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Effects of binary variables in mixed integer linear programming based unit commitment in large-scale electricity markets

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ABSTRACT

Mixed integer linear programming is one of the main approaches used to solve unit commitment problems. Due to the computational complexity of unit commitment problems, several researches remark the benefits of using less binary variables or relaxing them for the branch-and-cut algorithm. However, integrality constraints relaxation seems to be case dependent because there are many instances where applying it may not improve the computational burden. In addition, there is a lack of extensive numerical experiments evaluating the effects of the relaxation of binary variables in mixed integer linear programming based unit commitment. Therefore, the primary purpose of this work is to analyze the effects of binary variables and compare different relaxations, supported by extensive computational experiments. To accomplish this objective, two power systems are used for the numerical tests: the IEEE118 test system and a very large scale real system. The results suggest that a direct link between the relaxation of binary variables and computational burden cannot be easily assured in the general case. Therefore, relaxing binary variables should not be used as a general rule-of-practice to improve computational burden, at least, until each particular model is tested under different load scenarios and formulations to quantify the final effects of binary variables on the specific UC implementation. The secondary aim of this work is to give some preliminary insight into the reasons that could be supporting the binary relaxation in some UC instances.

1. Introduction

The unit commitment (UC) calculation is extensively used in day-ahead power system operation planning and comprises the pre-dispatch of power generation units to satisfy the electricity demand and unit operation constraints. It is an exercise of large-scale, time-varying, non-convex, mixed-integer modeling and optimization. The average size of the UC problem for large electricity market applications is commonly conformed by hundred thousands of variables and constraints [1]. The size of this challenging problem is one of the primary motivations to develop different strategies to improve the algorithm's performance. Not only regarding computational times but also, to obtain more economy efficient global solutions.

Currently, the mathematical optimization technique known as Mixed-Integer Linear Programming (MILP, Branch-and-Cut and Heuristics based algorithms) is one of the main solvers applied to UC problems. MILP applied to the UC problem is not new [2–4]; however, it recently becomes popular due to new advances [5,6] that permit solving problems for the sizes of electricity markets and allow to develop more complex formulations. Different current day-ahead electricity market clearing applications support its applicability, for example [7–9]. MILP model flexibility and accuracy, as well as state-of-the-art algorithms, have evolved over the years to become robust and effectively enough to fulfill the electricity market necessities, overpassing the performance of Lagrange Relaxation (LR) based applications, and in some cases replacing them, as the most popular algorithms used in the past.

Some works [10,11] have compared the computational behavior between the MILP and LR, UC based algorithms. The general conclusion extracted is that the LR algorithm has a linear computational behavior and the MILP algorithm has an exponential behavior with the increase of the UC size. Based on this exponential behavior, the number of binary variables in a MILP based UC is often used as an indicator of the computational difficulty in several works [12–14]. This intuitive assumption, probably not based

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on exhaustive numerical observation, gives reasonable room to the suggestion of using fewer or relaxing some of the binary variables to obtain the consequent improvement in computational performance.

In the general case, the Branch-and-Cut (B&C) based algorithms discard large sections of the tree of potential solutions from further examination. This is the main difference between B&C algorithms and exhaustive enumeration. The B&C algorithms detect infeasible solutions or out-of-bounds solutions and sequentially prune the branches that form the tree of potential solutions. In this process, the solution space is gradually reduced until reaching the convergence criteria. This criterion is usually a time limit or a gap distance between the lower and upper bounds. These bounds are defined by the last best integer solution and its relaxation (i.e., the linear program solution). The solution obtained after convergence is always a feasible solution. This fact might be the most remarkable feature of B&C based algorithms, compared to LR based ones.

Nevertheless, modern B&C solvers (CPLEX,GUROBI) implement pre-solver heuristics mainly taking advantage of fixing variables, tightening bounds of variables, guessing preliminary values of variables, eliminating redundant rows and columns of the constraints matrix, changing the sense of constraints, reducing symmetry. The main purpose of the pre-solver heuristics is to find fast a feasible solution of the problem (integer solution) before starting the regular B&C process. The heuristics of the pre-solver eventually facilitate the problem resolution. However, it can be experimentally observed that relaxing binary variables to achieve better computational performance, produces the risk of losing the benefits of the pre-solver strength. Therefore, this binary-relaxation rule-of-thumb method should not be considered as a general rule-of-practice, because it may not take the advantages of the pre-solver. In consequence, the number of binary variables might become a poor indicator of the computational difficulty when solving a MILP based realistic UC [15].

The commercial success of solvers like CPLEX or GUROBI, is mainly due to their robustness and fast performance. This solver evolution has produced that many market operators become to develop their own market clearing tools; therefore, nowadays they focus their efforts in modeling and not in algorithms. Consequently, it is really important to evaluate different model performances under different circumstances like the number of binary variables used, always thinking on real life applications.

Although there are many current works [16–19] focused on model tightness and “good” constraints, the focus of our work is in the number of binary variables (and in the relaxation of the integrality constraints that bind them) utilized in the UC model.

In this regard, the primary objective of this work is to evaluate through extensive numerical simulations the effects of relaxing the binary variables when solving a MILP based UC for a real large-scale electricity market. The secondary aim of this work is to give some preliminary insight on the reasons that could be supporting the binary relaxation in some UC instances.

The work is organized as follow: Section 3 formulates the UC model used for the purpose of this work. Section 5 presents the numerical results and the discussion about them; Finally, Section 7 resumes the main conclusions of this work.

2. Nomenclature

Sets

T, G	Time and unit sets.
t, g	Hour and unit index set.
B	Piece-wise block set.
b	Power block index set.
n, N	Index and set for the multi-steps start-up costs.

Variables

Cp, Cs	Unit's production and start-up costs.
δ_{bgt}	Unit's piece-wise block variable.
p_{gt}	Unit's active power variable.
r_{gt}	Unit's reserve variable.
u_{gt}	Unit's state binary variable.
j_{bgt}	Unit's power block binary variable.
s_{gt}, h_{gt}	Unit's start-up/shut-down binary variables.
z^*	Objective value for feasible solution.
$z_{(u,\ldots)}$	Simulation instances for different binary variables.

Parameters

D_t	Hourly system demand.
R	Reserve requirement.
τ	Unit's number of steps for start-up costs.
$K_{g\tau}$	Unit's start-up cost for step τ.
K_g	Unit's constant start-up cost.
c_g	Unit's generation fix cost.
F_{bg}	Unit's power block slope.
Tr_{bg}	Unit's min-max power block limits.
UT_g, DT_g	Unit's min on/off service times.
T_g^{on}, T_g^{off}	Unit's initial on/off hours of service.
$\underline{P}_g$, $\bar{P}_g$	Unit's min-max power limits.
RU_g, RD_g	Unit's up-down ramp limits.

3. Unit commitment model

For real market applications, the traditional UC problem [12,10,20] is formulated as a cost minimization function subject to system, units and a representative set of some critical network constraints. In general, the minimization function considers the generation costs, including production costs, start-up costs, and no-load costs.

Currently, many works present different UC formulations with improved computational efficiency [22–24]. In [22] the authors propose a projection technique to simplify some constraints of the UC, allowing the solvers to exploit it. The effectiveness of the methodology is demonstrated through a set of randomly generated UC cases, with sizes ranging from 10 to 200 units over a 24 hours scheduling horizon. In [23] the authors present a MILP model for an accurate representation of the main technical and operating characteristics of thermal generation units on a day-ahead market, incorporating non-convex production costs, time-dependent start-up costs, and inter-temporal constraints. The effectiveness of the model is demonstrated with an UC case conformed by 15 thermal units and 24 h scheduling horizon. In [24], the authors present two improved formulations for reducing the approximation errors produced by the traditional interpolation function for costs. The proposed optimal linear approximation and the optimal piecewise linear approximation are more precise and reduce the computational times. The effectiveness of the formulation is demonstrated with a set of 36 thermal units and up to 96 scheduling periods.

However, none of these publications present results for large-scale real power market applications [1]. The model used in this work is based on traditional models proposed in [12,10,20] which have been used effectively in the electricity industry. Network constraints were not included in the model in order to make the explanation simpler without losing generality since the main objective of this work is not altered by the inclusion of network constraints. The reason for this, depends on the mathematical nature of the network constraints; they can be linear or nonlinear relationships among continuous variables. These constraints do not affect the branching process of a MIP algorithm because this process exclusively depends on the binary variables.

Additionally, in market practices can be considered that the most important feature of “network constraints” is related to the addition of security constraints [21]. In general, for large scale real power systems applications, it is non-viable to directly consider the large set of likely contingencies; therefore, decomposition techniques are commonly utilized because they permit to separate the