Automatica 63 (2016) 302–310

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stabilization of a class of nonlinear systems with actuator saturation via active disturbance rejection control[☆]

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ARTICLE INFO

Article history:
Received 7 October 2014
Received in revised form
1 July 2015
Accepted 18 September 2015

Keywords:
Nonlinear systems
Actuator saturation
Active disturbance rejection control
Stabilization
Domain of attraction
Linear matrix inequalities

ABSTRACT

The stabilization problem of a class of nonlinear systems with actuator saturation is investigated via active disturbance rejection control (ADRC). We first present results for systems with nonlinear ADRC and show that local stabilization can be achieved in a region including the origin. Then, for the linear ADRC, the linear matrix inequality (LMI) conditions for determining the estimate of the domain of attraction of the resulting closed-loop system are formulated based on a quadratic candidate Lyapunov function and a generalized sector condition. An LMI-based algorithm is correspondingly established to design the linear ADRC controller. The obtained results suggest a new way to stabilize the saturated nonlinear system, even in the case that the state of the system is not fully available for measurement and system nonlinear dynamics are largely unknown. An illustrative example validates the effectiveness of the proposed approach.

1. Introduction

Actuator saturation affects virtually all practical control systems. It may lead to performance degradation and even induce instability. In the past few decades, the problems of analysis and design of linear systems with actuator saturation have been extensively studied (see, e.g., Dai, Hu, Teel, & Zaccarian, 2009; Lin & Saberi, 1993; Sontag, 1984; Teel, 1992; and Weston & Postlethwaite, 2000). It is now well-recognized that only local stabilization can be achieved if a linear system is open-loop unstable. In this case, it is quite natural to design a control law with the objective of enlarging the domain of attraction of the resulting closed-loop system. Since the exact characterization of the domain of attraction is almost impossible, many researchers have devoted to enlarging an estimate of it (Cao, Lin, & Ward, 2002; Gomes da Silva & Tarbouriech, 2001; Li & Lin, 2013; Lu & Lin, 2010; Zhou, Zheng, & Duan, 2010).

However, most practical control systems are inherently nonlinear, and the number of available results by taking actuator saturation into account in the design and analysis of nonlinear control systems is still limited. Due to the difficulty of the problem itself, most researchers have focused their attention on particular classes of nonlinear systems and designed dedicated controllers for that class. For example, Coutinho and Gomes da Silva (2007) devised a generic method for obtaining the estimates of the domain of attraction of a class of nonlinear systems that can be put in a rational algebraic-differential form. In Castelan, Tarbouriech, and Queinnec (2008), the stability and stabilization problems of a class of nonlinear systems consisting of a linear system affected by a state-dependent nonlinearity belonging to a general class of sectors and subject to actuator saturation were considered. Valmórbida, Tarbouriech, and Garcia (2010) proposed a method to design stabilizing state feedback control laws for nonlinear quadratic systems with actuator saturation. In addition, several papers have appeared on input-constrained feedback linearizable nonlinear systems under nonlinear dynamic inversion control (Gußner, Jost, & Adamy, 2012; Herrmann, Menon, Turner, Bates, & Postlethwaite, 2010; Yoon, Park, & Yoon, 2008).

On the other hand, active disturbance rejection control (ADRC) is an effective method to deal with nonlinear system, especially when its dynamics are largely unknown. ADRC was

☆ This work was supported by the National Natural Science Foundation of China (Nos. 61273083 and 61374012). The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Zongli Lin under the direction of Editor Andrew R. Teel.

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http://dx.doi.org/10.1016/j.automatica.2015.10.010
0005-1098/

originally[2] proposed by Han in 1980s (Han, 1989, 1995, 1998). This methodology requires very little information about the plant dynamics and is easy to be implemented. The essential of ADRC is to treat the unknown dynamics as an extended state of the plant, and then estimate it using an extended state observer (ESO) and compensate for it in real time. ADRC has found wide industrial applications even though its theoretical justification was lagging behind for quite some time (see, e.g., Li, Yang, & Yang, 2009; Talole, Kolhe, & Phadke, 2010; Wu & Chen, 2009 and Zheng, Dong, Lee, & Gao, 2009). In a pair of recent papers, Guo and Zhao (2011, 2013) gave a rigorous proof of the convergence of ESO and ADRC. For more applications of ADRC and the progress of its theoretical analysis, one can refer to a recent survey paper (Huang & Xue, 2014).

This paper studies the stabilization problem of a class of input-constrained nonlinear systems that incorporate ADRC laws. Since we consider a general class of nonlinear systems, we will restrict ourselves to the local stabilization problem. We first show that, with the application of the nonlinear ADRC, the considered nonlinear system is asymptotically stable in a region including the origin. Then, we consider a class of special ADRC, the linear ADRC. By using a quadratic candidate Lyapunov function and the generalized sector condition established in Tarbouriech, Prieur, and Gomes da Silva (2006), the conditions developed to address local stabilization problem are formulated in linear matrix inequalities (LMIs) form. As a result, an LMI-based algorithm is correspondingly established to design the linear ADRC controller, with the objective of enlarging the domain of attraction of the closed-loop system.

Our results are motivated by the works in Freidovich and Khalil (2008), Guo and Zhao (2011, 2013), Han (2009), Nazrulla and Khalil (2011) and Tarbouriech et al. (2006). The first contribution of this paper is to prove that local stabilization can be achieved for a class of saturated nonlinear systems by using an ADRC law. Note that Freidovich and Khalil (2008) and Guo and Zhao (2013) have shown that semi-global stabilization can be achieved via a bound ADRC law. However, the saturation bound they considered is utilized to avoid the peaking phenomenon caused by the high gain in the ESO, and its value depends on the initial state of the closed-loop system. In this paper, we consider the saturation caused by the inherent actuator limitations and its bound value is in general fixed. The second contribution is to establish an LMI-based framework for the analysis and design of the linear ADRC controller for input-constrained nonlinear systems.

Notation. $\mathbb{R}$ is the set of real numbers. A^{T} is the transpose of a real matrix A. $A_{(i)}$ denotes the ith row of A. The matrix inequality $A > B$ ($A \geq B$) means that $A - B$ is positive (semi-) definite. $\lambda_{\max}(P)$ ($\lambda_{\min}(P)$) denotes the maximum (minimum) eigenvalue of a real symmetric matrix P. A block diagonal matrix with sub-matrices $X_1, X_2, \ldots, X_n$ in its diagonal will be denoted by $\mathrm{diag}\{X_1, X_2, \ldots, X_n\}$. I and $\mathbf{0}$ denote the identity matrix and zero matrix with appropriate dimensions, respectively. To reduce clutter, off-diagonal entries in symmetric matrices are occasionally replaced by '$*$'.

2. Problem statement

Consider the following n-dimensional SISO nonlinear system with actuator saturation

$$y^{(n)} = f(y, \dot{y}, \ldots, y^{(n-1)}) + b\mathrm{sat}(u) \tag{1}$$

where $y \in \mathbb{R}$ is the plant output, $f(y, \dot{y}, \ldots, y^{(n-1)})$, or simply denoted as f, is a C^1-function that represents the possibly unknown nonlinear dynamics of the system, b is a given constant, $u \in \mathbb{R}$ is the control input, $\mathrm{sat}(\cdot)$ is the saturation function defined as $\mathrm{sat}(u) = \mathrm{sign}(u)\min\{|u|, 1\}$. Here, we have assumed the unity saturation level. Nonunity saturation levels can be readily absorbed into the constant b. In addition, we assume $f(0) = 0$ henceforth.

Let $f = x_{n+1}$ be an extended state of the system and assume $h = \dot{f}$. Then, system (1) can be written as the following equivalent form:

$$\begin{cases} \dot{x}_1 = x_2, \\ \vdots \\ \dot{x}_{n-1} = x_n, \\ \dot{x}_n = x_{n+1} + b\mathrm{sat}(u), \\ \dot{x}_{n+1} = h, \\ y = x_1 \end{cases} \tag{2}$$

where $x = [x_1, x_2, \ldots, x_n]^{\mathrm{T}} \in \mathbb{R}^n$ is the state of the system. An ESO is correspondingly designed for (2),

$$\begin{cases} \dot{\hat{x}}_1 = \hat{x}_2 + \varepsilon^{n-1} g_1\left(\dfrac{x_1 - \hat{x}_1}{\varepsilon^n}\right), \\ \dot{\hat{x}}_2 = \hat{x}_3 + \varepsilon^{n-2} g_2\left(\dfrac{x_1 - \hat{x}_1}{\varepsilon^n}\right), \\ \vdots \\ \dot{\hat{x}}_n = \hat{x}_{n+1} + g_n\left(\dfrac{x_1 - \hat{x}_1}{\varepsilon^n}\right) + b\mathrm{sat}(u), \\ \dot{\hat{x}}_{n+1} = \varepsilon^{-1} g_{n+1}\left(\dfrac{x_1 - \hat{x}_1}{\varepsilon^n}\right) \end{cases} \tag{3}$$

where $\left[\hat{x}^{\mathrm{T}}, \hat{x}_{n+1}\right]^{\mathrm{T}} = \left[\hat{x}_1, \ldots, \hat{x}_n, \hat{x}_{n+1}\right]^{\mathrm{T}} \in \mathbb{R}^{n+1}$ is the ESO state, ε is a small positive constant, and g_i, $i = 1, 2, \ldots, n+1$, are some nonlinear or linear functions. The above is a special form of the general ESO proposed in Han (1995), and was also considered in Guo and Zhao (2011, 2013). We anticipate that $\hat{x}_i \to x_i$ as $\varepsilon \to 0$ and $t \to \infty$ for all $i = 1, 2, \ldots, n+1$. Then, by using the output of the ESO, the control law is given as

$$u = \frac{1}{b}\left(\varphi(\hat{x}) - \hat{x}_{n+1}\right) \tag{4}$$

where $\varphi(\cdot)$ is some nonlinear or linear function to be decided. Substitution of (4) into (1) yields

$$y^{(n)} = f + b\mathrm{sat}\left(\frac{1}{b}\left(\varphi(\hat{x}) - \hat{x}_{n+1}\right)\right). \tag{5}$$

In the absence of actuator saturation, the equation above becomes

$$y^{(n)} = (f - \hat{x}_{n+1}) + \varphi(\hat{x}). \tag{6}$$

It can be observed that in this case the possibly unknown dynamics f is asymptotically on-line canceled out by the term $-\frac{1}{b}\hat{x}_{n+1}$ in the control action, and as a consequence, the closed-loop system is reduced to a linear one that incorporates with the estimated state feedback $\varphi(\hat{x})$. The estimated state feedback $\varphi(\hat{x})$ is designed to guarantee the stability of the closed-loop system and to render a highly desirable performance when saturation is not accounted for at the plant's input.

Due to the actuator saturation, the actual control signal to be injected into the system is a saturated one, $\mathrm{sat}(u)$, instead of u, namely, the states of the controller may achieve different values from those in the absence of saturation. Thus performance deterioration and even instability may happen. In particular, in ADRC, the nonlinear dynamics cannot be accurately compensated, and consequently, the desired response is no longer guaranteed. The goal of this paper is then to provide a solution to the stabilization problem of the saturated nonlinear system (1) by using the ESO-based control law (4).

[2] Han's original papers on ADRC are mainly appeared in Chinese in 1980s–1990s. The essentials of ADRC are summed up in English in Han (2009).