follows:

(i) Different from the existing literature on stochastic feedforward systems, many unknowns occur in the systems. Owing to this fact, the adaptive methods adopted in the literature (Liu & Xie, 2013; Liu et al., 2014; Zha et al., 2014; Zhang et al., 2015; Zhao & Xie, 2014) are not applicable. Therefore, in this paper, we will introduce a novel time-varying gain, instead of a static gain used in Liu and Xie (2013), Liu et al. (2014), Zha et al. (2014) and Zhang et al. (2015), into the backstepping framework. Later, a time-varying output feedback controller is designed recursively. Meanwhile, we also analyze the convergence of the solution to the close-loop system, rather than the stability (e.g., globally asymptotic stability Liu and Xie, 2013, Liu et al., 2014, Zhao and Xie, 2014, Zhang et al., 2015 or finite-time stability in probability Zha et al., 2014).

(ii) The systems considered in this paper own upper-triangular structures and unknown output function. It means that the time-varying high-gain observers (for example, the observers in Liu (2013) or the K-filters in Li and Liu, 2015) are invalid here. Hence, we need to design a new form of K-filters with low-gain to compensate the unmeasurable/unknown states.

The rest of this paper is organized as follows. In Section 2, several useful lemmas, problem formulation and three assumptions are presented. In Section 3, a coordinate transformation and a new form of K-filters are given explicitly. Then, a time-varying output feedback controller is designed in Section 4 by using the time-varying technique and backstepping method. In Section 5, a simulation is given to illustrate our theoretical results. Finally, Section 6 provides some conclusions.

Note: Throughout this paper, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{Z}_+$ denotes the set of positive integers, I_n denotes the $n \times n$ identity matrix, $\mathbb{R}^n$ denotes the Euclidean space, and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrix. For any vector $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm of vector x. $A = (a_{ij})_{N\times N}$ denotes a matrix of N-dimension, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ mean the largest and smallest eigenvalues of matrix A, respectively, A^T denotes the transpose of A, $\mathrm{tr}\{A\}$ denotes its trace and $\|A\| = \sqrt{\mathrm{tr}(A^TA)}$.

2. Preliminaries

Let $w = (w_1(t), \ldots, w_r(t))^T$ be an r-dimensional Brownian motion defined in a complete probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. In this paper, we consider the following stochastic feedforward system

$$\begin{cases} \mathrm{d}\zeta_i = (d_i\zeta_{i+1} + \bar{f}_i(t,\zeta,u))\mathrm{d}t + \bar{g}_i^T(t,\zeta,u)\mathrm{d}w, \\ \mathrm{d}\zeta_n = d_n u\mathrm{d}t + \bar{g}_n^T(t,\zeta,u)\mathrm{d}w, \quad i = 1,\ldots,n-1, \\ y = \delta(\zeta_1), \end{cases} \tag{1}$$

where $\zeta = (\zeta_1, \ldots, \zeta_n)^T \in \mathbb{R}^n$ is the system state with the initial data $\zeta(t_0) = \zeta_0$ and t_0 being the initial time. $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are system's control input and output, respectively. The unknown constants d_i, $i = 1,\ldots,n$, are the system's control coefficients. $\delta(\cdot) : \mathbb{R} \to \mathbb{R}$ is an unknown function satisfying $\delta(0) = 0$. $\bar{f}_i : [t_0, +\infty) \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$, $\bar{g}_i : [t_0, +\infty) \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^r$, $i = 1,\ldots,n-1$, and $\bar{g}_n : [t_0, +\infty) \times \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^r$ are unknown continuous functions, which are assumed to be locally Lipschitz[1] in ζ and u with $\bar{f}_i(t,0,0) = 0, \bar{g}_i(t,0,0) = 0, \bar{g}_n(t,0,0) = 0$.

Consider the following stochastic nonlinear system

$$\mathrm{d}x = f(t,x)\mathrm{d}t + g(t,x)\mathrm{d}w, \quad x(t_0) = x_0 \in \mathbb{R}^n, \tag{2}$$

where functions $f : [t_0, +\infty) \times \mathbb{R}^n \to \mathbb{R}^n$ and $g : [t_0, +\infty) \times \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are continuous in all arguments and locally Lipschitz in x with $f(t,0) = 0$ and $g(t,0) = 0$.

Let $\mathcal{C}^2(\mathbb{R}^n; \mathbb{R}_+)$ be the family of all nonnegative functions V on $\mathbb{R}^n$ which are continuously twice differentiable in x. For each $V \in \mathcal{C}^2(\mathbb{R}^n; \mathbb{R}_+)$, define an operator $\mathcal{L}V$ associated with (2) as follows:

$$\mathcal{L}V = \frac{\partial V}{\partial x^T} f(t,x) + \frac{1}{2}\mathrm{tr}\left\{ g^T(t,x)\frac{\partial^2 V}{\partial x^2} g(t,x) \right\}. \tag{3}$$

Lemma 1 (Zhao and Xie, 2014)**.** *Suppose that c and d are two positive real numbers, $x, y \in \mathbb{R}$. Then, for any real-valued function $\gamma(x,y) > 0$, we have $|x|^c|y|^d \leq \frac{c}{c+d}\gamma(x,y)|x|^{c+d} + \frac{d}{c+d}(\gamma(x,y))^{-\frac{c}{d}}|y|^{c+d}$.*

Definition 1 (Jiang et al., 2017, Li et al., 2017)**.** A function $\delta : \mathbb{R} \to \mathbb{R}$ is said to belong to the sector $[\beta_1, \beta_2]$ if $(\delta(s) - \beta_1 s)(\delta(s) - \beta_2 s) \leq 0$, where β_1 and β_2 are constants with $\beta_2 > \beta_1$.

Lemma 2 (Li et al., 2017)**.** *If a function δ belongs to some a sector $[\beta_1, \beta_2]$ with $\beta_2 > \beta_1$, then $|\delta(s) - s| \leq \max\{|\beta_1 - 1|, |\beta_2 - 1|\}|s|$, for all $s \in \mathbb{R}$.*

To achieve the stabilization of system (1), we need the following lemma, which can be seen as an extension of Theorem 1 in Liu et al. (2008):

Lemma 3. *For system (2), if there exists a function $V \in \mathcal{C}^2(\mathbb{R}^n; \mathbb{R}_+)$ such that for some constant $K > 0$ and any $t \geq 0$,*

$$\mathcal{L}V \leq K(1 + V(t,x)), \quad \liminf_{|x|\to\infty} V(t,x) = \infty,$$

then, there exists a unique global solution on $[t_0, \infty)$.

Proof. Since both of terms f and g are locally Lipschitz in x, it follows from Lemma 4 in Zhao and Deng (2016) that system (2) exists a unique local strong solution. The rest of the proof can be obtained by the similar method used in Theorem A.1 (Mao, 2002), and thus we omit it for brevity.

In what follows, we state three important assumptions.

Assumption 1. For all $i = 1,\ldots,n-1$, there exists an unknown growth rate $\vartheta > 0$ and two known constants $\alpha_1 \in [0,1)$, $\alpha_2 \in [0,1/2)$ such that for all $t \geq t_0$, $\zeta \in \mathbb{R}^n$, and $u \in \mathbb{R}$,

$$\begin{aligned} |\bar{f}_i(t,\zeta,u)| &\leq \vartheta(1 + t^{\alpha_1})(|\zeta_{i+2}| + \cdots + |\zeta_n| + |u|), \\ |\bar{g}_i(t,\zeta,u)| &\leq \vartheta(1 + t^{\alpha_2})(|\zeta_{i+2}| + \cdots + |\zeta_n| + |u|), \\ |\bar{g}_n(t,\zeta,u)| &\leq \vartheta(1 + t^{\alpha_2})|u|. \end{aligned}$$

Assumption 2. The unknown control coefficients d_i $(i = 1,\ldots,n)$ have known signs and satisfy $\underline{d}_i \leq |d_i| \leq \bar{d}_i$, where $\underline{d}_i, \bar{d}_i > 0$ are known constants.

Assumption 3. The output function $\delta(\cdot)$ is Lipschitz continuous.

[1] Throughout the paper, the local Lipschitz condition is defined as that proposed in Zhao and Deng (2016), i.e., consider a Borel measurable function $h : [t_0, +\infty) \times \mathbb{R}^n \to \mathbb{R}^n$ and for each $n = 1, 2, \ldots,$ there exists a continuous function $c_n(t) \geq 0$ such that for all $t \geq t_0$, $|x_1| \leq n$ and $|x_2| \leq n$, we have $|h(t,x_1) - h(t,x_2)| \leq c_n(t)|x_1 - x_2|$.