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Brief paper

Dissipative approach to sliding mode observers design for uncertain mechanical systems☆

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ABSTRACT

A class of nonlinear uncertain mechanical systems with the Coriolis term, is considered. Since these systems generally do not satisfy the bounded-input-bounded-state property, a global sliding-mode observer with theoretically exact finite-time convergence using dissipative properties, is proposed.

1. Introduction

The control of mechanical systems requires information of both variables: position and velocity. Since usually only the position is measured, it is necessary to estimate the velocity by means of an observer. When the model of the system is nonlinear, the parameters and the inputs of the system are well known there is an extensive literature providing global and asymptotically converging velocity estimation, see e.g. Astolfi, Ortega, and Venkatraman (2010), Besançon (2000, 2007) and Gauthier, Hammouri, and Othman (1992). However, in the presence of uncertainties/perturbations (U/P) (e.g. dry friction, unknown torque, etc.) the challenge of estimating globally, exactly the value of the velocity becomes more difficult even more if finite-time convergence is required. If the perturbation in the system is *arbitrary*, the unknown input observer theory (Hautus, 1983; Rocha-Cózatl & Moreno, 2004, 2011) requires relative degree one of the measured output with respect to (w.r.t.) the perturbation, but mechanical systems with U/P have relative degree two w.r.t. the measured position. To allow the estimation of the velocity in this paper we assume that the uncertainties/perturbations are *bounded*.

For this purpose a discontinuous estimation algorithm is required, such as sliding-mode observers (Barbot, Boukhobza, & Djemai, 2003; Edwards, Spurgeon, & Tan, 2002; Spurgeon, 2008). One of their advantages is that they provide theoretically exact convergence to the true system's states, even in the presence of bounded perturbations and under the condition that the nonlinear system has a Bounded-Input-Bounded-State (BIBS) property w.r.t. the perturbations. Moreover, HOSM observers (Barbot & Floquet, 2010; Bejarano, Pisano, & Usai, 2011; Efimov, Zolghadri, Cieslak, & Henry, 2012; Fridman, Shtessel, Edwards, & Yan, 2008; Pisano & Usai, 2011) ensure this convergence in finite time.

In particular, for nonlinear mechanical systems with bounded U/P the sliding-mode observers/differentiators (Davila, Fridman, & Levant, 2005; Levant, 1998; Moreno, 2009; Xian, de Queiroz, Dawson, & McIntyre, 2004) require the system to be BIBS. To overcome this restriction Apaza-Perez, Fridman, and Moreno (2017) propose a strategy connecting two observers in cascade: (i) A Luenberger observer ensuring that the estimation error converges to a neighborhood of zero; (ii) A higher-order sliding mode differentiator guarantees global finite-time theoretically exact convergence to zero of estimation error. However, this design strategy grows twice the order of observer, and requires restrictive conditions for gains design. These restrictive conditions were partially overcoming in Apaza-Perez, Moreno, and Fridman (2016).

For observation of nonlinear systems that do not necessarily have the BIBS property, a dissipative approach (Moreno, 2004, 2005) for systems with known inputs, and in Rocha-Cózatl and Moreno (2004, 2011) with the presence of U/P, results to be efficient. This technique contains as particular cases well-known

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observer design methods, as e.g. Lipschitz (Rajamani, 1998) and high gain observers (Gauthier et al., 1992). For systems with U/P satisfying the conditions for existence of an observer, among them the relative degree one condition, the dissipative observer is able to estimate globally and exponentially the true states (Rocha-Cózatl & Moreno, 2004, 2011). But, when the relative degree condition is not met but the U/P are bounded the dissipative observer assures the convergence to a neighborhood of the origin of the estimation error (Moreno, 2005).

Summarizing, one can conclude that observer design for mechanical systems with U/P has the following difficulties: (i) relative degree two w.r.t. U/P; (ii) the system could be not BIBS, i.e. the sliding-mode differentiators cannot be used directly; (iii) the Coriolis term depending quadratically on the velocity; (iv) there are uncertainties on the parameters of model, e.g. dry friction, hysteresis, etc.; (v) the system can be affected by external perturbations.

Main contribution. We consider one-degree-of-freedom mechanical systems with Coriolis term, dry friction, bounded uncertainties/perturbations and other nonlinearities. These systems do not require to have the BIBS property. For this class of systems, a global sliding-mode observer estimating the velocity theoretically exactly in finite time, is proposed.

The rest of the paper is organized as follows. Section 2 contains a motivating example of the system class for which sliding-mode differentiators cannot ensure finite-time estimation. The problem statement is presented in Section 3. Section 4 presents a state transformation, to deal with the Coriolis term, and the proposed observer. The main results are presented in Section 5. Section 6 illustrates the main results with computer simulations. Section 7 provides some conclusions. All proofs are located in Appendix.

Notations. Throughout this paper we avail of the following notations: $\lceil\cdot\rfloor^p := |\cdot|^p sign(\cdot)$; $\lambda_M(D)$ and $\lambda_m(D)$ are the largest and the smallest eigenvalue of a square matrix D.

2. Motivation example

The following Lagrangian system was considered by Besançon (2000)

$$(1+\cos^2(q))\ddot{q} - \frac{1}{2}\sin(2q)\dot{q}^2 + g\sin(q) = \tau,$$

where $q \in \mathbb{R}$ is the position, $(1+\cos^2(q))$ is the inertial term, $-\frac{1}{2}\sin(2q)\dot{q}^2$ is the Coriolis force. Consider a more general system adding a continuous nonlinear term $-\frac{\sin^2(q)+1}{3}\dot{q}$, a discontinuous term (e.g. dry friction) $0.5\,\mathrm{sign}(\dot{q})$ and a bounded perturbation $\tilde{\delta}(t)$ in the form:

$$(1+\cos^2(q))\ddot{q} - \frac{1}{2}\sin(2q)\dot{q}^2 + \\ + g\sin(q) - \frac{\sin^2(q)+1}{3}\dot{q} + 0.5\,\mathrm{sign}(\dot{q}) = \tau + \tilde{\delta}(t). \quad (1)$$

This system has relative degree two w.r.t. the measured output q and the perturbation $\tilde{\delta}(t)$.

Let us apply the generalized super-twisting (GST) algorithm (Moreno, 2011) as an observer to estimate the unmeasured variable in finite time

$$\dot{z}_1 = -6.7\lceil e_1\rfloor^{1/2} - 3.4\lceil e_1\rfloor + z_2, \quad (2)$$
$$\dot{z}_2 = -20\lceil e_1\rfloor^{0} - 33\lceil e_1\rfloor^{1/2} - 11\lceil e_1\rfloor,$$

where $e_1 = z_1 - q$ and the gains are obtained according to its methodology.

For the simulations, consider the perturbation in the form

$$\tilde{\delta}(t) = 0.4\sin(3t)\cos(4t^3) + 0.5\cos(\pi t) + 0.6, \quad (3a)$$

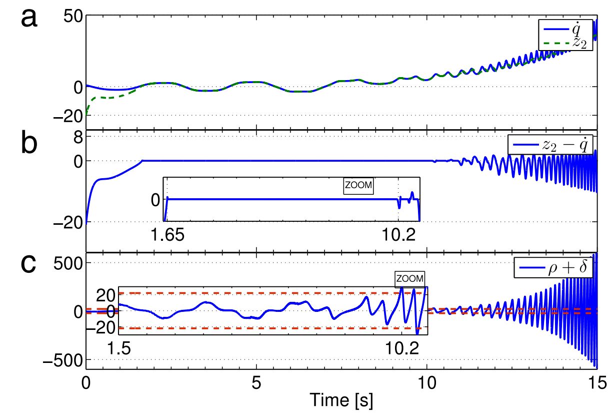

Fig. 1. (a) The estimation state z_2 of differentiator (2) and the true state $\dot{q}$ of (1). (b) The estimation error $z_2 - \dot{q}$. (c) Nonlinearity and the perturbation $\rho + \tilde{\delta}$ overcome the gain 20.

$\tau = 0$ and the initial conditions as

$$(q(0), \dot{q}(0)) = (1, 1), \quad (z_1(0), z_2(0)) = (-20, -20), \quad (3b)$$

Fig. 1:(a) illustrates that trajectories of system (1) with initial condition (3b) are not bounded. Fig. 1:(a)–(b) illustrate that the differentiator state (2) converges at $t = 1.65$ [s] to the true state $\dot{q}$, but after $t = 10.2$ [s] the differentiator (2) loses convergence. This is because at this time the nonlinearity

$$\rho(q,\dot{q}) = \frac{\sin^2(q)+1}{3(1+\cos^2(q))}\dot{q} + \frac{\sin(2q)}{2(1+\cos^2(q))}\dot{q}^2 \\ - \frac{9.8\sin(q) + 0.5\,sign(\dot{q})}{1+\cos^2(q)}$$

with the unknown input $\tilde{\delta}(t)$ exceeds the value 20 corresponding to the gain of the discontinuous term in the differentiator (2), see Fig. 1:(c).

From this example one can conclude that for mechanical systems with U/P and without BIBS property, the observer convergence is lost even if the GST algorithm based differentiator is applied. Hence, it is necessary to design an observer for systems not possessing BIBS property.

3. Problem statement

Consider one-degree-of-freedom mechanical systems with uncertainties/perturbations given as

$$m(q)\ddot{q} + c(q)\dot{q}^2 + H(q,\dot{q}) + \eta\cdot\mathrm{sign}(\dot{q}) + g(q) = \tau + \tilde{\delta}(t,q,\dot{q}) \quad (4)$$

where $q \in \mathbb{R}$ is the (measured) generalized position, $\dot{q}$ is the generalized velocity; $m(q)$ is the inertia term; $c(q)\dot{q}^2$ is Coriolis and centrifugal force; $H(q,\dot{q})$ is a continuous nonlinearity (e.g. continuous frictions, air resistance, etc.); $\eta \in \mathbb{R}$ and $\eta \cdot sign(\dot{q})$ is the dry friction, which possibly contains relay terms depending on $\dot{q}$, $g(q)$ denotes gravitational forces; $\tilde{\delta}(t,q,\dot{q})$ contains U/P and τ is the measured torque.

Suppose that the family of one-degree-of-freedom mechanical systems with uncertainties/perturbations represented by (4) satisfies the following assumptions:

A1. The inertia term $m(q)$ satisfies

$$\exists\, a_1, a_2 > 0\,;\, \forall q,\;\; a_1 \le m(q) \le a_2, \quad (5)$$
$$\dot{m}(q) = 2c(q)\dot{q}. \quad (6)$$