Automatica ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical Communique

A new geometric proof of super-twisting control with actuator saturation☆

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ARTICLE INFO

Article history:
Received 5 December 2016
Received in revised form 21 May 2017
Accepted 4 August 2017
Available online xxxx

Keywords:
Super-twisting control
Actuator saturation
Finite-time stability

ABSTRACT

In this note, the stability of an uncertain system with actuator saturation using super-twisting controller (STC) is analysed. First, a new proof of STC ensuring finite-time stability of the system is proposed using geometric method which gives a new gain conditions. Then, using the proposed proof the domain of attraction (DOA) is explicitly calculated for the system with bounded control.

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1. Introduction

Sliding mode control (SMC) is popularly used for stabilizing the uncertain dynamical systems by a discontinuous control (Utkin, 1977). However, the discontinuous control signal causes wear and tear of the actuator. In the early nineties, a continuous SMC, known as super-twisting control (STC), is proposed that also ensures a sliding mode in finite-time. This control structure is given by

$$u(t) = -K|s(t)|^{\frac{1}{2}}\operatorname{sign}(s(t)) - \int_0^t L\operatorname{sign}(s(\tau))\,\mathrm{d}\tau \tag{1}$$

that stabilizes an uncertain scalar dynamical system

$$\dot{s}(t) = a(t) + b(t)u(t) \tag{2}$$

in finite-time, where $a(t)$ and $b(t)$ are unknown but continuously differentiable scalar functions, and K and L are some positive constants. The control law (1) is studied widely in literature (e.g., Moreno & Osorio, 2012; Mu & Sun, 2015; Levant, 1993; Levant, 1998; Polyakov & Poznyak, 2009; Utkin, 2013; Utkin, 2016) due to its ability to reject the disturbance completely with continuous control signal. Similarly, the multi-input case is also reported in Nagesh and Edwards (2014). Despite of the continuous control, STC may result in a high amplitude oscillations of state trajectory in certain cases (Utkin, 2016). Nevertheless, STC is still considered as an alternative approach to design a continuous SMC. The solutions of the system are absolutely continuous functions that satisfy (2) almost everywhere, and are understood in the Filippov's sense on discontinuous manifold (Filippov, 1988).

In this paper, the stability of the system is analysed using STC with actuator saturation which is one of the major concerns in many practical applications. First, a new geometric proof is proposed to show the finite-time stability of STC which is different from the existing ones, e.g., see (Levant, 1993; Moreno & Osorio, 2012; Mu & Sun, 2015; Polyakov & Poznyak, 2009; Utkin, 2013). The main advantage of this proposed proof is that here no difficulty arises for the points on the line $s = 0$. The similar proof for super-twisting observer is presented recently in Kumar, Behera, and Bandyopadhyay (2017) but with a different gain conditions. Then, using the proposed proof the domain of attraction (DOA) is explicitly computed for the actuator saturation such that the system is finite-time stable within this DOA. It is to be noted that the stability of STC with saturating actuator is presented using Lyapunov method in Castillo, Steinberger, Fridman, Moreno, and Horn (2016). However, in this paper the stability of STC under actuator saturation is analysed using the proposed geometric proof with an aim of achieving the largest DOA.

The proposed proof follows the idea of constructing system trajectories in the original coordinate instead of in the phase plane. So, the difficulty incurred for the points on the line $s = 0$ is avoided. Then, using this technique DOA is computed for any given saturation limit.

2. Main results

First, we state some assumptions on the system (2) which hold throughout this paper.

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Maria Letizia Corradini under the direction of Editor André L. Tits.

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https://doi.org/10.1016/j.automatica.2017.09.015
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Assumption 1. The function a and its rate are bounded, i.e., $|a| < \bar{a}$ and $|\dot{a}| \leq A$. The function b is bounded and sign definite, i.e., $b \neq 0$, and without loss of generality, $0 < \underline{b} \leq b \leq \bar{b}$. Further, we also assume $|\dot{b}| \leq B$.

The STC given in (1) is rewritten as

$$u = -K|s|^{\frac{1}{2}}\text{sign}(s) + L_1 L_2 v \tag{3}$$

$$\dot{v} = -\text{sign}(s) \tag{4}$$

where $L = L_1 L_2$ for some positive constants L_1 and L_2. Here, the gains L_1 and L_2 allow the flexibility in the design of L as we shall see later. The closed loop system with the control law (3) and (4) is given as

$$\dot{s} = b\left(-K|s|^{\frac{1}{2}}\text{sign}(s) + L_1\mu\right) \tag{5}$$

$$\dot{\mu} = -L_2\text{sign}(s) + \frac{\gamma}{L_1} \tag{6}$$

where $\mu = L_2 v + \frac{1}{L_1}(\frac{a}{b})$ and $\gamma = \frac{\mathrm{d}}{\mathrm{d}t}(\frac{a}{b})$. It is easy to see that $|\gamma| \leq \Gamma^+$ where $\Gamma^+ = \frac{A\bar{b}+B\bar{a}}{\underline{b}^2}$. The classical notions of solution are not applicable since the system (5)–(6) is discontinuous for the points on $s = 0$. So, the differential equation on the discontinuous manifold is replaced by an inclusion which is nonempty, closed and bounded, convex and upper semi-continuous in its argument. Then, there exists an absolutely continuous function which satisfies the inclusion almost everywhere, and is regarded as a solution to the system in the Filippov's sense (Filippov, 1988).

2.1. Stability of super-twisting control

The following theorem gives the proof of STC without assuming any bound on control which is used later for calculation of DOA.

Theorem 1. *Consider the system (5) and (6). Then, the system is finite-time stable if*

$$K > 1.8\sqrt{\frac{L_1\left(L_2 + \frac{\Gamma^+}{L_1}\right)}{\underline{b}}} \quad \textit{and} \quad L_2 > \frac{\Gamma^+}{L_1} \tag{7}$$

where $L_1 > 0$.

Proof. The proof follows by the construction of geometrical trajectories in each quadrant separately in (s, μ) plane. Note that every solution of the system (5)–(6) satisfies Filippov's inclusion for all points on the line $s = 0$. The system trajectory leaves the line $s = 0$ whenever it crosses $s = 0$ for nonzero μ due to (5).

Define the curves $\Sigma_1 \equiv L_1\mu - K|s|^{\frac{1}{2}}\text{sign}(s) = 0$ and $\Sigma_2 \equiv L_1\mu + K|s|^{\frac{1}{2}}\text{sign}(s) = 0$ as shown in Fig. 1. Clearly, the curve $\Sigma_1 = 0$ divides the (s, μ) plane into two parts namely $\Sigma_1 > 0$ and $\Sigma_1 < 0$ such that any trajectory in $\Sigma_1 > 0$ crosses $\Sigma_1 = 0$ before entering $\Sigma_1 < 0$ and vice versa. Similarly for $\Sigma_2 = 0$.

We now proceed to find the trajectory of the system (5) and (6) as shown in Fig. 1. Consider the first quadrant, $s > 0$ and $\mu > 0$. Any trajectory starting in the region $\Sigma_1 > 0$ with initial condition $(0, \mu(0))$ is bounded by the line segment I due to $L_2 > \frac{\Gamma^+}{L_1}$. The equation of segment I is governed by

$$\dot{s} > 0 \quad \text{and} \quad \dot{\mu} = 0.$$

So, the line segment starting from the point $(0, \mu(0))$ hits $\Sigma_1 = 0$ at $(s(t_1), K|s(t_1)|^{\frac{1}{2}}/L_1)$. Then, it enters the region $\Sigma_1 < 0$ in the same quadrant. Similarly, all the trajectories in this region remain bounded by the line segment II which drops from $(s(t_1), K|s(t_1)|^{\frac{1}{2}}/L_1)$ to $(s(t_1), 0)$. This is because both $\dot{s} < 0$ and $\dot{\mu} < 0$ as $\Sigma_1 < 0$ and $L_2 > \frac{\Gamma^+}{L_1}$, respectively.

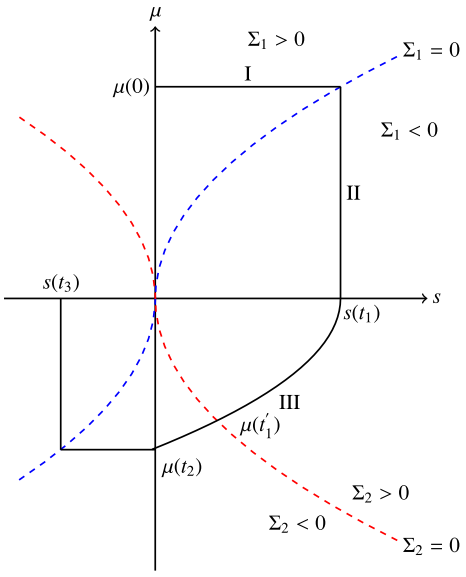

Fig. 1. Majorant curve of STC in (s, μ) plane.

Then, the system trajectory enters into the fourth quadrant ($s > 0$ and $\mu < 0$) where $\dot{s} < 0$ and $\dot{\mu} < 0$. It is easy to see that in this quadrant all the trajectories remain bounded by the segment III which is governed by

$$\dot{s} = \underline{b}\max\left\{L_1\mu, -K|s|^{\frac{1}{2}}\right\} \quad \text{and} \quad \dot{\mu} = -\left(L_2 + \frac{\Gamma^+}{L_1}\right).$$

Clearly, the dynamical equations of curve segment III until it reaches the curve $\Sigma_2 = 0$ are represented by $\dot{s} = \underline{b}L_1\mu$ and $\dot{\mu} = -\left(L_2 + \frac{\Gamma^+}{L_1}\right)$ as $\Sigma_2 > 0$ and $s > 0$, respectively. On solving these two, the equation of motion of this segment is obtained as

$$\mu^2(t) = \frac{2\left(L_2 + \frac{\Gamma^+}{L_1}\right)}{\underline{b}L_1}(s(t_1) - s(t)) \tag{8}$$

for all $t \in [t_1, t_1']$ where t_1' is the time instant at which the trajectory reaches the curve $\Sigma_2 = 0$. A simple calculation shows that the segment of curve III intersects the curve $\Sigma_2 = 0$ at $(s(t_1'), -K|s(t_1')|^{\frac{1}{2}}/L_1)$ where

$$s(t_1') = \frac{2L_1\left(L_2 + \frac{\Gamma^+}{L_1}\right)}{\underline{b}K^2 + 2L_1\left(L_2 + \frac{\Gamma^+}{L_1}\right)}s(t_1).$$

Once the trajectory reaches $\Sigma_2 = 0$, it moves towards the line $s = 0$ due to both $\dot{s} < 0$ and $\dot{\mu} < 0$. During this, the dynamics of segment III is governed by

$$\dot{s} = -\underline{b}K|s|^{\frac{1}{2}} \quad \text{and} \quad \dot{\mu} = -\left(L_2 + \frac{\Gamma^+}{L_1}\right)$$

as $\Sigma_2 < 0$. It is seen that with the above dynamical equations the majorant curve now traverses from $(s(t_1'), -K|s(t_1')|^{\frac{1}{2}}/L_1)$ to $(0, \mu(t_2))$ and is given as

$$\mu(t) = \mu(t_1') - \frac{2}{\underline{b}K}\left(L_2 + \frac{\Gamma^+}{L_1}\right)\left(|s(t_1')|^{\frac{1}{2}} - |s(t)|^{\frac{1}{2}}\right) \tag{9}$$

for all $t \in [t_1', t_2]$.

This curve hits the line $s = 0$ in finite time with the intercept $\mu(t_2) < 0$. Using the above relation, we compute the value of this

Please cite this article in press as: Behera, A.K., et al., A new geometric proof of super-twisting control with actuator saturation. Automatica (2017), https://doi.org/10.1016/j.automatica.2017.09.015.

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