Automatica 69 (2016) 242–249

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs[☆]

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ARTICLE INFO

Article history:
Received 11 April 2014
Received in revised form
4 February 2016
Accepted 23 February 2016

Keywords:
Decentralized event-triggering
Consensus control
Multi-agent systems
General linear dynamics
Directed graphs

ABSTRACT

In this paper, the event-triggered consensus problem is studied for multi-agent systems with general linear dynamics under a general directed graph. Based on state feedback, we propose a decentralized event-triggered consensus controller (ETCC) for each agent to achieve consensus, without requiring continuous communication among agents. Each agent only needs to monitor its own state continuously to determine when to trigger an event and broadcast its states to its out-neighbors. The agent updates its controller when it broadcasts its states to its out-neighbors or receives new information from its in-neighbors. The ETCC can be implemented in multiple steps. it is proved that under the proposed ETCC there is no Zeno behavior exhibited. To relax the requirement of continuous monitoring of each agent's own states, we further propose a self-triggered consensus controller (STCC). Simulation results are given to illustrate the theoretical analysis and show the advantages of the event-triggered and self-triggered controllers proposed in this paper.

1. Introduction

In the last decade, the consensus problem of continuous-time multi-agent systems (MAS) has been attracting much attention due to its wide applications. Many significant works have been obtained, e.g., see Jadbabaie, Lin, and Morse (2013), Li, Duan, Chen, and Huang (2010), Olfati-Saber, Fax, and Murray (2007), Ren and Beard (2008), Xiao and Wang (2008), and Yu, Chen, and Cao (2010), just to name a few. Note that in the above works the agents need to continuously employ their own and neighbors' states and hence these states need to be obtained continuously. To avoid this disadvantage, some researchers has begun to study the centralized/distributed event-triggered consensus problem. The event-triggered average-consensus problem was considered for MAS with single-integrator dynamics in Dimarogonas and Johansson (2009) and Fan, Feng, Wang, and Song (2013). The event-triggered consensus problem for MAS with general linear dynamics was investigated in Zhang, Feng, Yan, and Chen (2014). Periodic event/self-triggered consensus for general linear MAS and distributed convex optimization problem based on event-triggered algorithms have been studied in Chen and Ren (2016) and Yang, Ren, and Liu (2014). While the controllers in Dimarogonas and Johansson (2009), Fan et al. (2013), and Zhang, Feng et al. (2014) are often updated less by using the event-triggered algorithms than using the time-triggered ones, they still require the agents to communicate with their neighbors continuously.

It is well known that unnecessary communication can lead to a waste of energy. Continuous communication would also cause the communication resource competition among agents. To remove the requirement for continuous communication and hence reduce the communication cost, researchers have begun to study the quantized consensus or the event-/self-triggered consensus. For example, the event triggering idea was used to solve the continuous-time quantized average consensus problem in Ceragioli, Persis, and Frasca (2011) with the graph being

☆ This work was supported by National Natural Science Foundation of China (61433003, 60904003, 61174213, 61528301, 61120106010), the Program for New Century Excellent Talents in University (NCET-10-0665), and was partially supported by the National Science Foundation under grant ECCS-1307678. The material in this paper was partially presented at the 53rd IEEE Conference on Decision and Control, December 15–17, 2014, Los Angeles, CA, USA. This paper was recommended for publication in revised form by Associate Editor Dimos V. Dimarogonas under the direction of Editor Christos G. Cassandras.

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http://dx.doi.org/10.1016/j.automatica.2016.03.003
0005-1098/

weight-balanced and weakly-connected. Besides, periodic event-triggered consensus algorithm was studied in Meng and Chen (2013) for single-integrator agents over undirected connected communication topologies. A self-triggered control algorithm for single-integrator agents was given in Dimarogonas, Frazzoli, and Johansson (2012). In Seyboth, Dimarogonas, and Johansson (2013), a decentralized event-triggered consensus algorithm was considered for single- and double- integrator agents. To remove the requirement of global information or synchronous broadcasting as in sampled-data approaches, Garcia et al. (2013) studied the event-triggered control based on only local state errors. In addition, the consensus problem of double-integrator MAS with intermittent communications was investigated in Wen, Duan, Yu, and Chen (2013). Note that during the disconnected communicating time intervals, the communication is still continuous, but not discrete. An event-triggered distributed consensus optimization algorithm was proposed in Chen and Ren (2016). However, in Ceragioli et al. (2011), Chen and Ren (2016), Dimarogonas et al. (2012), Garcia et al. (2013), Meng and Chen (2013), Seyboth et al. (2013), and Wen et al. (2013), the agents were assumed to be with single- or double- integrator dynamics. For MAS with general linear dynamics, Garcia, Cao, Giua, and Casbeer (2014), Zhang, Feng et al. (2014), Zhang, Hao, Zhang, and Wang (2014), and Zhu, Jiang, and Feng (2014) have recently considered the event-triggered consensus problem. However, in Garcia et al. (2014) and Zhu et al. (2014) the final consensus error could only converge to a neighborhood of zero and Zhang, Feng et al. (2014), Zhang, Hao et al. (2014), and Zhu et al. (2014) required continuous communication of neighbors' states to check the triggering conditions. The communication topology in Zhang, Hao et al. (2014) was assumed to be undirected. Also Seyboth et al. (2013) only considered the event-triggered consensus for double-integrator dynamics under undirected graphs. In short, the event-triggered consensus problem with zero final consensus error for general linear MAS without continuous communication under directed graphs has not been addressed.

Motivated by the above discussion, we consider the consensus problem for MAS with general linear dynamics under a general directed graph based on an event-triggered broadcasting scheme by expanding on our preliminary work reported in Yang et al. (2014). The communication topology among the agents is assumed to be a general directed graph containing a directed spanning tree. With state feedback, we first propose a decentralized event-triggered consensus controller (ETCC) implemented in multiple steps for each agent to achieve consensus. Under our proposed controller, there is no continuous communication required among agents. We further prove that there is no Zeno behavior exhibited during the control process, that is, the event would not be triggered continuously. Note that if the Zeno behavior cannot be avoided, then essentially continuous communication is required again. Note that under the ETCC, each agent needs to monitor its own state continuously. To relax this limitation, we further propose a self-triggered consensus controller (STCC), where the next triggering instant is preset by the agent itself at the previous triggering instant. Here the stability analysis and the exclusion of the Zeno behavior of the closed-loop systems are partly inspired by Li et al. (2010) and Seyboth et al. (2013). However, due to the significant challenges caused by the coupling of non-monotonically changing measurement errors, general linear dynamics, and directed communication topologies, the convergence of the consensus errors and the non-Zeno analysis are more difficult than before. The primary contributions of the paper are summarized as follows.

(1) We solve the event-triggered/self-triggered consensus problem for multi-agent systems with general linear dynamics under general directed graphs without the need for continuous communication in either controller update or triggering condition monitoring. In addition, our results guarantee zero final consensus error. Most works in the existing literature have some limitations such as agents' dynamics, communication topologies, nonzero final consensus error, and continuous communication. So, the methods proposed in the literature cannot be directly used in our paper.
(2) By using the matrix exponential function e^{At}, a piecewise continuous control input is designed in the proposed ETCC to estimate the current states of the agents and solve the dynamic consensus problem, where the final consensus states can be time varying. In contrast, Dimarogonas et al. (2012), Fan et al. (2013), and Meng and Chen (2013) adopted a piecewise constant control input to solve the static consensus problem, where the final consensus states are constant. Note that, for single-integrator agents, the event-based controller can be obtained directly from the traditional continuous consensus controller. But to solve the dynamic consensus problem for double-integrator or general linear agents, the continuous controllers cannot be directly implemented in the event-triggered form. Introducing the matrix exponential function e^{At} in the controller design and the threshold function is an innovative point of our research. The results in Seyboth et al. (2013) dealing with single- and double-integrator dynamics can be regarded a special case of our result. It is worth mentioning that the analysis for convergence and exclusion of the Zeno behavior in our framework is nontrivial and there exist significant challenges.

The rest of this paper is organized as follows. Some useful results and the dynamics are introduced in Section 2. The event-triggered consensus is investigated in Section 3 and the self-triggered scheme is discussed in Section 4. Simulation examples are given in Section 5. Section 6 concludes the paper.

2. Preliminaries

2.1. Notation and graph theory

Let $\mathbf{R}^{m\times n}$ and $\mathbf{C}^{m\times n}$ be, respectively, the set of $m \times n$ real and complex matrices. Let $\mathbf{1}_m$ and $\mathbf{0}_m$ denote, respectively, the $m \times 1$ column vector of all ones and all zeros. Let $\mathbf{0}_{m\times n}$ denote the $m \times n$ matrix with all zeros and I_m denote the $m \times m$ identity matrix. The superscript T means the transpose for real matrices. We denote by $\lambda_i(\cdot)$ the ith eigenvalue of a matrix. By $\mathrm{diag}(A_1, \ldots, A_n)$, we denote a block-diagonal matrix with matrices A_i, $i = 1, \ldots, n$, on its diagonal. A matrix $A \in \mathbf{C}^{m\times m}$ is Hurwitz if all of its eigenvalues have strictly negative real parts. The matrix $A \otimes B$ denotes the Kronecker product of matrices A and B. Let $\|\cdot\|$ denote, respectively, the Euclidean norm for vectors and the induced 2-norm for matrices. Let $\|\cdot\|_F$ denote the Frobenius norm of a matrix. Let $\dim(\cdot)$ describe the dimension of a square matrix. For a complex number, $\mathrm{Re}(\cdot)$ denotes its real part.

A directed graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_N\}$ is a nonempty finite set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of edges, in which an edge is represented by an ordered pair of distinct nodes. An edge (v_i, v_j) means that node v_j can receive information from node v_i or equivalently node v_i can broadcast information to node v_j. Here we call v_i an in-neighbor of v_j and v_j an out-neighbor of v_i. A directed path from node v_{i_1} to node v_{i_l} is a sequence of ordered edges of the form $(v_{i_k}, v_{i_{k+1}})$, $k = 1, \ldots, l-1$. A directed graph contains a directed spanning tree if there exists a node called the root such that there exist directed paths from this node to every other node. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{N\times N}$ associated with the directed graph $\mathcal{G}$ is defined by $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbf{R}^{N\times N}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. The graph $\mathcal{G}$ is undirected if $a_{ij} = a_{ji}, \forall i, j = 1, \ldots, N$ and directed otherwise.