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Formulations for the apparent and unbalanced power vectors in three-phase sinusoidal systems

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ABSTRACT

Buchholz's apparent power and its derived unbalanced power, determined from the quadratic difference between the apparent power and the positive-sequence apparent power, are formulated in this paper in vector notation for three-phase sinusoidal systems. The proposed unbalanced power vector holds three components that measure the imbalance effects caused by the active and reactive currents, and the voltage imbalance effects, respectively. The total apparent and unbalanced powers of several subsystems (sources or loads) can be respectively obtained as the norm of the vector sums of their apparent and unbalanced power vectors, according to these new formulations. The correctness of the apparent and unbalanced power vector formulations are verified in a three-phase sinusoidal installation formed by two linear loads supplied from a 400 kVA delta/wye distribution transformer, as an application example.

1. Introduction

The apparent power is of significant interest in power systems because it has numerous applications in electrical engineering ranging from the design of generators, transformers, and various other devices in electrical facilities to the determination of the correct operation of power systems, through the value of the power factor (PF).[1]

Steinmetz [1], a number of power approaches with different apparent power formulations have been established. In our opinion, the most important formulations for the three-phase power systems are the apparent power formulations established in references [2–10,14–16]. However, the apparent power is expressed as a scalar quantity in these approaches. As the apparent power is not a conservative but a formal quantity, the total apparent power of two or more subsystems (loads or sources) in a power system cannot be generally obtained by the addition of the apparent powers of these subsystems.

Complex power is previously known and has been applied in the theory of circuits to calculate the apparent, active, and reactive powers in sinusoidal single-phase and three-phase balanced circuits long since. This quantity is expressed as a complex notation and represented in the Gauss plane; its module is the well-known Steinmetz's apparent power and its components are the active and reactive powers.

A remarkable property of the complex power is that the total complex power of several single-phase or three-phase balanced subsystems (loads or sources) can be determined by the sum of the complex powers corresponding to each.

The complex power is used in certain approaches such as the currents' physical components (CPC) [9,10] for determining certain apparent power components in three-phase systems.

In our opinion, a formulation for the apparent power in vector notation is required for simplifying the determination of the apparent powers and their components in three-phase power systems with distributed loads, at least. Thus, in this paper, an apparent power formulation in vector notation has been established for three-phase sinusoidal systems.

The norm (S) of the apparent power vector ($\bar{S}$) should be,

$$S = \sqrt{S_+^2 + S_u^2} \tag{1}$$

where S_+ is the fundamental-frequency positive-sequence apparent power and S_u is the unbalanced power. An expression similar to (1) is included in the IEEE Standard 1459-2010 [16] but it uses the effective apparent power (S_e). In our opinion, the apparent power formulation used in (1) should be the well-known Buchholz's apparent power [2],

$$S = \sqrt{\left(V_1^2 + V_2^2 + V_3^2\right) \cdot \left(I_1^2 + I_2^2 + I_3^2\right)} \tag{2}$$

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[1] PF, power factor; CPC, currents' physical components; PCC, point of common coupling.

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because this expression leads to formulations for the apparent and unbalanced powers in vector notation that provide a consistent explanation of the power phenomena in three-phase sinusoidal systems, as demonstrated in this paper.

In order to satisfy (1), the apparent power vector ($\bar{S}$) proposed in this paper should be the vector sum of the fundamental positive-sequence complex power ($\bar{S}_+$) and the unbalanced power expressed in vector notation ($\bar{S}_u$). The first apparent power vector component ($\bar{S}_+$) is well-known in three-phase systems as,

$$\bar{S}_+ = 3\,\bar{V}_+\bar{I}_+^* = P_+ + \bar{Q}_+ \tag{3}$$

where $\bar{V}_+$ and $\bar{I}_+^*$ are the CRMS of the fundamental positive-sequence voltage and the conjugate current, respectively, in the part of the system where the apparent power is calculated; P_+ and $\bar{Q}_+ = \pm jQ_+$ are the fundamental positive active and reactive powers, respectively, which are represented in a Gauss plane. The unbalanced power, S_u, was formulated in vector notation in an earlier paper [12] and designated as a complex unbalanced power ($\bar{A}_u$); however, the application of $\bar{A}_u$ is restricted to systems with balanced voltages. More recently, a general formulation of the unbalanced power vector, ($\bar{S}_u$), applicable to power systems with balanced and unbalanced voltages was used in reference [13] for analysing the operation of a 4WD service, although the unbalanced power vector formulation was not justified in [13]. Therefore, a proposed general formulation for the unbalanced power in vector notation has been established in Section 2 of this paper using Buchholz's apparent power formulation. The proposed unbalanced power vector ($\bar{S}_u$) is represented in a three-dimensional space defined by three unitary orthogonal vectors ($\bar{p}$, $\bar{q}$, $\bar{z}$) and its components in the space directions ($\bar{S}_{uip}, \bar{S}_{uiq}, \bar{S}_{uv}$) are also expressed in Section 2. $\bar{S}_{uip}$ and $\bar{S}_{uiq}$ measure the effects of the active and reactive current imbalances, whereas, $\bar{S}_{uv}$ quantifies the effects of the voltage imbalances.

Once the proposed unbalanced power vector has been expressed, a formulation for the apparent power vector ($\bar{S}$) is proposed in Section 3 as a function of the fundamental positive complex power ($\bar{S}_+$) and the proposed unbalanced power vector ($\bar{S}_u$).

In Section 4, the power vector formulations established in Sections 2 and 3 are applied for formulating the total apparent and unbalanced power vectors of several subsystems (loads and sources) in sinusoidal power systems. The apparent and unbalanced power vector formulations described in Section 4 are verified through an application example in Section 5 in which the combined apparent and unbalanced powers of a three-phase installation formed by two resistive, unbalanced, and linear loads supplied by a 400 kVA delta/wye distribution transformer are calculated under two conditions: balanced and unbalanced source voltages. Finally, the conclusions are presented in Section 6.

2. Proposed unbalanced power vector and its components

Let us consider a part of a sinusoidal and unbalanced three-phase system, where the set of the CRMS phase voltages and currents are given by ($\bar{V}_1, \bar{V}_2, \bar{V}_3$) and ($\bar{I}_1, \bar{I}_2, \bar{I}_3$), respectively and the symmetrical components are ($\bar{V}_+, \bar{V}_-, \bar{V}_0$) and ($\bar{I}_+, \bar{I}_-, \bar{I}_0$), respectively, defined by Fortescue's theorem [11]. According to (1), the norm of our proposed unbalanced power vector ($\bar{S}_u$) in this part can be expressed as:

$$S_u = ||\bar{S}_u|| = \sqrt{S^2 - S_+^2} \tag{4}$$

where S_+ is the module of the complex fundamental positive apparent power expressed by (3) and S is the Buchholz's fundamental-frequency apparent power expressed as a function of the RMS symmetrical components of the voltages and currents

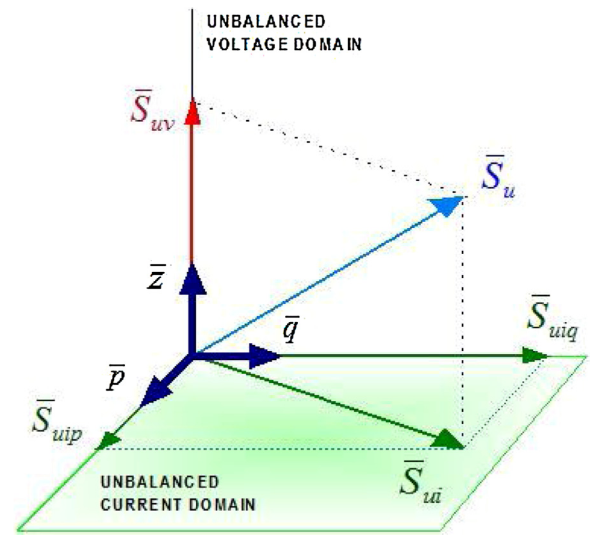

Fig. 1. Unbalanced power vector and its components.

(positive, +, negative, −, and zero-sequence, 0), according to Fortescue's [11]:

$$S = 3\sqrt{(V_+^2 + V_-^2 + V_0^2)\cdot(I_+^2 + I_-^2 + I_0^2)} \tag{5}$$

The last apparent power formulation provides the same results as those obtained through the previously known Buchholz's apparent power (2).

The square of the unbalanced power, from (3) to (5), satisfies:

$$\begin{aligned} S_u^2 &= S^2 - S_+^2 = \\ &= 3^2\cdot\left(V_+^2 + V_-^2 + V_0^2\right)\cdot\left(I_+^2 + I_-^2 + I_0^2\right) - 3^2V_+^2I_+^2 = \\ &= 3^2\cdot\left(V_+^2 + V_-^2 + V_0^2\right)\cdot\left(I_-^2 + I_0^2\right) + 3^2\cdot(V_-^2 + V_0^2)\cdot I_+^2 = \\ &= \left(1 + \delta_U^2 + \delta_A^2\right)\cdot 3^2V_+^2\cdot\left(I_-^2 + I_0^2\right) + (\delta_U^2 + \delta_A^2)\cdot 3^2V_+^2I_+^2 = \\ &= S_{ui}^2 + S_{uv}^2 \end{aligned} \tag{6}$$

where

$$\delta_U = \frac{V_-}{V_+} \qquad \delta_A = \frac{V_0}{V_+} \tag{7}$$

are the unbalanced and asymmetrical degrees of the phase voltages, respectively.

It is observed from (6) that the square of the unbalanced power (S_u) is the quadratic sum of the two quantities, S_{ui} and S_{uv},

$$\begin{aligned} S_{ui} &= \left(1 + \delta_U^2 + \delta_A^2\right)^{1/2}\cdot 3V_+\sqrt{I_-^2 + I_0^2} \\ S_{uv} &= (\delta_U^2 + \delta_A^2)^{1/2}\cdot 3V_+I_+ = (\delta_U^2 + \delta_A^2)^{1/2}\cdot S_+ \end{aligned} \tag{8}$$

As the scalar product of S_{ui} and S_{uv} is zero, they are orthogonal quantities in the space of the unbalanced power (Fig. 1); therefore, they are the components of our proposed unbalanced power vector ($\bar{S}_u$). The component, S_{ui}, becomes zero when the phase currents are balanced; thus, it measures the effects of the current imbalances (unbalanced current domain). The other component, S_{uv}, is nil when the phase voltages are balanced; hence, it determines the exclusive effects of the voltage imbalances (unbalanced voltage domain).

2.1. Expressions for the $\bar{S}_{ui}$ component of the unbalanced power vector

The $\bar{S}_{ui}$ component of the unbalanced power vector has to measure the effects of the current imbalances, i.e., those owing to the load imbalances and current imbalances caused by the unbalanced