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Appointed-time consensus: Accurate and practical designs☆

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This paper investigates the appointed-time consensus problem under directed and periodical switching topologies. Here, the appointed finite time means that the consensus arriving time can be off-line pre-assigned according to task requirements without conservative estimation and is independent of the initial conditions of agents' states. From a motion-planning perspective, a novel distributed appointed-time algorithm is developed for a multi-agent system with double-integrator dynamics. Under the assumption that the union of the directed periodical switching topologies contains a directed spanning tree, the proposed distributed algorithms can guarantee the multi-agent system to achieve consensus at an appointed time. Furthermore, the robustness and the practicability of the proposed algorithms are extended. It is the first time here to solve the finite- and appointed-time consensus problem for double-integrator systems under directed switching topologies.

1. Introduction

Over the past two decades, consensus as a most fundamental research topic for multi-agent coordination has been extensively investigated and applied (Bechlioulis & Rovithakis, 2017; Chen, Lewis, & Xie, 2011; Cortés, 2006; Hong, Chen, & Bushnell, 2008; Hui, Haddad, & Bhat, 2009; Li, Du, & Lin, 2011; Liu, Zhao, & Chen, 2016, 2017; Macellari, Karayiannidis, & Dimarogonas, 2016; Shim & Trenn, 2015; Wang & Hong, 2008; Zhao, Duan, Wen, & Chen, 2016; Zhao, Liu, Li, & Duan, 2017; Zhao, Liu, Wen, & Chen, 2017). Some interesting and meaningful results were derived to improve the properties for consensus problems. In Bechlioulis and Rovithakis (2017), Macellari et al. (2016) and Shim and Trenn (2015), the bounded error consensus problems of heterogeneous agents with prescribed transient behavior were investigated. The optimal consensus tracking problems were studied in Zhao, Liu, Li et al. (2017) and Zhao, Liu, Wen et al. (2017).

As an effective way to improve the rate of convergence for consensus problems, the finite-time consensus problem was firstly studied in Cortés (2006). Since then, a variety of finite-time consensus algorithms were proposed to solve the finite-time consensus problem in different scenarios (see Chen et al., 2011, Hong et al., 2008, Hui et al., 2009, Li et al., 2011, Liu et al., 2016, 2017, Wang and Hong, 2008, Zhao et al., 2016, and references therein). In Chen et al. (2011) and Hui et al. (2009), the finite-time average consensus problem was investigated for multiple single-integrator systems. Further, finite-time consensus algorithms for multiple double-integrator systems were developed in Hong et al. (2008), Li et al. (2011), Wang and Hong (2008) and Zhao et al. (2016). Also, the finite-time consensus problem for multiple non-identical second-order nonlinear systems was studied in Zhao et al. (2016) with the settling time estimation. However, the settling time functions in Zhao et al. (2016) depended on the initial states of the agents, which prohibited their practical applications if the initial conditions were unavailable in advance. In Liu et al. (2016), finite-time formation control was studied for nonlinear systems.

Recently, in Zuo and Tie (2012) a novel class of nonlinear consensus algorithms, called fixed-time consensus, are presented assuming uniform boundedness of the settling time regardless of the initial conditions. For double-integrator systems, decentralized fixed-time consensus was studied in Zuo (2015). Further, in Fu and Wang (2016), a distributed algorithm was designed under an undirected topology, which depended only on the relative measurements of the neighboring agents. Moreover, in Fu and Wang (2016), Zuo (2015) and Zuo and Tie (2012), the upper bound of the settling time was estimated based on the conservation, which means that the estimated upper bound of the settling time was

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always larger than the real settling time. It is also worth noting that most of the above-mentioned finite- and fixed-time consensus works were derived under undirected (Fu & Wang, 2016; Zuo, 2015; Zuo & Tie, 2012) and fixed topologies (Liu & Zhao, 2016). In practical applications, periodic processes widely exist in nature and engineering (Nayfeh & Mook, 1979). In some cases, the communication among agents exhibits periodic phenomena, which implies that the topology among agents is periodically changing. Therefore, compared with the undirected and fixed topologies, it is meaningful to study the appointed-time consensus problem under periodically time-varying and directed topologies.

Motivated by the above observations, from a motion-planning perspective, this paper investigates the appointed-time consensus problem of double-integrator systems under directed switching topologies. The main results of this paper extend the existing works in three aspects. Firstly, from a motion-planning perspective, a novel framework is formulated to solve the finite-time consensus problems. Compared with the existing results in Zhao et al. (2016) and Zuo (2015), where the settling time could only be estimated, in this paper, with the proposed appointed-time consensus algorithms, the settling time can be off-line pre-assigned according to task requirements. Furthermore, unlike the conservative estimation in Fu and Wang (2016), Zuo (2015) and Zuo and Tie (2012), here the settling time can be appointed in advance without estimation or conservatism. Secondly, the existing results on finite-time consensus issues were solved under undirected (Fu & Wang, 2016; Zuo & Tie, 2012) or fixed topologies (Liu & Zhao, 2016). Compared with Fu and Wang (2016), Liu and Zhao (2016) and Zuo and Tie (2012), the topology here is greatly extended. It is assumed that the topology is directed and periodically switching, the union of which contains a directed spanning tree. To the best of the authors' knowledge, it is the first time to solve finite- and appointed-time consensus problems under directed and periodical switching topologies for double-integrator systems. Thirdly, the first two algorithms designed in this paper are based only on sampling measurements of the relative states among each agent's neighbors, which greatly reduces the cost of the network integration (Yu, Zheng, Chen, Ren, & Cao, 2011).

2. Preliminaries

For a multi-agent system with N agents, a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to model the interaction among these agents, where $\mathcal{V} = \{1, 2, \ldots, N\}$ is the node set and $\mathcal{E} \subset \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\}$ is the edge set. An edge (v_i, v_j) is an ordered pair of vertices in $\mathcal{V}$, which means that agent j can receive information from agent i. If there is an edge from i to j, i is defined as the parent node and j is defined as the child node. A directed tree is a directed graph, where every node, except for the root, has exactly one parent. A directed spanning tree of a directed graph is a directed tree formed by edges that connect all the nodes of the graph. We say that a graph has a directed spanning tree if a subset of the edges forms a directed spanning tree. The interaction topology may be dynamically changing. Therefore, let $\overline{\mathcal{G}} = \{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_s\}$ denote the set of all possible directed graphs defined for the N agents. In applications, the possible interaction topologies will likely be a subset of $\overline{\mathcal{G}}$. Obviously, $\overline{\mathcal{G}}$ has finite elements. The union of a group of directed graphs $\{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_m\} \subset \overline{\mathcal{G}}$ is a directed graph with nodes given by $\mathcal{V}$ and edge set given by the union of the edge sets of $\mathcal{G}_i$, $i = 1, \ldots, m$. The adjacency matrix A associated with $\mathcal{G}$ is defined such that $a_{ij} = 1$ if there is an edge from j to i, and $a_{ij} = 0$ otherwise. The Laplacian matrix of the graph associated with the adjacency matrix A is given as $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$, where $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. For $t_k \leqslant t \leqslant t_{k+1}$, the neighbors of node i are denoted by $\mathcal{N}_i(t_k) = \{j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\}$, and $|\mathcal{N}_i(t_k)|$ is the cardinality of $\mathcal{N}_i(t_k)$. Given a matrix $M = [m_{ij}] \in \mathbb{R}^{N\times N}$, it is said that M is nonnegative if all its elements m_{ij} are nonnegative, and M is positive if all its elements m_{ij} are positive. Further, if a nonnegative matrix $M \in \mathbb{R}^{N\times N}$ satisfies $M\mathbf{1} = \mathbf{1}$, where $\mathbf{1}$ represents $[1, 1, \ldots, 1]^T$ with an appropriate dimension, then it is said to be stochastic (Horn & Johnson, 1985).

3. Appointed-time consensus under directed switching topologies

Consider the multi-agent system with N agents labeled as $1, 2, \ldots, N$. The dynamics of each agent is described by

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ are, respectively, the position and velocity of agent i, and $u_i(t) \in \mathbb{R}^n$ the control input.

Definition 1 (*Appointed-time Consensus*). For multi-agent systems (1), the appointed-time consensus problem is said to be solved if and only if, for an appointed settling time $T_s > 0$, for any initial conditions, the positions and velocities of multi-agent systems (1) satisfy $\lim_{t\to T_s} \|x_i(t) - x_j(t)\| = 0$, $\lim_{t\to T_s} \|v_i(t) - v_j(t)\| = 0$, $\forall i, j \in \mathcal{V}$, and $x_i(t) = x_j(t)$, $v_i(t) = v_j(t)$, when $t \geq T_s$.

Assumption 1. For a time series $\{t_k\}$ with $t_0 = 0$, there exists a corresponding directed topologies set $\overline{\mathcal{G}} = \{\mathcal{G}_0, \mathcal{G}_1, \ldots, \mathcal{G}_{m-1}\}$. The topology among agents is periodically time-varying with the period m, (i.e. $\mathcal{G}_{k+m} = \mathcal{G}_k$, $k = 0, 1, \ldots$, and the topologies only exist at the time instant) such that the union of the directed interaction graphs at discrete times $\{t_k, t_{k+1}, \ldots, t_{k+m-1}\}$ has a directed spanning tree.

To achieve appointed-time consensus, a motion-planning approach is used to design the control algorithm. First, consider a cost function $J_{i,k} = \frac{1}{2}\int_{t_k}^{t_{k+1}} u_i^T(t) R_i u_i(t) dt$ and the related Hamiltonian function $H_{i,k}(t) = \frac{1}{2} u_i^T(t) R_i u_i(t) + (p_{x_i}^T(t) v_i(t) + p_{v_i}^T(t) u_i(t))$, with the predicted state in the $k+1$th step

$$\begin{aligned} x_i(t_{k+1}) &= \frac{1}{|\mathcal{N}_i(t_k)|+1}\left[\sum_{j\in\mathcal{N}_i(t_k)} x_j(t_k) + x_i(t_k)\right] \\ &\quad + \frac{t_{k+1}-t_k}{|\mathcal{N}_i(t_k)|+1}\left[\sum_{j\in\mathcal{N}_i(t_k)} v_j(t_k) + v_i(t_k)\right], \\ v_i(t_{k+1}) &= \frac{1}{|\mathcal{N}_i(t_k)|+1}\left[\sum_{j\in\mathcal{N}_i(t_k)} v_j(t_k) + v_i(t_k)\right], \\ &\quad i = 1, 2, \ldots, N, \end{aligned} \tag{2}$$

where both $p_{x_i}(t) \in \mathbb{R}^n$ and $p_{v_i}(t) \in \mathbb{R}^n$ represent the covariant. Then, it follows from Pontryagin's principle (Bryson & Ho, 1975) that the necessary condition of optimality can be given by (1) and

$$\dot{p}_{x_i}(t) = -\frac{\partial H_{i,k}}{\partial x_i(t)} = 0, \tag{3}$$

$$\dot{p}_{v_i}(t) = -\frac{\partial H_{i,k}}{\partial v_i(t)} = -p_{x_i}(t). \tag{4}$$

Then, consider the extremal condition $\frac{\partial H_{i,k}}{\partial u_i(t)} = R_i u_i(t) + p_{v_i}(t) = 0$. One has that the optimal motion-planning controller for systems (1) is $u_i(t) = -R_i^{-1} p_{v_i}(t)$. Since $p_{x_i}(t)$ is a constant. One has $p_{x_i}(t) = c_{i,k}$, with an unknown constant $c_{i,k}$ in $t \in [t_k, t_{k+1})$. Integrating (4) from t_k to t, one obtains $p_{v_i}(t) = -c_{i,k}(t - t_k) + p_{v_i}(t_k)$. Then, one gets

$$u_i(t) = R_i^{-1} c_{i,k}(t - t_k) - R_i^{-1} p_{v_i}(t_k). \tag{5}$$

Substitute (5) into (1) and integrate it from t_k to t, one gets $v_i(t) = -R_i^{-1} c_{i,k} \frac{(t-t_k)^2}{2} - R_i^{-1} p_{v_i}(t_k)(t - t_k) + v_i(t_k)$. According to (1), one has