Automatica 51 (2015) 80–84

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Observer design for uncertain nonlinear systems with unmodeled dynamics[☆]

Zhengqiang Zhang[a], Shengyuan Xu[b,1]

[a] School of Engineering, Qufu Normal University, Rizhao 276826, China

[b] School of Automation, Nanjing University of Science and Technology, Nanjing 210094, China

ARTICLE INFO

Article history:
Received 18 June 2013
Received in revised form
2 June 2014
Accepted 30 August 2014

Keywords:
Nonlinear systems
Uncertain dynamic systems
Unmodeled dynamics
Adaptive observer
Robustness

ABSTRACT

This paper is concerned with the observer design problem for a class of multi-input–multi-output nonlinear systems with the unmodeled dynamics, unknown parameters and external disturbance. A dynamic signal, which can dominate the unmodeled dynamics, is firstly constructed. Then, two types of observer schemes, that is, adaptive observer and robust observer, are respectively proposed. The observation error in the two schemes can be made arbitrarily small by choosing the appropriate design parameters. An illustrative example is provided to demonstrate the validity of the proposed design methods.

1. Introduction

The unmodeled dynamics often appears in many control systems due to the modeling errors or the modeling simplifications. The stability analysis and feedback control problems for linear or nonlinear systems with the unmodeled dynamics have attracted the attention of many researchers (see, e.g. Chen & Huang, 2010, Ikhouane & Krstic, 1998, Jiang & Praly, 1998, and references therein).

On the other hand, the problem of state observer design has been well studied in the past decades. The observer schemes have been developed for all kinds of nonlinear systems, such as time-delay systems (Ghanes, De Leon, & Barbot, 2013; Ibrir, 2009; Wu, 2009), Lipschitz nonlinear systems (Ekramian, Sheikholeslam, Hosseinnia, & Yazdanpanahd, 2013) and other class of nonlinear systems (Boizot, Busvelle, & Gauthier, 2010; Churilov, Medvedev, & Shepeljavyi, 2012; Farza, M'Saad, Maatoug, & Kamoun, 2010; Grip, Saberi, & Johansen, 2012; Hammouri, Bornard, & Busawon, 2010; Marino, Santosuosso, & Tomei, 2001; Menard, Moulay, & Perruquetti, 2010; Stamnes, Aamo, & Kaasa, 2011). However, few observer results have been reported for nonlinear systems with the unmodeled dynamics. In the presence of the unmodeled dynamics, the observer design problem becomes more challenging. In a recent paper by Liu (2009), robust adaptive observers were designed for a class of nonlinear systems with the unmodeled dynamics. It has been shown that the proposed observers can guarantee the boundedness of the estimation error and the adaptive gain.

In this paper, we address the observer design problem for a class of nonlinear systems with unmodeled dynamics, unknown system parameters and unknown time-varying disturbance. Since a parameter in the assumption for the unmodeled dynamics is not employed in the newly defined dynamic signal, the developed observer does not require it to be known. Moreover, in the bounding function of the uncertain nonlinear function, the coupling terms related to the unmodeled dynamics and the system states are fully considered. The first technical result presented in this paper allows us to show the dominating property of the introduced dynamic signal with respect to the unmodeled dynamics. Then, two types of observer schemes are proposed. The first scheme is adaptive observer, which has the same spirit as in Liu (2009). In the second scheme, adaptive methodology is not employed, which has a great significance for reducing the system complexity and saving the cost for observer implementation.

☆ This work was supported by the National Natural Science Foundation of China under Grant Nos. 61104007, 61473171 and 61374087, the Program for Changjiang Scholars and Innovative Research Team in University under Grant No. IRT13072 and China Postdoctoral Science Foundation funded project under Grant Nos. 2012M511465 and 2014T70622. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor James Lam under the direction of Editor Ian R. Petersen.

E-mail addresses: qufuzzq@126.com (Z. Zhang), syxu@njust.edu.cn (S. Xu).

[1] Tel.: +86 25 84315872; fax: +86 25 84315872.

http://dx.doi.org/10.1016/j.automatica.2014.10.068
0005-1098/

2. Problem formulation and preliminaries

We consider a class of uncertain nonlinear systems with the unmodeled dynamics in the form of

$$\dot{x} = Ax + Bf(x, u, \omega, \theta, d) + g(y, u),$$
$$y = Cx, \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^m$ is the control input, $y \in R^p$ is the measured output, $\omega \in R^r$ is the unmodeled dynamics, $\theta \in R^q$ is the unknown parameter vector, $d \in R^k$ is the unknown time-varying disturbance, $f(x, u, \omega, \theta, d) \in R^l$ represents the unknown nonlinear perturbation, $g(y, u)$ is the known nonlinear function, and A, B, C are the known constant matrixes of appropriate dimensions. The unmodeled dynamics is described by

$$\dot{\omega} = q(y, \omega), \tag{2}$$

where $q(y, \omega)$ is the unknown nonlinear function.

Our objective is to design observer with the output $y(t)$ and input $u(t)$ to estimate the system states. We make the following assumptions.

Assumption 1. The pair $\{A, C\}$ given in (1) is detectable. That is, there exists a matrix $K \in R^{n\times p}$ such that the matrix $A_m = A - KC$ is Hurwitz.

Assumption 2. There exist positive definite matrixes P, Q such that $A_m^T P + PA_m = -Q$, $PB = C^T$.

Assumption 3. For the uncertain function $f(\cdot)$, there exist known nonnegative functions $\xi_i(\cdot)$, $i = 1, 2, 3, 4$, known class K_∞ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$, and unknown constants c_i, $i = 1, 2, \ldots, 8$, such that the following inequality holds: $\|f(x, u, \omega, \theta, d)\| \le c_1 + c_2\|x\| + c_3\xi_1(y, u) + c_4\alpha_1(\|\omega\|) + c_5\|x\|\xi_2(y, u) + c_6\|x\|\alpha_2(\|\omega\|) + c_7\xi_3(y, u)\alpha_3(\|\omega\|) + c_8\|x\|\xi_4(y, u)\alpha_4(\|\omega\|)$, where $\|\cdot\|$ denotes the Euclidean norm of a vector.

Assumption 4. For the unmodeled dynamics (2), there exists a Lyapunov function $V_\omega(\omega)$ such that the following conditions are satisfied: $V_\omega(\omega) \ge \alpha(\|\omega\|)$, $\frac{\partial V_\omega(\omega)}{\partial \omega} q(y, \omega) \le -\gamma_1 V_\omega(\omega) + \rho(y) + \gamma_2$, where $\alpha(\cdot)$ is a known function of class K_∞, $\rho(\cdot)$ is a known nonnegative function, $\gamma_1 > 0$, $\gamma_2 \ge 0$, are constants, and γ_2 is not required to be known.

Then, in order to handle the unmodeled dynamics $\omega(t)$, we introduce a new dynamic signal $\delta(t)$, which is generated by

$$\dot{\delta} = -\gamma_0\delta + \rho(y), \qquad \delta(t_0) = \delta_0, \tag{3}$$

where γ_0 is a design constant satisfying $0 < \gamma_0 < \gamma_1$, and δ_0 represents the initial condition of $\delta(t)$ and is nonnegative design constant.

Lemma 1. *The dynamic signal $\delta(t)$ has the following properties: (i) $\delta(t) \ge 0$, $\forall t \ge t_0 \ge 0$; (ii) $V_\omega(\omega(t)) \le \delta(t) + D$, $\forall t \ge t_0 \ge 0$, where $D = V_\omega(\omega_0) + \gamma_2/\gamma_1$, ω_0 is the initial condition of $\omega(t)$ in (2), i.e. $\omega_0 = \omega(t_0)$.*

Proof. Since $\rho(\cdot)$ is nonnegative function and δ_0 is nonnegative constant, Property (i) holds. We define $U(t) = V_\omega(\omega(t))$. Thus, we have $U(t_0) = V_\omega(\omega_0) \ge 0$. From (2), (3) and Assumption 4, the derivative of $U(t)$ is given by $\dot{U}(t) = \frac{\partial V_\omega}{\partial \omega} q(y, \omega) \le -\gamma_1 U(t) + \dot{\delta} + \gamma_0\delta + \gamma_2$, $\forall t \ge t_0 \ge 0$, which together with the formula of integration by parts leads to

$$\begin{aligned} U(t) &\le e^{-\gamma_1(t-t_0)}U(t_0) \\ &\quad + \int_{t_0}^{t} e^{-\gamma_1(t-\tau)}(\dot{\delta}(\tau) + \gamma_0\delta(\tau) + \gamma_2)d\tau \\ &= e^{-\gamma_1(t-t_0)}U(t_0) + \delta(t) + \gamma_2/\gamma_1 - e^{-\gamma_1(t-t_0)}\delta_0 \\ &\quad - (\gamma_1 - \gamma_0)\int_{t_0}^{t} \delta(\tau)e^{-\gamma_1(t-\tau)}d\tau - \frac{\gamma_2}{\gamma_1}e^{-\gamma_1(t-t_0)}. \end{aligned}$$

Using Property (i), $\delta_0 \ge 0$ and $\gamma_1 > \gamma_0$, we get $U(t) \le \delta(t) + U(t_0) + \gamma_2/\gamma_1$, that is, (ii) holds.

3. Observer design and analysis

3.1. Scheme I: adaptive observer

We propose the following observer with adaptive gain:

$$\dot{\hat{x}} = A\hat{x} + g(y, u) - K\bar{e} - \beta_{11}(t)\beta_{12}(t)B\bar{e}, \tag{4}$$

where $\hat{x}$ is the state estimate, $e = \hat{x} - x$ is the observation error, $\bar{e} = C\hat{x} - y = Ce$ is the output error, $\beta_{11}(t) = 1 + k_1 + \|\hat{x}\|^2(1 + \xi_2^2(y, u) + \xi_4^2(y, u) + \xi_4^2(y, u)\alpha_4^2(2\alpha^{-1}(2\delta))) + \xi_1^2(y, u) + k_2\xi_2^2(y, u) + \xi_3^2(y, u) + k_5\xi_4^2(y, u) + \alpha_1^2(2\alpha^{-1}(2\delta)) + (1 + k_3)\alpha_2^2(2\alpha^{-1}(2\delta)) + \xi_3^2(y, u)\alpha_3^2(2\alpha^{-1}(2\delta)) + k_4\xi_4^2(y, u)\alpha_4^2(2\alpha^{-1}(2\delta))$, $\beta_{12}(t)$ is adaptive gain, which is updated by

$$\dot{\beta}_{12}(t) = \gamma(\|\bar{e}\|^2\beta_{11}(t) - \sigma\beta_{12}(t)), \tag{5}$$

k_i, $i = 1, 2, 3, 4, 5, \gamma, \sigma$, are positive design constants, k_i, $i = 1, 2, 3, 4, 5$, are chosen to be sufficiently large such that $\mu := (\lambda_{\min}(Q) - \sum_{i=1}^{5} 1/(2k_i))/(2\lambda_{\max}(P)) > 0$ for any given P, Q satisfying Assumption 2, and $\lambda_{\min}(\cdot)$, $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively. Then, we have the following theorem.

Theorem 1. *The system (3)–(5) is an adaptive observer for the system (1). For any given initial condition $e(t_0) = \hat{x}(t_0) - x(t_0)$, the observation error $e(t)$ exponentially converges to a neighborhood of the origin, which can be made arbitrarily small by choosing the appropriate design parameters k_i, $i = 1, 2, 3, 4, 5, \gamma, \sigma$.*

Proof. From (1), (4) and Assumption 1, the dynamics of $e(t)$ is governed by

$$\dot{e} = A_m e - Bf(x, u, \omega, \theta, d) - \beta_{11}(t)\beta_{12}(t)B\bar{e}. \tag{6}$$

Define the Lyapunov function as $V_1 = 1/2e^T Pe + 1/2\gamma^{-1}(\beta_{12}(t) - k_0)^2$, where k_0 is a constant satisfying $k_0 \ge \max\{\bar{c}_2^2, c_5^2, c_6^2, c_8^2, c_8^2\alpha_4^2(2\alpha^{-1}(2D))\}$, and $\bar{c}_2$ is defined below. From Assumption 2, (5) and (6), it can be obtained that

$$\begin{aligned} \dot{V}_1 &= -1/2e^T Qe - \bar{e}^T f - \beta_{11}(t)\beta_{12}(t)\|\bar{e}\|^2 \\ &\quad + \gamma^{-1}(\beta_{12}(t) - k_0)\dot{\beta}_{12}(t). \end{aligned} \tag{7}$$

From Assumption 3, we have

$$\begin{aligned} |\bar{e}^T f| \le \|\bar{e}\|[&c_1 + c_2\|x\| + c_3\xi_1(y, u) + c_4\alpha_1(\|\omega\|) + c_5 \\ &\cdot \|x\|\xi_2(y, u) + c_6\|x\|\alpha_2(\|\omega\|) + c_7\xi_3(y, u) \\ &\cdot \alpha_3(\|\omega\|) + c_8\|x\|\xi_4(y, u)\alpha_4(\|\omega\|)]. \end{aligned} \tag{8}$$

From Assumption 4 and Lemma 1, we get $\alpha(\|\omega\|) \le \delta(t) + D$, $\|\omega\| \le \alpha^{-1}(2\delta(t)) + \alpha^{-1}(2D)$, $\alpha_i(\|\omega\|) \le \alpha_i(2\alpha^{-1}(2\delta(t))) + \alpha_i(2\alpha^{-1}(2D))$, $i = 1, 2, 3, 4$, which together with $\|x\| \le \|\hat{x}\| + \|e\|$ results in

$$\begin{aligned} |\bar{e}^T f| \le \|\bar{e}\|[&\bar{c}_1 + \bar{c}_2\|\hat{x}\| + c_3\xi_1(y, u) + c_4\alpha_1(2\alpha^{-1}(2\delta)) \\ &+ c_5\|\hat{x}\|\xi_2(y, u) + c_6\|\hat{x}\|\alpha_2(2\alpha^{-1}(2\delta)) \\ &+ c_7\xi_3(y, u)\alpha_3(2\alpha^{-1}(2\delta)) + c_7\alpha_3(2\alpha^{-1}(2D)) \\ &\cdot \xi_3(y, u) + c_8\xi_4(y, u)\|\hat{x}\|\alpha_4(2\alpha^{-1}(2\delta)) \\ &+ c_8\alpha_4(2\alpha^{-1}(2D))\xi_4(y, u)\|\hat{x}\|] + \bar{c}_2\|\bar{e}\| \cdot \|e\| \\ &+ c_5\|\bar{e}\| \cdot \|e\|\xi_2(y, u) + c_6\|\bar{e}\| \cdot \|e\|\alpha_2(2\alpha^{-1}(2\delta)) \\ &+ c_8\|\bar{e}\|\xi_4(y, u)\|e\|\alpha_4(2\alpha^{-1}(2\delta)) \\ &+ c_8\alpha_4(2\alpha^{-1}(2D))\|\bar{e}\|\xi_4(y, u)\|e\|, \end{aligned} \tag{9}$$