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Stochastic Adaptive Control - Integrating Research and Teaching

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Abstract: This paper focuses on innovative methods of teaching stochastic adaptive control with students who represent all science, technology, engineering and mathematics (STEM) disciplines. The Stochastic Adaptive Control course has been developed based on the authors' research area and it demonstrates the power, beauty and excitement of stochastic adaptive control as a field that spans STEM. Teaching is shown as a stochastic process that changes in time. The course has been taught at the mathematics department for the last 20 years by the team of mathematics and engineering faculty. The course is very popular, attracts junior and senior undergraduate and graduate students, and leads towards honors theses for undergraduates, and masters and doctoral theses for graduate students. Best practices leading to the success are presented.

Keywords: stochastic systems, adaptive control, STEM education.

1. INTRODUCTION

The authors of the paper have an established record of research collaboration in the area of stochastic systems and control, as well as estimation and stochastic adaptive control. Together with some other international collaborators the authors have been working on solving stochastic control problems in which a system is described by either stochastic differential equations or stochastic partial differential equations with a noise modeled by an ordinary Brownian motion or by a fractional Brownian motion. The theory of stochastic adaptive control generates tremendously many attractive problems for mathematics, engineering and computer science as well as economics and business students. There are many open mathematical problems such as the existence of solutions of stochastic differential and partial differential equation and there are many challenging numerical and computational problems such as approximate solutions or convergence and rate of convergence of estimators of unknown parameters as well as estimates of the so called a Hurst parameter that characterizes a fractional Brownian motion. The Math 750 course on stochastic adaptive control was developed and designed for motivated students from all science, technology, engineering, and mathematics (STEM) disciplines with the purpose to increase in a friendly and welcoming way the general awareness of the importance of control and control technology and its cross-disciplinary nature. Stochastic adaptive control has been recognized naturally by students as a field that spans science, technology, engineering and mathematics (STEM), therefore it has attracted strongly STEM students. The course became quickly popular as a powerful resource for ideas, methods, techniques, and topics which when carried on beyond the course lead initially to research projects, and then with further developments and advances to honors undergraduate, masters and doctoral theses and dissertations. The course has been offered every year and has been adapted to current advances in stochastic adaptive control following closely the authors research developments in the area.

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2. DESCRIPTION OF THE MATH 750–STOCHASTIC ADAPTIVE CONTROL COURSE

Stochastic adaptive control theory is concerned with recursive estimation of unknown parameters and control for systems with uncertainties modeled as random variables or random processes. The stochastic adaptive control problem is understood as identification and control of unknown stochastic systems. The solution to the stochastic adaptive control problem consists of the strong consistency of a family of estimators for an identification problem and of self optimality of an adaptive control that uses the family of estimates as the true parameters for a control problem. The theory is motivated by applications in such diverse areas as aerospace guidance and control, signal processing and communications, manufacturing processes, and financial economics. The mathematical theory of identification, control and stochastic adaptive control for models based on stochastic difference equations such as autoregressive processes and stochastic differential equations as Markov diffusion processes have been developed and are presented.

The course main topics:

(1) Conditional Expectation and their Properties.

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(2) Introduction to Autoregressive and ARMAX models.
(3) Linear Systems and their Properties.
(4) Linear Filtering Theory, Kalman Filter.
(5) Introduction to the Optimal Control Theory. Feedback Control.
(6) Martingales and Limit Theorems of Probability
(7) Estimation Methods: Least Squares and Maximum Likelihood Method.
(8) Introduction to Stochastic Processes: Markov Chains and Brownian Motion as well as fractional Brownian motions.
(9) Introduction to System Identification. Identification of Markov Chains and Identification of Linear Systems.
(10) Adaptive Control of Markov Chains and Adaptive Control of Linear Systems.
(11) Applications of Stochastic Adaptive Control to finance, economics models, telecommunication networks, actuarial sciences, biomedical sciences, and all engineering areas.

Recommended textbooks include:

(1) *A First Course in Stochastic Processes*, S. Karlin, H.M. Taylor
(2) *Introduction to Stochastic Processes*, S. Ross
(3) *Discrete Time Stochastic Systems*, T. Sodestrom
(4) *Mathematical Theory of Statistics*, Hogg/Tanis or Hogg/Craig or any other book
(5) *Identification and Stochastic Adaptive Control*, H.F. Chen, L. Guo
(6) *Stochastic Systems, Estimation, Identification and Adaptive Control*, P.R. Kumar, Pravin Varaiya, Prentice-Hall, 1986; new edition: SIAM, 2016
(7) *Stochastic Modeling and Control*, M.H.A. Davis, R.B. Vinter
(8) *On Adaptive Control*, (O Sterowaniu Adaptacyjnym) Research Monograph, Habilitation Doctorate Dissertation, (in Polish) B. Pasik, 1986
(9) *Linear Systems*, P. Antsaklis, A. Michel, McGraw Hill, 1997
(10) *Introduction to Mathematical Systems Theory, Linear Systems, Identification and Control*, Christian Heij, Andre Ran, Freek van Schagen, Birkhauser Verlag, 2007
(11) *Feedback Systems, An Introduction for Scientists and Engineers*, K. J. Astrom and R. M. Murray, 2008

The course has been developed by Bozenna Pasik-Duncan based on the author's habilitation doctorate dissertation written in Polish, and translated into English for the purpose of being used as the lecture notes for the course. The lecture notes have been available to students and have been adapted each year to the current advances in stochastic adaptive control and technology. The course has been taught for the last 25 years by Bozenna Pasik-Duncan with collaboration of Tyrone Duncan. The course requires a very good mathematical understanding in real analysis, probability and statistics as well as good computational skills. The students diversity differs from year to year therefore the way how the course is taught, its content, and illustrative examples of applications differ too.

3. CLASSROOM TEACHING AS A STOCHASTIC ADAPTIVE CONTROL PROBLEM

The classroom with students and their instructor is considered as a controlled system. It is stochastic because there is a lot of randomness in the classroom. Students talk, fall asleep or bring a baby to the classroom, or there is a fire alarm suddenly on during the exam and we must leave the building. In systems theory, we analyze every system carefully. We analyze the existence of a solution and computational aspects of it, we simulate stochastic equations, we collect information, we compose results, etc. We do the same in our teaching. A classroom becomes a scientific laboratory. We collect information. As we introduce ourselves as the course instructors, we ask students to introduce themselves too. The students in the class are unknown to the instructor at the beginning of a semester. They come from different departments with different academic background. The only prerequisites for this course are: good knowledge of real analysis, probability and statistics, very good academic standing, and permission of the instructor. The challenge for the instructor is to learn about students by collecting relevant course information about them. Too many unknowns in the system make the system unknown so learning or identifying the system is a critical issue in the stochastic adaptive control.

4. SHORT BIO AS THE FIRST ASSIGNMENT

One of the best practices for learning about the students from the beginning of a semester is to ask them to prepare a well done short bio with information that will help us as instructors to find optimal adaptive strategies. The information should contain students family and academic background, math and science courses taken, any significant recognitions, motivation for taking this particular course, short and long term goals for studying and career, research and real world problem interests, hobbies and favorite things to do during free time. These short bios should be revisited by the instructor as a semester progresses, and students should have an opportunity to update them twice: during the middle and at the end of a semester. We as teachers need to know the students and their interests. This information is important when we design projects for them. There are many unknowns in this system, so we need to estimate (learn) them as in the theory, and at each instant a controller/teacher uses this new estimate in control strategies and adapts the system. In theory we called it adaptation. It is the same in teaching. We collect information, we build a portfolio, we analyze our reports and data after every class, and we also want to do better each time so that in the long run we will do as well as if we knew the system perfectly. In the theory of stochastic adaptive control, this property of adaptive control is called self-tuning. We call this method of teaching scholarship in teaching. We treat teaching as a stochastic process that changes over time, a process with several components such as vision, design, data collection, and data analysis. We integrate teaching and learning. As in the theory, the controller has to learn, so a teacher, as controller, has to learn too, and the system has to learn, meaning the students have to learn.