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Time domain generalized telegrapher's equations for the electromagnetic field coupling to finite-length wires buried in a lossy half-space

Dragan Poljak [a], Silvestar Šesnić [a,*], Khalil El Khamlichi Drissi [b], Kamal Kerroum [b], Sergey Tkachenko [c]

[a] Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, University of Split, Ruđera Boškovića 32, 21000 Split, Croatia
[b] Blaise Pascal University, Clermont-Ferrand, France
[c] Institute for Fundamental Electrical Engineering and EMC, Otto-von-Guericke University Magdeburg, Magdeburg, D-39106, Germany

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ABSTRACT

The paper deals with a derivation of a time domain variant of the generalized telegrapher's equations for transient electromagnetic field coupling to buried wires. The formulation is based on the thin wire antenna theory in the time domain. The influence of a lossy half-space is taken into account by means of the time dependent reflection coefficient approximation arising from the modified image theory (MIT), while the per-unit-length surface impedance accounts for the conductor losses. The obtained space-time integral expressions are handled analytically. The concept of the scattered voltage is naturally included in the full model formulation. Computational examples are given for the induced currents along the buried wire and compared to the results obtained by using the transmission line (TL) approximation.

1. Introduction

The electromagnetic field coupling to buried wires is of appreciable interest in many electromagnetic compatibility (EMC) applications, such as the analysis of power and communications cables, submarine dipoles, geophysical probes, grounding systems, ground penetrating radars (GPR), etc. (e.g., Refs. [1–16]). Buried wires, subjected to transient external electromagnetic fields inducing current along the wire and generating scattered fields are usually analysed by means of the transmission line (TL) approach in either frequency or time domain, e.g. Refs. [1,12].

An important feature of the TL approximation is its formulation simplicity and relatively low computational cost when compared to the antenna theory approach. However, TL models suffer from some serious limitations, as the wire length is required to be significantly larger than the wire cross section, separation of wires, and the height above a lossy ground, or a burial depth, respectively. In some cases, when there are no losses and radiation resistance included in the model, the current grows to infinity at resonant points [1].

Therefore, the TL approximation basically fails to provide a complete solution if the wavelengths existing in the spectrum of the transient electromagnetic field exciting a transmission line are comparable to or less than the transverse electrical dimensions of the conductor.

On the other hand, antenna theory (or full-wave)-based approaches to the analysis of finite length lines below a lossy ground, account for the radiation effects [1–10,12,14].

The principal drawbacks of the wire antenna theory applied to buried wires of finite length are the complexity of formulation and the relatively high computational cost if longer wires are analysed [14], particularly if the analysis is carried out directly in the time domain [15].

There are a number of papers improving standard TL models to overcome their limitations to a certain extent. This includes analysis in the frequency domain [17,18] and in the time domain [19]. Some recent work on the subject can be found in Refs. [20–23].

* Corresponding author.
E-mail addresses: dpoljak@fesb.hr (D. Poljak), ssesnic@fesb.hr (S. Šesnić), drissi@univ-bpclermont.fr (K. El Khamlichi Drissi), kamal.kerroum@lasmea.univ-bpclermont.fr (K. Kerroum), sergey.tkachenko@etechnik.uni-magdeburg.de (S. Tkachenko).

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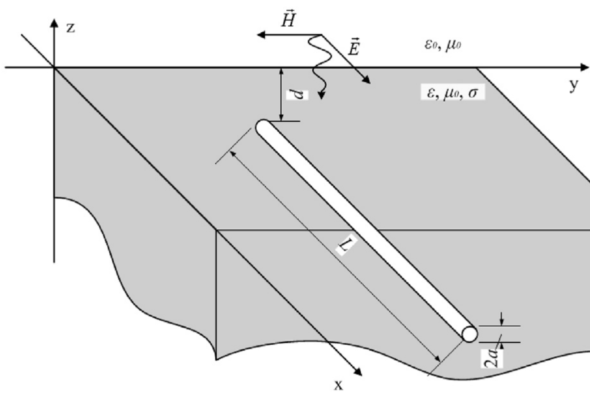

Fig. 1. Finite length wire below a lossy ground.

Furthermore, a theoretical relationship between the standard telegrapher's equations and integral equations arising from the wire antenna theory including the effect of a lossy ground has been studied in the last decade in a few papers by Poljak et al. [13,14,24]. Thus, frequency domain studies of overhead and buried wires of finite length, including the derivation of the corresponding generalized telegrapher's equations, have been reported in Refs. [13] and [14], respectively. Finally, a direct time domain analysis of transient excitation of finite length wires above a lossy ground using the concept of the time domain generalized telegrapher's equations is given in Ref. [20].

The present work deals with a theoretical relationship between the rigorous antenna theory approach to the analysis of finite length wires buried in a lossy ground based on the time domain Pocklington integral equation formulation and the standard time domain TL approach, thus completing the research previously carried out in Refs. [13,14,24]. The influence of a lossy ground is taken into account via the space-time reflection coefficient arising from the modified image theory (MIT) appearing within the integral equation kernel function. The conductor losses are taken into account via the per-unit-length surface impedance [1]. The generalized telegrapher's equations for buried wires derived directly in the time domain is of both theoretical and practical interest in lightning electromagnetics and particularly in the transient analysis of grounding systems. Namely, the scattered voltage is naturally included in the formulation which is not the case in standard antenna theory. This concept enables one to determine important parameters of grounding systems, such as transient voltage at the feeding point, step-voltage at the interface of the air-earth surface or input impedance. Thus, in addition to providing a correlation between the antenna theory and transmission line approach at the level of mathematical formulation of the finite length buried wire problems, the paper significantly extends the analysis carried out in the frequency domain (reported in Ref. [14]) to the time domain.

The related time domain integro-differential expressions are solved analytically using the method reported elsewhere, e.g. in Ref. [25]. The corresponding transmission line equations are treated using the Finite Difference Time Domain (FDTD) Method [1].

Finally, some illustrative computational examples pertaining to a straight buried wire are given in this work.

2. Time domain formulation

A single wire transmission line of a finite length L and radius a, buried at depth d inside a lossy medium, see Fig. 1, is considered.

The wire is subjected to transient fields from a distant source transmitted through a lossy medium, inducing a current along the line.

2.1. Continuity conditions at the buried wire surface

The governing integro-differential expressions for the transient current induced along the buried wire and related scattered voltage are derived starting from the interface condition for the tangential components of the electric field.

If lossy conductors are considered, the tangential component of the electric field along the wire differs from zero and in the Laplace domain the tangential component of the total field at the conductor surface is equal to the product of the unknown wire current $I(x,s)$ and per-unit-length internal impedance $Z_s(x,s)$ of the wire, i.e.

$$E_x^{exc}(x,s) + E_x^{sct}(x,s) = Z_s(x,s)\, I(x,s) \tag{1}$$

where $E_x{}^{exc}$ and $E_x{}^{sct}$ is the excitation and scattered component, respectively, while s denotes the Laplace variable. The details of the surface internal impedance $Z_s(x)$ of a thin wire can be found elsewhere, e.g. in Ref. [1].

The convolution operator applied to Eq. (1) yields

$$E_x^{exc}(x,t) + E_x^{sct}(x,t) = \int_0^t z_s(x,\tau)\, i(x, t-\tau)\, d\tau \tag{2}$$

where $z_s(x,t)$ denotes the inverse Laplace transform of $Z_s(x,s)$, $i(x,t)$ is the space-time current along the buried conductor.

2.2. Generalized telegrapher's equations for buried wires

There are several advantages in using generalized telegrapher's equations to analyse straight buried wires instead of solving the corresponding Pocklington's equation, such as; a clear correlation of the antenna theory (AT) model with TL formulation and a direct inclusion of scattered voltage into the formulation.

Derivation of telegrapher's type equations for the buried wire requires the scattered voltage to be included in the formulation which raises a conceptual difficulty in handling the half-space problem arising from the definition of the line voltage [1]. This matter has also been discussed in detail in Refs. [13,14,24]. The tangential scattered field component due to the current induced along the wire can be expressed by means of the axial component of the vector potential A_x and the scalar potential φ [25].

$$E_x^{sct}(x,t) = -\frac{\partial A_x(x,t)}{\partial t} - \frac{\partial \varphi(x,t)}{\partial x}, \tag{3}$$

where the space-time vector potential is given by Ref. [26].

$$\begin{aligned} A_x(x,t) = {} & \frac{\mu}{4\pi}\int_0^L i\left(x', t - R/v\right) \frac{e^{-\frac{t}{\tau_g}\frac{R}{v}}}{R} dx' \\ & - \frac{\mu}{4\pi}\int_{-\infty}^{t}\int_0^L \Gamma_{ref}^{MIT}(\tau)\, i\left(x', t - R^*/v - \tau\right) \frac{e^{-\frac{t}{\tau_g}\frac{R^*}{v}}}{R^*} dx' d\tau, \\ \tau_g = {} & \frac{2\varepsilon}{\sigma},\; v = \frac{1}{\sqrt{\mu\varepsilon}} \end{aligned} \tag{4}$$

where R is the distance from the source point to the observation point, both located at the buried wire, while R^* is the distance from the source point located at the image wire in the air to the observation point located at the wire immersed in a lossy medium, τ_g is the time constant and v is the propagation velocity in the lossy medium.