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Passive fault tolerant perfect tracking with additive faults☆

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ABSTRACT

An architecture of a control system with additive faults is given, with a controller consisting of three blocks, and a problem of fault-tolerant (FT) perfect tracking is formulated. It is shown that there is a passive FT controller such that its performance is not worse than the performance of the nominal controller, while in presence of faults, it practically rejects the faults, on expense of a big magnitude of the control in times when faults appear. A numerical simulation is given.

1. Introduction

One of the most important control problems for systems with faults is to find a controller such that the behavior of the closed loop system is tolerant to faults. We call *nominal controller* the controller designed under the absence of faults. The main requirement for the FT controller is that the performance of this controller at least approximates the performance of the nominal controller in times when the faults are absent (see item (2) on page 1616 of Zhou & Ren, 2001). Indeed, since the faults happen rarely, we do not want to lose the nominal performance (the performance of the closed-loop system with applied nominal controller) in times when there are no faults. However, when faults appear, for safety reasons, we allow a worse than nominal performance (for example, more spent energy), while retaining some basic system properties (for example, stability).

The problem of FT control can be solved using active and passive controllers (see a comparison of passive versus active FT control, page 347 of Blanke, Kinnaert, Lunze, & Staroswiecki, 2016). Active controllers are reconfigurable ones, with the reconfiguration made on the basis of the information obtained from a fault detector/estimator. Passive controllers are not reconfigurable, they are designed under the criterion of a pre-defined performance and a pre-defined insensitivity to faults, of the closed loop system. Before we proceed with the literature review, we explain the used notation.

Remarks on the notation. The matrices are denoted by upper-case letters, and vectors and scalars are denoted by lower-case letters. All functions of s are real rational, will be bold-faced, and if not ambiguous, without the argument. The abbreviations RM and FT mean rational matrix and fault-tolerant. The poles and zeros (including poles and zeros at infinity) of a RM are defined through its McMillan form. If a RM is without poles in $\Re[s] \geq 0$ (the closed right complex half-plane), then we say that it is stable. By $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ we denote the transfer matrix $D + C(sI - A)^{-1}B$. By $\boldsymbol{T}_{yr}$ we denote the transfer matrix from r to y.

Drawbacks of the active FT controllers are that they are complex, that they require a certain time for fault detection/estimation before another controller is computed and applied, and that the variables (outputs and state) are not smooth in time (see Section 9.5 of Blanke et al., 2016).

There are different FT controller architectures in the literature, (Ding, Yang, Zhang, Ding, Jeinsch, Weinhold, & Schulalbers, 2010; Lan & Patton, 2016; Stoustrup & Niemann, 2001; Zhou & Ren, 2001) and Section 15 of Ding (2013). All of them are based on a residual generator included in the control loop. It is known that, even the passive controllers include (implicitly) a residual generator (see Fig. 6 in Ding et al., 2010).

Our controller will be a passive one. The performance will be perfect tracking with a minimal norm of the control. We shall use some ideas of the existing theory of tracking with disturbance rejection, in the construction of the controller. The existing works can be grouped in two global groups: The first one elaborates on the class of plants and inputs in which ideal tracking and ideal disturbance rejection is achievable, i.e. the tracking error tends to zero when $t \to \infty$, for all assumed inputs (Chen, Lin, & Liu, 2002; Stefanovski, 2007; Willems & Mareels, 2004, Chapter 13 of Saberi, Stoorvogel, & Sannuti, 2000, and Section VII of Wolovich, 1974).

The second group elaborates on the plants and/or inputs in which the ideal behavior is not achievable. The authors set some

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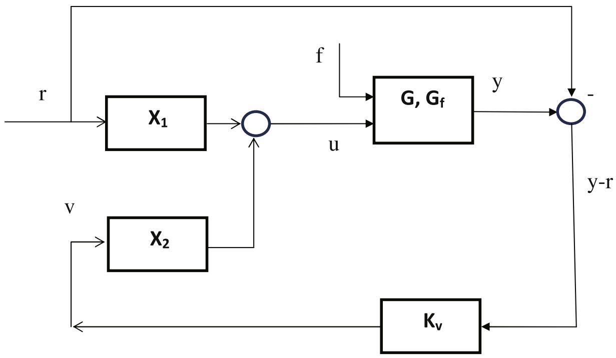

Fig. 1. Control system architecture.

analytic criteria (for a distance to the ideal performance), and minimize the criteria (Cirka, Fikar, & Mikle, 2005; Hoover, Longchamp, & Rosenthal, 2004; Leva & Bascetta, 2007; Tsai, Yang, & Chen, 2014; Wang, Guan, & Yuan, 2011; Xie, Xue, & Shiu Kit Tso, 2000).

The first group is closer to our work, therefore we give more details. There are at least three classes of plants and inputs of ideal tracking with disturbance rejection:

(1) The most restrictive one (in respect to the class of inputs, but the most general in respect to the class of plants) requires a strong condition, that $\bar{\boldsymbol{Q}}^{-1}\boldsymbol{r}$ is a proper stable rational vector, where $\boldsymbol{G} = \bar{\boldsymbol{Q}}^{-1}\bar{\boldsymbol{P}}$ is a left coprime factorization of the plant transfer matrix $\boldsymbol{G}$, and $\boldsymbol{r}$ is the Laplace transform of the assumed *shape-deterministic* reference signal input $r(t)$ (see Stefanovski, 2007, Xie et al., 2000 and references therein). An analogous condition holds for the ideal disturbance rejection.

(2) Another class pre-assumes a model of the inputs (see Theorem 2.4.1 in Saberi et al., 2000, the so called *internal model principle*). The tracking and disturbance rejection is ideal under a necessary and sufficient condition that is satisfied generically if the plant transfer matrix $\boldsymbol{G}$ is right-invertible. A drawback is that if the model changes in time, i.e. when different reference or disturbance inputs are applied in real time, or if the model of the plant is not precise, or changes in time, then the tracking with disturbance rejection can be unsatisfactory.

(3) The *perfect tracking* (see Chen et al., 2002 and references therein) is feasible under the right-invertibility of the plant transfer matrix $\boldsymbol{G}$ and that it has no zeros in $\Re[s] \geq 0$ and infinity. The *exact disturbance decoupling* is feasible under very restrictive conditions on the disturbance dynamics of the plant (see Theorems 13.2.1 and 13.2.2 of Saberi et al., 2000). Although the elaborated conditions on the plant are most restrictive, there are no constraints on the inputs.

In this paper we adopt a performance criterion that is most close to the latter one. We find a passive FT controller such that the output perfectly tracks the given reference input, and is practically insensitive to faults. When the faults have magnitudes much greater than the magnitudes of the reference signal and the initial values, the output is still insensitive to the faults, but on expense of a big magnitude of the control. Unlike the strong conditions in Theorem 13.2.1 of Saberi et al. (2000), we require only that the RM $\boldsymbol{G}_{\mathrm{f}}$ has no zeros on the extended imaginary axis. Finally, our FT controller meets the requirement of Zhou and Ren (2001), stated in the first paragraph of this paper.

2. Main result

Consider a linear time-invariant plant with the following inputs and output: $f(t)$ is m_{f}-dimensional input called fault, $u(t)$ is m-dimensional input called control, and $y(t)$ is p-dimensional output called measurement. Denote by $\boldsymbol{f}(s)$, $\boldsymbol{u}(s)$ and $\boldsymbol{y}(s)$ the Laplace transforms of $f(t)$, $u(t)$ and $y(t)$. We consider the system architecture given in Fig. 1, where the plant is given by the RMs $\boldsymbol{G}$ and $\boldsymbol{G}_{\mathrm{f}}$, and $\boldsymbol{X}_1$, $\boldsymbol{X}_2$ and $\boldsymbol{K}_{\mathrm{v}}$ are unknown RMs, which constitute the controller. We take that $\boldsymbol{K}_{\mathrm{v}}$ has m_{f} rows. Denote by $e = y - r$ the tracking error. Introducfe a controlled variable z by $z = \begin{bmatrix} e \\ \beta u \end{bmatrix}$, for some weighting design parameter $\beta > 0$. We pose the following problem.

Problem 1. Find a controller for the system given in Fig. 1 such that

(1) the system is stable,
(2) $\boldsymbol{T}_{\mathrm{yr}} = I_p$,
(3) $\|\boldsymbol{T}_{\mathrm{ur}}\|_\infty$ is minimal, and
(4) $\|\boldsymbol{T}_{\mathrm{zf}}\|_\infty$ is minimal.

By $\boldsymbol{X}_1$ we satisfy the requirements (2) and (3) of Problem 1, and by $\boldsymbol{X}_2$ and $\boldsymbol{K}_{\mathrm{v}}$ we satisfy the requirements (1) and (4).

Remark. The block-scheme in Fig. 1 coincides with the block-scheme in Fig. 15.6 of Ding (2013) (The feedforward controller $\boldsymbol{K}_1$ in Ding, 2013 coincides with $\boldsymbol{X}_1$, and feedback controller $\boldsymbol{K}_2$ coincides with $\boldsymbol{X}_2\boldsymbol{K}_{\mathrm{v}}$). The contribution of this paper in respect to Ding (2013) is a detailed design of the blocks to satisfy the requirements in Problem 1, with arbitrary small $\|\boldsymbol{T}_{\mathrm{ef}}\|_\infty$. By considering that $\boldsymbol{K}_{\mathrm{v}}$ has m_{f} rows, we pre-determine that $\boldsymbol{X}_2\boldsymbol{K}_{\mathrm{v}}$ is not a full rank RM, at least in the most frequent case that $m_{\mathrm{f}} < \min(m, p)$.

Consider the following system state-space model:

$$\begin{aligned} \dot{x} &= Ax + Bu + B_{\mathrm{f}}f\ , \quad x(0) = x_0 \in \mathbb{R}^n\ , \\ y &= Cx + Du + D_{\mathrm{f}}f\ . \end{aligned} \tag{1}$$

Then the plant transfer matrices $\boldsymbol{G}$ and $\boldsymbol{G}_{\mathrm{f}}$ are

$$[\boldsymbol{G}, \boldsymbol{G}_{\mathrm{f}}] = \left[\begin{array}{c|cc} A & B & B_{\mathrm{f}} \\ \hline C & D & D_{\mathrm{f}} \end{array}\right].$$

It is easy to see that the following assumption is necessary for Problem 1.

Assumption 1. (1) The pair (C, A) is detectable, the pair (A, B) is stabilizable, and (2) the RM $\boldsymbol{G}$ is right-invertible and has no zeros in $\Re[s] \geq 0$ and infinity.

We have

$$\boldsymbol{y} = \boldsymbol{G}\boldsymbol{u} + \boldsymbol{G}_{\mathrm{f}}\boldsymbol{f}\ , \tag{2}$$

$$\boldsymbol{u} = \boldsymbol{X}_1\boldsymbol{r} + \boldsymbol{X}_2\boldsymbol{v}\ , \tag{3}$$

$$\boldsymbol{v} = \boldsymbol{K}_{\mathrm{v}}(\boldsymbol{y} - \boldsymbol{r})\ . \tag{4}$$

Define the RM $\boldsymbol{M}$ by

$$\begin{aligned} \boldsymbol{M} &= [\boldsymbol{M}_1,\ \boldsymbol{M}_2] = [I_p,\ \boldsymbol{M}_2] \\ &= \left[\begin{array}{c|cc} A & B_{\mathrm{M1}} & B_{\mathrm{M2}} \\ \hline C & D_{\mathrm{M1}} & D_{\mathrm{M2}} \end{array}\right] = \left[\begin{array}{c|cc} A & 0 & B_{\mathrm{M2}} \\ \hline C & I_p & D_{\mathrm{M2}} \end{array}\right], \end{aligned}$$

where the matrices B_{M2} and D_{M2} are left arbitrary, temporarily. Let $\boldsymbol{X}$ be a proper stable RM that satisfies $\boldsymbol{G}\boldsymbol{X} = \boldsymbol{M}$. Introduce the following partitions of $\boldsymbol{X}$, compatible to that of $\boldsymbol{M}$: $\boldsymbol{X} = [\boldsymbol{X}_1,\ \boldsymbol{X}_2]$.

With respect to (2)–(4), we have $\boldsymbol{y} = \boldsymbol{T}_{\mathrm{yr}}\boldsymbol{r} + \boldsymbol{T}_{\mathrm{yf}}\boldsymbol{f}$, where

$$\boldsymbol{T}_{\mathrm{yr}} = (I - \boldsymbol{G}\boldsymbol{X}_2\boldsymbol{K}_{\mathrm{v}})^{-1}(\boldsymbol{G}\boldsymbol{X}_1 - \boldsymbol{G}\boldsymbol{X}_2\boldsymbol{K}_{\mathrm{v}})\ , \tag{5}$$

$$\boldsymbol{T}_{\mathrm{yf}} = (I - \boldsymbol{G}\boldsymbol{X}_2\boldsymbol{K}_{\mathrm{v}})^{-1}\boldsymbol{G}_{\mathrm{f}}\ ,$$

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