Automatica 87 (2018) 404–411

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stochastic feedback coupling synchronization of networked harmonic oscillators ☆

Jingyi Wang [a], Chen Xu [a], Michael Z.Q. Chen [b], Jianwen Feng [a], Guanrong Chen [c]

[a] College of Mathematics and Statistics, Shenzhen University, Shenzhen 518060, PR China
[b] School of Automation, Nanjing University of Science and Technology, Nanjing, Jiangsu 210094, PR China
[c] Department of Electronic Engineering, City University of Hong Kong, Kowloon, Hong Kong

ARTICLE INFO

Article history:
Received 3 November 2016
Received in revised form 14 May 2017
Accepted 29 August 2017

Keywords:
Almost sure exponential synchronization
Networked harmonic oscillator
Noisy sampled-data
Stochastic feedback

ABSTRACT

This paper presents a stochastic feedback coupling strategy for almost sure exponential synchronization of networked harmonic oscillators. Differing from most of existing publications, in which noise is not beneficial to synchronization in complex dynamical networks, a stochastic coupling strategy is designed here by using discrete-time noisy sampled-data to achieve the almost sure exponential synchronization of networked harmonic oscillators. Both leaderless synchronization and leader-following synchronization of networked harmonic oscillators are considered. Some sufficient conditions are established to reach two types of synchronization by employing algebraic graph theory, Itô stochastic integral, stochastic control technique and mathematical analysis. Two numerical simulations are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

In the past decade, various synchronization problems in complex dynamical networks have attracted growing attention and have been widely investigated in different fields due to their board applications, including power systems, automatic control, wireless sensor networks, flight formation and electrical networks, see Boccaletti, Kurths, Osipov, Valladares, and Zhou (2002), Dörfler and Bullo (2014), Olfati-Saber, Fax, and Murray (2007), Song, Liu, Wen, Cao, and Tang (2016), Su, Chen, Lam, and Lin (2013), Tanner, Jadbabaie, and Pappas (2007), Tuna (2017), Wang and Chen (2002), Wang, Chen, and Wang (2015), Wang, Su, Wang, and Chen (2017) and Wang, Xu, et al. (2015). In a network of dynamical nodes, synchronization is the process in which all nodes converge to a common behavior driven by some prescribed coupling or/and control protocols.

Synchronization of networked harmonic oscillators is interesting and important in both theoretical research and practical applications. There are three reasons (Zhao, Zhou, & Wu, 2016): (1) it provides a fundamental mechanics model for the study of consensus problems of multi-agent systems and synchronization problems of complex dynamical networks; (2) it can be regarded as a canonical Hamiltonian system; (3) it plays a significant role in various applications of multi-agent systems involving coordinate behaviors, including motion coordination and consensus tracking (Ballard, Cao, & Ren, 2010; Casau, Sanfelice, Cunha, Cabecinhas, & Silvestre, 2015; Chiorescu et al., 2004; Li & Ding, 2015; Ren, 2008; Ren & Beard, 2005; Tuna, 2017; Wang et al., 2016; Zhang, Yang, & Zhao, 2013; Zhou, Zhang, Xiang, & Wu, 2012). Several different synchronization protocols have been proposed for networked harmonic oscillators from various perspectives. For instance, Ren (2008) first investigated the synchronization of networked harmonic oscillators with continuous-time interactions, while the problem with discrete-time interactions was considered by Ballard et al. (2010), with instantaneous/impulsive interactions by Sun, Lü, Chen, and Yu (2014) and Zhou et al. (2012). Moreover, Song et al. (2016) proposed two distributed synchronization protocols for networked harmonic oscillators by utilizing current and past relative position data, respectively. Su, Wang, and Lin (2009) studied synchronization with coupling between a pair of oscillators being effective only when they are close enough. Su, Chen, Wang, Wang, and Valeyev (2013) discussed cluster synchronization of networked harmonic oscillators with adaptive coupling. In addition, Zhang and Zhou (2012) investigated the synchronization of networked harmonic oscillators using sampled-data measurements subject to controller failures, and Zhou, Zhang, Xiang, and

☆ This work was supported in parts by the National Natural Science Foundation of China under Grants 61603260, 61472257, 61374053 and 61273220, and the Postdoctoral Science Foundation of China under Grant 2016M590811, and the Hong Kong Research Grants Council under the GRF Grant CityU 11208515. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Michael V. Basin under the direction of Editor Ian R. Petersen.

E-mail addresses: wangjingyi@szu.edu.cn (J. Wang), xuchen_szu@szu.edu.cn (C. Xu), mzqchen@outlook.com (M.Z.Q. Chen), fengjw@szu.edu.cn (J. Feng), eegchen@cityu.edu.hk (G. Chen).

https://doi.org/10.1016/j.automatica.2017.10.011
0005-1098/

Wu (2013) studied sampled-data subject to controller failure and communication delays. Furthermore, Sun, Yu, Lü, and Chen (2015) addressed synchronization of networked harmonic oscillators with random noise. Recently, Wang et al. (2016) considered synchronization of instantaneously networked harmonic oscillators using noisy sampled-data. Tuna (2017) studied the synchronization of identical pendulums coupled via dampers and springs, and applied it to the collective behavior of coupled oscillators in LTI passive electrical networks. To the best of our knowledge, some important issues in synchronization of networked harmonic oscillators still remain for further research, such as almost sure synchronization of networked harmonic oscillators based on noisy sampled-data, which will be investigated in the present paper.

Generally speaking, noise is not considered beneficial to achieve synchronization for networked harmonic oscillators in the existing literature, see Djaidja, Wu, and Cheng (2017), Sun et al. (2015) and Wang et al. (2016). However, it is well known that noise can be used to stabilize a given unstable system and that stochastic feedback control has been widely used in engineering to model systems with abrupt random changes in control parameters (Mao, 2007, 2016). Compared with the deterministic control, the biggest advantage of the stochastic state feedback control is that the system under the stochastic control achieves sample-path stabilization, while the expectation of the state is equal to the state of the original uncontrolled system (Mao, 2016). Hence, how to design a coupling protocol via stochastic feedback to achieve synchronization is interesting and deserves consideration. On the other hand, the oscillators need to measure the states of their neighbors in the current time and update themselves continuously (Chen, Zhang, Su, & Li, 2015; Ren, 2008; Song et al., 2016; Su et al., 2009; Sun et al., 2015). The continuous measurements and updates are hard to implement continuously, and it is more realistic to do so at discrete sampling instants (Huang, Duan, Wen, & Zhao, 2016). In terms of control cost, discrete-time sampling is cheaper than the continuous-time setting (Ballard et al., 2010; Chen, Zhang, Su, & Chen, 2016). In the past, the synchronization problems of networked harmonic oscillators were considered by different sampled-data schemes, such as stochastic sampling (Shen, Wang, & Liu, 2012), aperiodic sampling (Wen, Yu, Chen, Yu, & Chen, 2014; Wu, Park, Su, Song, & Chu, 2012; Zhou et al., 2013) and periodic sampling (Sun et al., 2014; Wen et al., 2014).

Motivated by the above observations, this paper is devoted to studying the synchronization problem of networked harmonic oscillators via stochastic feedback coupling using noisy sampled-data. The main objective is to investigate a model of networked harmonic oscillators for using noisy sampled-data, and analyze both leaderless and leader-following synchronization scenarios of the model. In practical applications, almost sure synchronization is usually the more desirable property because we can only observe a sample path of the system and the synchronization in mean square conditions can sometimes be too conservative to be practically useful. This is one of the main reasons to study the stochastic feedback coupling protocols. The main contributions of this paper are three-fold. First, we model the dynamical process of a class of harmonic oscillators that update their states via stochastic feedback control by using the neighbor's sampled-data which contain the relative position and velocity information. Second, we establish sufficient conditions for almost sure synchronization of the proposed model with or without a dynamic leader. Finally, we provide two numerical simulations to show the performance of the discrete-time sampling on synchronization of the model and illustrate the effectiveness of the proposed stochastic feedback coupling protocols.

The rest of this paper is organized as follows. In Section 2, we present some preliminaries and the general model formulation. In Section 3, we establish some synchronization criteria for the proposed model with or without a dynamic leader. We then illustrate the effectiveness of the theoretical results with two numerical examples in Section 4. Finally, we draw some conclusions in Section 5.

2. Preliminaries

2.1. Notations and lemmas

Let $\mathbb{R} = (-\infty, +\infty)$ be the set of real numbers, $\mathbb{N} = \{1, 2, 3, \dots\}$ be the set of natural numbers, $\mathbb{R}^n$ be the n-dimensional Euclidean space and $\mathbb{R}^{n\times n}$ be the set of all $n \times n$ real matrices. Let $\mathbf{1}_n = [1, 1, \dots, 1]^\top \in \mathbb{R}^n$, $O_n = [0] \in \mathbb{R}^{n\times n}$ and I_n be the n-dimensional identity matrix. For a vector $x = [x_1, x_2, \dots, x_n]^\top \in \mathbb{R}^n$, denote $|x|^p = [|x_1|^p, |x_2|^p, \dots, |x_n|^p]^\top$ for a positive constant $p \in \mathbb{R}$, $\|x\| = \max_{1\le j\le n}|x_j|$. For $X = [x_{ij}] \in \mathbb{R}^{n\times n}$, $|X| = [|x_{ij}|]$, $\|X\| = \max_{1\le i\le n}\sum_{j=1}^{n}|x_{ij}|$, and $\rho(X)$ is the spectral radius of X. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge0}$ which is right continuous and $\mathcal{F}_0$ contains all P-null sets. Let $\mathcal{M}^2([a, b]; \mathbb{R})$ denote the family of processes $\{f(t)\}_{a\le t\le b}$ in $\mathcal{L}^2([a, b]; \mathbb{R})$ such that $\mathrm{E}\{\int_a^b |f(t)|^2 dt\} < \infty$. Let $\mathcal{L}^2([a, b]; \mathbb{R})$ denote the family of $\mathbb{R}$-valued $\mathcal{F}_t$-adapted processes $\{f(t)\}_{a\le t\le b}$ such that $\int_a^b |f(t)|^2 dt < \infty$ a.s.

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph for a set of nodes $\mathcal{V} = \{1, 2, \dots, n\}$ and a set of edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and let $A = [a_{ij}] \in \mathbb{R}^{n\times n}$ be the adjacency matrix associated with $\mathcal{G}$ such that a_{ij} is a positive weight for $(i, j) \in \mathcal{E}$ (so that node i receives information from node j with j being the parent of i) and $a_{ij} = 0$ for $(i, j) \notin \mathcal{E}$. Define the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n\times n}$ associated with $\mathcal{G}$ by $l_{ii} = \sum_{i=1, j\ne i}^{n} a_{ij}$, where $l_{ij} = -a_{ij}$ for $i \ne j$. A directed graph is called a directed tree, if every node, except a node called root, has exactly one parent. A spanning tree of a directed graph is a directed tree formed by edges that connect all the nodes of the graph. To study synchronization problems of networked harmonic oscillators, the oscillator interactions are modeled by a graph.

The following lemmas will be required throughout this paper.

Lemma 1 (*Li, 2008; Ren & Beard, 2005*). *If $\mathcal{G}$ has a directed spanning tree with an associated Laplacian matrix L, then*

(1) *$\mathbf{1}_n = (1, 1, \dots, 1)^\top$ is a right eigenvector of L corresponding to the eigenvalue $\lambda_1 = 0$ with multiplicity 1 (i.e. $L\mathbf{1}_n = 0$) and all the other right eigenvalues $\lambda_2, \dots, \lambda_n$ have positive real parts;*
(2) *if $\xi = (\xi_1, \xi_2, \dots, \xi_n)^\top$ is a left eigenvector of L corresponding to the eigenvalue 0 (i.e. $\xi^\top L = 0$), then $\xi_i \ge 0$ for all $i = 1, 2, \dots, n$ and ξ has multiplicity 1 (in the following, always assume that $\sum_{i=1}^{n}\xi_i = 1$);*
(3) *there exists a nonsingular matrix P, where the first column of P is $\mathbf{1}_n$ and the first row of P^{-1} is $\xi^\top$, such that $L = PJP^{-1}$ is the Jordan decomposition of L, where $J = diag\{0, \hat{J}\}$ and $\hat{J}$ is the Jordan upper diagonal block matrix corresponding to the other distinct eigenvalues λ_r $(r = 2, \dots, n)$ of the matrix L.*

Lemma 2 (*Raw Absolute Moments, Winkelbauer, 2014*). *Let X be a normal random variable with mean $\mu = \mathrm{E}\{X\}$ and variance $\sigma^2 = \mathrm{E}\{X^2\} - \mu^2$, and denote $X \sim N(\mu, \sigma^2)$. Then, the raw absolute $p > 0$ moments are*

$$\mathrm{E}\{|X|^p\} = \sigma^p 2^{\frac{p}{2}} \Gamma\Big(\frac{p+1}{2}\Big)\Phi\Big(-\frac{p}{2}, \frac{1}{2}; -\frac{\mu^2}{2\sigma^2}\Big)/\sqrt{\pi},$$

where $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$, $\Phi(\alpha, \gamma; z) = \sum_{n=0}^{\infty} \frac{\alpha^{\bar{n}}}{\gamma^{\bar{n}}}\frac{z^n}{n!}$ and $z^{\bar{n}} = z(z+1)\cdots(z+n-1)$, $n \in \mathbb{N} \cup \{0\}$. Obviously, $z^{\bar{0}} = 1$, $z^{\bar{1}} = z$, and if $\mu = 0$ then $\mathrm{E}\{|X|^p\} = \sigma^p 2^{\frac{p}{2}} \Gamma(\frac{p+1}{2})/\sqrt{\pi}$. Moreover, if $X \sim N(\mu, \sigma^2)$ and $Y \sim N(-\mu, \sigma^2)$, then $\mathrm{E}\{|X|^p\} = \mathrm{E}\{|Y|^p\}$. In particular, if $0 < p < 1$ and $X \sim N(1, \sigma^2)$, then there exists a content σ_0 such that $\mathrm{E}\{|X|^p\} < 1$ for $\sigma^2 < \sigma_0^2$.

The Itô stochastic integrate, which was first defined by K. Itô in 1949, is the main tool to deal with the almost sure synchronization problems, and some lemmas about the stochastic integral with respect to Brownian motion will be given in the next.