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Brief paper

State estimation and sliding mode control for semi-Markovian jump systems with mismatched uncertainties☆

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ABSTRACT

This paper is concerned with the state estimation and sliding mode control problems for phase-type semi-Markovian jump systems. Using a supplementary variable technique and a plant transformation, a finite phase-type semi-Markov process has been transformed into a finite Markov chain, which is called its associated Markov chain. As a result, phase-type semi-Markovian jump systems can be equivalently expressed as its associated Markovian jump systems. A sliding surface is then constructed and a sliding mode controller is synthesized to ensure that the associated Markovian jump systems satisfy the reaching condition. Moreover, an observer-based sliding mode control problem is investigated. Sufficient conditions are established for the solvability of the desired observer. Two numerical examples are presented to show the effectiveness of the proposed design techniques.

1. Introduction

Markovian jump systems (MJS) are a special class of stochastic dynamic systems which are popular for modeling the random abrupt variations in their structures, since in practice many dynamical systems may subject to frequent unpredictable structural changes, such as random failures, repairs of sudden environment disturbances and abrupt variation of the operating point. Research into this class of systems and their applications have spanned several decades. For some representative work on this general topic, we refer to Costa and De Oliveira (2012), Gao, Fei, Lam, and Du (2011), Ji and Chizeck (1990), Mahmoud (2004), Shi, Boukas, and Agarwal (1999a,b), Shi and Yu (2009) and the references therein.

However, MJS have many limitations in applications, since the jump time of a Markov chain is, in general, exponentially distributed, and the results obtained for MJS are intrinsically conservative due to constant transition rates (Huang & Shi, 2013). Unlike the MJS, semi-Markovian jump systems (S-MJS) are characterized by a fixed matrix of transition probabilities and a matrix of sojourn time probability density functions (Hou, Luo, Shi, & Nguang, 2006). Due to their relaxed conditions on the probability distributions, S-MJS have much broader applications than the conventional MJS. Indeed, it is expected that most of the modeling, analysis and design results for MJS could be regarded as special cases of S-MJS. Hence, this area is significant not only in theory, but also in practice.

Sliding mode control (SMC) is an effective control approach due to its excellent advantage of strong robustness against model uncertainties, parameter variations and external disturbances. It is worthwhile to mention that the SMC strategy has been successfully applied to a variety of practical systems such as robot manipulators, aircraft navigation and control, and power system stabilizers. Consequently, the SMC design problem has received increasing research attention and there are a large number of significant results in the literature (see, for example, Basin, Ferreira, & Fridman, 2007; Basin & Rodriguez-Ramirez, 2011, 2012; Barambones, Alkorta, & de Durana, 2013; Niu, Ho, & Lam, 2005; Niu, Ho, & Wang, 2007; Soltanpour, Zolfaghari, Soltani, & Khooban, 2013; Wu & Shi, 2010; Wu &

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Zheng, 2009, and the references therein). Furthermore, the system states are not always available. Thus, sliding mode observer technique has been developed to deal with the state estimation problems for linear or nonlinear uncertain systems. Nevertheless, there are still a few results related to state estimation and SMC problems for S-MJS.

Motivated by the above discussion, in this paper, we investigate the state estimation and sliding mode control problems for semi-Markovian jump systems. This paper addresses three open questions: (1) how a phase-type semi-Markov process can be replaced by a Markov chain, which is equivalent to transforming a phase-type semi-Markovian jump system into its associated Markovian jump system; (2) how to design the appropriate sliding surface function to adjust the effect of the jumping phenomenon in the plant; and (3) how to perform the reachability analysis for the resulting sliding mode dynamics. Thus, sliding surface function design and reachability analysis of the resulting sliding mode dynamics are the main issues to be addressed in this paper. Numerical examples are given to illustrate the effectiveness of the proposed control scheme.

Notations. The notations used throughout the paper are standard. The superscripts 'T' and '-1' denote matrix transposition and matrix inverse, respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The notation $X > 0$ ($\geqslant 0$) means that matrix X is positive definite (semi-definite); and $\lambda_{\min}(X)$ denotes the minimum eigenvalue of the symmetric matrix X. The notation $(\Omega, \mathcal{F}, \mathcal{P})$ represents the probability space, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. In addition, $\mathbb{E}\{\cdot\}$ denotes the expectation operator. Symbol $\|\cdot\|$ stands for the Euclidean norm of a vector and its induced norm of a matrix. We use diag$\{A_1, \ldots, A_n\}$ to denote the block-diagonal matrix with $A_1, \ldots, A_n$ on the diagonal. In symmetric block matrices, we use an asterisk $*$ to represent a term that is induced by symmetry.

2. Phase-type semi-Markov processes and Markovization

Consider a class of stochastic systems in the probability space $(\Omega, \mathcal{F}, \mathcal{P})$ for $t \geqslant 0$

$$\dot{x}(t) = \big[\hat{A}(\hat{r}_t) + \Delta\hat{A}(\hat{r}_t, t)\big]x(t) + \hat{B}(\hat{r}_t)\big[u(t) + \varphi(t)\big],$$

$$y(t) = \hat{C}(\hat{r}_t)x(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $u(t) \in \mathbb{R}^p$ is the control input, $y(t) \in \mathbb{R}^q$ is the system output, and $\varphi(t) \in \mathbb{R}^p$ is uncertainty disturbance. $\hat{A}(\hat{r}_t)$, $\hat{B}(\hat{r}_t)$ and $\hat{C}(\hat{r}_t)$ are matrix functions of the random process $\{\hat{r}_t, t \geqslant 0\}$; and $\Delta\hat{A}(\hat{r}_t, t)$ is system uncertainty. Let $\{\hat{r}_t, t \geqslant 0\}$ be a continuous time stochastic process on the state space $\{1, 2, \ldots, m+1\}$, where the states $1, 2, \ldots, m$ are transient and the state $m+1$ is absorbing. The infinitesimal generator is $Q = \begin{bmatrix} T & T^0 \\ 0_{1\times m} & 0 \end{bmatrix}$, where the matrix $T = (T_{ij})_{m\times m}$ satisfies $T_{ii} < 0, T_{ij} \geqslant 0, i \neq j$; and $T^0 = \begin{bmatrix} T_1^0 & T_2^0 & \cdots & T_m^0 \end{bmatrix}^T$ is a non-negative column vector such that $Te + T^0 = 0$, where e denotes an appropriately dimensioned column vector with all components equal to one. The initial distribution vector is $(\mathbf{a}, a_{m+1})$, where $\mathbf{a} = (a_1, a_2, \ldots, a_m)$ satisfies $\mathbf{a}e + a_{m+1} = 1$. In addition, we have the following assumption.

Assumption 1. Assume that the absorbing state is reached with probability one for a finite time.

Definition 1 (*Neuts, 1975*)**.** A probability distribution is said to be of phase-type if it is the absorption time distribution of a finite Markov chain having an absorbing state and all the other states transient. This distribution is defined by the pair $(\mathbf{a}, T)$ and we say that the pair $(\mathbf{a}, T)$ is a representation of this distribution.

Definition 2 (*Hou et al., 2006*)**.** Let E be a finite set. A stochastic process $\hat{r}_t$ on the state space E is called a phase-type semi-Markov process, if the following conditions hold.

(i) The sample paths of $\hat{r}_t$ are right-continuous functions and have left-hand limits with probability one;
(ii) Denote the sth jump point of the process $\hat{r}_t$ by τ_s, where $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_s < \cdots$, and τ_s $(s = 1, 2, 3, \ldots)$ are Markovian of the process $\hat{r}_t$;
(iii) $F_{ij}(t) \triangleq \mathcal{P}\big(\tau_{s+1} - \tau_s \leqslant t \,|\, \hat{r}_{\tau_s} = i, \hat{r}_{\tau_{s+1}} = j\big) = F_i(t), i, j \in E, t \geqslant 0$ do not depend on j and s; and
(iv) $F_i(t), i \in E$ is a phase-type distribution.

Remark 1. We considered the times between transitions are phase-type (PH) distributions. It is worth noting that the PH distribution is a generalization of the exponential distribution while still preserving much of its analytic tractability, and has been used in a wide range of stochastic modeling applications such as reliability theory, queueing theory and biostatistics. Furthermore, the family of PH distribution is dense in all the families of distributions on $[0, +\infty)$. So, for every probability distribution on $[0, +\infty)$, we may choose a PH distribution to approximate the original distribution in any accuracy (Neuts, 1975).

Let $(\mathbf{a}^{(i)}, T^{(i)})$, $i \in E$ denote the $m^{(i)}$ order representation of $F_i(t)$, and $E^{(i)}$ be the corresponding all transient states set (the number of the elements in $E^{(i)}$ is $m^{(i)}$), where

$$\mathbf{a}^{(i)} \triangleq \left(a_1^{(i)}, a_2^{(i)}, \ldots, a_{m^{(i)}}^{(i)}\right),$$

$$T^{(i)} \triangleq \left(T_{jk}^{(i)}, \ j, k \in E^{(i)}\right).$$

Also, let

$$p_{ij} \triangleq \Pr\left(\hat{r}_{s+1} = j |\, \hat{r}_s = i\right), \quad i, j \in E,$$

$$P \triangleq (p_{ij}), \quad i, j \in E$$

$$(\mathbf{a}, T) \triangleq (\mathbf{a}^{(i)}, T^{(i)}), \quad i \in E.$$

It is easy to see that the probability distribution of $\hat{r}_t$ can be determined only by $\{P, (\mathbf{a}, T)\}$. For every $s (s = 1, 2, \ldots)$, $\tau_s \leqslant t \leqslant \tau_{s+1}$, define

$$J(t) \triangleq \text{the phase of } F_{\hat{r}(t)}(\cdot) \quad \text{at time } t - \tau_s. \tag{2}$$

Also, for any $i \in E$, define

$$T_j^{(i,0)} \triangleq -\sum_{k=1}^{m^{(i)}} T_{jk}^{(i)}, \quad j = 1, 2, \ldots, m^{(i)}, \tag{3}$$

$$G \triangleq \left\{(i, k^{(i)}) |\, i \in E, \ k^{(i)} = 1, 2, \ldots, m^{(i)}\right\}. \tag{4}$$

Lemma 1 (*Hou et al., 2006*)**.** *$Z(t) = (\hat{r}_t, J(t))$ is a Markov chain with state-space G. The infinitesimal generator of $Z(t)$ given by $Q = (q_{\mu\nu})$, $\mu, \nu \in G$ is determined only by the pair of $(\hat{r}_t, J(t))$ given by $\{P, (\mathbf{a}, T)\}$ as follows:*

$$\begin{cases} q_{(i,k^{(i)})(i,k^{(i)})} = T_{k^{(i)}k^{(i)}}^{(i)}, & (i, k^{(i)}) \in G \\ q_{(i,k^{(i)})(i,\bar{k}^{(i)})} = T_{k^{(i)}\bar{k}^{(i)}}^{(i)}, & k^{(i)} \neq \bar{k}^{(i)}, (i, k^{(i)}) \in G, \\ & (i, \bar{k}^{(i)}) \in G \\ q_{(i,k^{(i)})(j,k^{(j)})} = p_{ij} T_{k^{(i)}}^{(i,0)} a_{k^{(j)}}^{(j)}, & i \neq j, (i, k^{(i)}) \in G, \\ & (j, k^{(j)}) \in G. \end{cases}$$

From (4), we obtain that G has $N = \sum_{i\in E} m^{(i)}$ elements, so the state space of $Z(t)$ has N elements. For convenience, we further