Electric Power Systems Research 163 (2018) 10–17

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

A three-dimensional downward leader model incorporating geometric and physical characteristics

Shijun Xie[a,*], F. D'Alessandro[b], Xiangen Zhao[c]

[a] State Grid Sichuan Electric Power Research Institute, Chengdu 610072, China
[b] PhysElec Solutions Pty. Ltd., Tasmania 7055, Australia
[c] Electric Power Research Institute, China Southern Power Grid, Guangzhou 510623, China

ARTICLE INFO

Keywords:
Lightning
Downward leader
Stochastic model
Charge simulation method
Electric field
Fractal dimension

ABSTRACT

In this paper, a three-dimensional, stochastic model of a lightning downward leader is proposed. The background electric field in which the downward leader propagates is used to determine the direction of propagation. The critical background electric field for the development of a downward leader was obtained by extrapolating the results of negative long air gap discharge experiments. The distribution of charge in the downward leader was modelled with the charge simulation method. The calculation took into account branching of the leader channel. By choosing a suitable value of the probability exponent, the simulation results of fractal dimension and charge in the downward leader are shown to closely match observational data of lightning. Finally, the model was used to reproduce a negative ground flash. The ground electric field calculated by the model was found to agree well with measurement results from the field.

1. Introduction

A valid lightning shielding analysis model is the major method for assessing the performance of lightning protection devices. Importantly, the model of the downward leader is a significant component of the analysis.

In many previous and current models of the downward leader [1,2], the leader is considered to develop vertically without branching. Under this consideration, the model cannot reflect the random development of the downward leader. This major omission means the model is unable to explain lower-probability lightning events, many of which are responsible for bypasses (shielding failures) that cause damages to assets.

Upon the development of fractal theory [3–5], a quantitative means of describing the stochastic nature of a downward leader was achieved and a variety of such models of the downward leader were proposed [6–16] However, in most former stochastic models of downward leader, the fractal dimension was the only parameter used to prove their validity. However, in practice, the charge distribution in the downward leader and the electric field it produces as it approaches the ground are also key parameters that play an important role in the lightning attachment process.

In order to establish a three-dimensional stochastic model of a downward leader whose simulation results can coincide with the observed results in terms of fractal dimension, charge distribution of the channel and electric field, the following studies have been carried out:

(i) A proposal is made for the development mode of the downward leader.
(ii) A series of experiments using negative discharges in long air gaps have been carried out to determine the critical background electric field for the development of the downward leader.
(iii) The charge simulation method was used to study the distribution characteristics of charges in the downward leader.
(iv) A suitable branching factor and probability exponent was chosen according to the fractal dimension and the total charge of the downward leader.
(v) The background electric field expected from the approach of a downward leader was reproduced using the model.

2. Development mode of the downward leader

The development mode of the downward leader is based on a fractal model [8]. The development of negative downward leaders takes place in steps [17], so the downward leader channel develops as a "stepped leader". In this paper, the tip of each leader step is defined as a "developed point", as shown in Fig. 1.

For each developed point, N "candidate points" are dispersed on a spherical surface, in which the developed point is its center and its

* Corresponding author.
E-mail address: sj-xie@163.com (S. Xie).

https://doi.org/10.1016/j.epsr.2018.05.023
Received 5 November 2017; Received in revised form 8 April 2018; Accepted 22 May 2018

Nomenclature

C	Branching factor
CSM	Charge simulation method
D	Fractal dimension
E_d	Internal electric field of the downward leader
E_r	Internal electric field of the return stroke channel
E_s	Critical electric field for negative streamers
E_{th}	Critical background electric field for the development of the downward leader
FEM	Finite element method
FD	Finite difference method
H	Gap length using in the long air gap experiment
H_c	Height of the thundercloud
I_p	Peak value of the stroke current
L	Length of step leader
L_{max}	Maximum length of step leaders
Q_{all}	Injected charge in the first 100 μs of the first return stroke current
Q_i	Positive induced charge in the return stroke channel when the first return stroke ends
Q_n	Negative charge in the downward leader at beginning of the first return stroke
Q_p	Point charge at the leader tip
R_{step}	Step length of negative downward leader
U_{50}	50% flashover voltage
U_c	Electric potential of the thundercloud
η	Probability exponent

radius is the step length R_{step}. Since the step length of the downward leader in natural lightning ranges from 3 m to about 200 m [18,19], a relative small value of 5 m was used in the simulations in Sections 5 and 6 to fully simulate the randomness of the downward leader. The probability of the j'th candidate point of the i'th developed point turning into the next developed point can be expressed as:

$$P_{i-j} = \frac{(\Delta E_{i-j})^{\eta}}{\sum_{i}^{N_d}\sum_{j}^{N_j}(\Delta E_{i-j})^{\eta}} \quad \text{and} \quad \begin{cases} \Delta E_{i-j} = E_{i-j} - E_{th} & for\ E_{i-j} \geq E_{th} \\ \Delta E_{i-j} = 0 & for\ E_{i-j} < E_{th} \end{cases} \tag{1}$$

where E_{th} is the critical background electric field for the development of the downward leader, $E_{i\text{-}j}$ is the background electric field in the j'th candidate point of the i'th developed point, $\Delta E_{i\text{-}j}$ is the difference between $E_{i\text{-}j}$ and E_{th}, N_i is the number of candidate point of the i'th developed point, N_d is the number of developed point, and η is the probability exponent which could be any positive rational number.

After calculating the probabilities for each candidate point, a random number between 0 and 1 is produced by a random function (rand()/(double)(MAX_RAND)) of the C++ programming language to decide which candidate point becomes the next developed point. Taking the new developed point as the tip of the downward leader, a new stepped leader is formed. If the tip of the streamer zone in front of the newborn stepped leader has reached the ground, the development process is stopped. Otherwise, the iterations continue to find the next developed point.

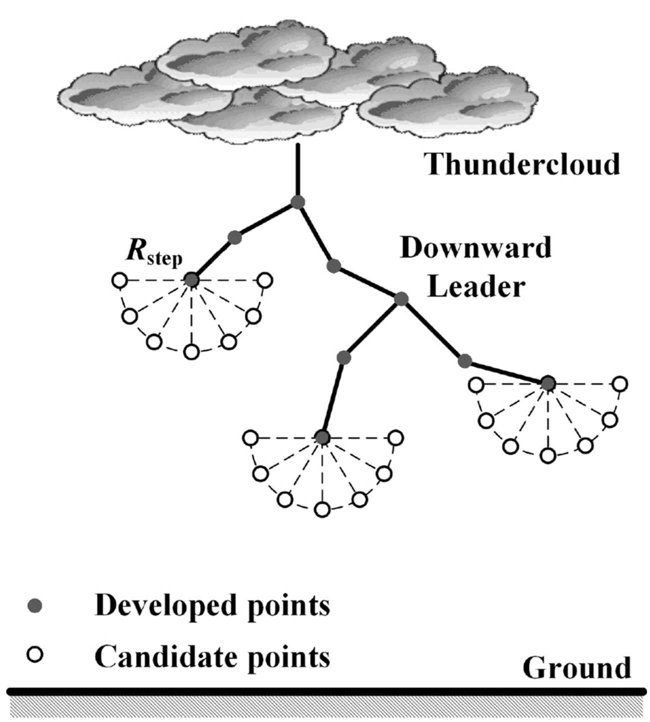

Fig. 1. Sketch of the development mode of the downward leader.

3. Critical background electric field for the development of the downward leader

According to Eq. (1), E_{th} is the minimum background electric field necessary for the development of the stepped leader. Unfortunately, it is almost impossible to obtain E_{th} directly from observational measurements of natural lightning due to the randomness of the downward leader.

However, negative leaders produced in laboratory tests with long air gaps display the same physical mechanism of stepping as natural lightning. Only the scales (step lengths, number of steps, etc.) are different. Therefore, it is reasonable to assume that E_{th} for natural lightning will be similar to the value obtained from laboratory tests on long negative discharges.

Based on the above considerations, a series of experiments using negative discharges in long air gaps were designed and carried out. In these experiments, a rod-plane gap of length H was used, where H was 4.0, 5.0, 6.0, 8.0 and 10.0 m. The rod electrode was cylindrical with a hemispherical tip, a length of 15 m and a diameter of 4.5 cm. Negative switching impulse voltages with a rise time of 80 μs, generated by a 7.5 MV impulse voltage generator, were applied to the rod electrode. The amplitude recorded as the applied voltage was the 50% flashover voltage (U_{50}) for each air gap length. These voltages are summarized in Table 1. For each air gap, enough impulses were applied to record at least 30 valid discharges.

A high-speed camera was used in framing mode to record the development processes of the negative discharges. Two static cameras (SCs) were positioned orthogonally and set in long exposure mode to record the last flashover channel. The applied voltages were measured with a capacitive divider that had a voltage ratio of 5358. The output was used to synchronize the high-speed camera.

A typical development process of the negative discharge across an air gap of 10 m is shown in Fig. 2, where three distinct steps of the negative leader can be observed. From these photos, the step length of each leader (L) can be estimated. The statistical results of measurements of L for different H values are shown in Table 2. For each value of L, 30 samples were used. These results indicate that L increases with

Table 1
50% flashover voltage values for different lengths of negative rod-plane discharges.

H (m)	4.0	5.0	6.0	8.0	10.0
U_{50} (MV)	2.07	2.38	2.59	2.94	3.32