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Application of direct collocation method in short-term line ampacity calculation

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ABSTRACT

Commonly for the calculation of line ampacity (rating) the steady-state heat balance equation is used. In this paper a novel method for the calculation of line ampacity is developed. This method is based on the definition of an optimal control problem and its solution represents the line ampacity in the desired time range. The solution of the defined optimal control problem is obtained by applying the direct collocation method. In this manner the short-term line ampacity is obtained by solving a nonlinear programming problem. Nowadays, many algorithms and hardware implementations are available for solving similar nonlinear programming problems. Thus, the usage of this method is suitable for simplifying the calculation of the short-term line ampacity in contemporary dynamical line rating systems. Finally, the developed is compared with existing methods for the short-term line ampacity calculation and the advantages and the disadvantages of each method are discussed. At the end of the paper for a 240/40 mm^2 aluminium steel-reinforced conductor, the methods are tested on several cases and the results compared. The effectiveness of each method is checked by simulating the conductor non-steady-state heat balance equation with the obtained results for the line ampacities. From the obtained results it is proven that the developed method effectively calculates the short-term line ampacity, and at the same time simplifies the calculation process. This paper is recommended for researchers focused in the field of dynamical rating systems.

1. Introduction

The line ampacity represents a limitation in magnitude and duration of the line current aiming at restricting the conductor temperature below the maximum allowable conductor temperature (T_{max}). From the aspect of calculation the term line ampacity could refer to the steady-state or short-term (dynamic) line ampacity.

The steady-state line ampacity represent a constant current which under an assumption of thermal equilibrium for a given set of climatic parameters and conductor characteristics would increase the conductor temperate to T_{max} [1]. This value is calculated from the steady-state heat balance equation (SSHBE).

The short-term line ampacity represent a constant current which under no thermal equilibrium, for the given time changes of climatic parameters and conductor characteristics, will increase the conductor temperature from an initial conductor temperature (T_{cin}) to T_{max} in a defined time interval.

In this paper a novel method for the short-term line ampacity calculation is presented. This method is based on defining an optimal control problem which is solved using the direct collocation method (described in Ref. [2]). Thus, the problem of the short-term line ampacity calculation is transformed into a nonlinear programming problem which can be solved by many current algorithms [3,4].

In contemporary systems basically two methods for the calculation of short-term line ampacity exists. The first one is based on iteratively solving the conductor non-steady-state heat balance equation (NSSHBE) with different values of line current until the maximum simulated conductor temperature in the defined time interval is equal to T_{max} [5–8]. Similarly, the second one is based on iteratively applying an analytical solution of the linearized NSSHBE [9–12].

Finally, on a typical aluminium-conductor steel-reinforced (ACSR) the values obtained by contemporary methods for the short-term line ampacity are compared with the developed method and

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with the value of the steady-state line ampacity. All the results are reversely checked by simulating the conductor NSSHBE with the obtained values of the line ampacity and the effectiveness of each method is discussed.

2. Conductor steady-state and non-steady-state heat balance equation

In order to understand the different methods and calculations made in this paper it is necessary to present the conductor steady-state and non-steady-state heat balance equation.

The steady-state line ampacity is calculated using the conductor SSHBE equation which in a simplified form could be defined as follows [1,13]:

$$I^2 \cdot R_{ac} + P_s = P_c + P_r \tag{1}$$

The components in Eq. (1) can be briefly explained as:

1) Joule heat gain per unit length ($I^2 \cdot R_{ac}$)

Conductors of overhead lines offer resistance to the passing alternating current. According to Joule's law, this is the main reason why they are heated. In practice, the conductor direct current (DC) resistance at 20 °C is given, which should be adjusted in terms of the conductor temperature, skin effect, transformer effect and core losses due to hysteresis and eddy currents (in case of ACSR) [14–16].

2) Solar heat gain per unit length (P_s)

The Sun emits radiation, which heats the overhead line conductor. Usually, this term is calculated from the data on the conductor diameter, global sun radiation intensity and conductor surface absorptivity coefficient [6].

3) Convective heat lost per unit length (P_c)

High temperature of the conductor surface heats the ambient air and reduces its density causing its rise if the wind speed is zero (or close to zero). On the other hand, the wind speed causes circulation of the air around the conductor. Thus, in both cases the cooler air replaces the warmer and cools the conductor. The convective heat lost is dependent on the ambient and conductor temperature, wind speed and angle, line height above sea level and other fixed parameters [17,18].

4) Radiative heat lost per unit length (P_r)

The radiative heat lost of the conductor is the total radiative energy transmitted from its surface. This element is calculated according to Stefan-Boltzmann law from the data on the ambient temperature, emissivity coefficient of conductor surface and conductor diameter [19,20].

The components of Eq. (1) are calculated by several different methodologies. The most commonly used methodologies are described in Refs. [1,20–22]. In this paper, all the calculation of components will be in accordance with the IEEE 738-2012 standard [1], with the exception of the calculation of the solar heat gained, which will be calculated by the following equation [21]:

$$P_s = D \cdot \alpha_s \cdot S \tag{2}$$

From Eq. (1) the steady-state line ampacity can be calculated according the following formula:

$$I_{dop} = \sqrt{\frac{P_c(T_{\max}, v, \delta, T_a) + P_r(T_{\max}, T_a) - P_s(S)}{R_{ac}(T_{\max})}} \tag{3}$$

When using Eq. (1) the balance between heat gained and lost by the conductor is assumed. When no heat balance is present the conductor non-steady thermal state arises. The non-steady heat balance state of a conductor, neglecting the radial and axial temperature distribution, is described by the following equation [1]:

$$m \cdot c_p \cdot \frac{dT_c}{dt} = I^2 \cdot R_{ac} + P_s - P_c - P_r \tag{4}$$

Eq. (4) represents a nonlinear differential equation which can be solved by using numerical methods such as Runge–Kutta method [5], Euler method [8], and other methods for solving nonlinear differential equations. All the components of Eq. (4) are calculated by the same methodologies as for the steady-state case, whereas $m \cdot c_p$ in case of ACSR conductors is calculated as follows [21]:

$$m \cdot c_p = m_a \cdot c_a + m_s \cdot c_s \tag{5}$$

In Eq. (5) the linear mass per unit length (m) is invariable up to 100 °C with the conductor temperature, while the specific heat capacity varies linearly according to the following equations:

$$c_a = c_{a20} \cdot \left(1 + \beta_a \cdot (T_c - 20)\right) \tag{6}$$

$$c_s = c_{s20} \cdot \left(1 + \beta_s \cdot (T_c - 20)\right) \tag{7}$$

3. Equivalent optimal control problem

From the optimal control theory point of view the problem of finding the current that taking into account Eq. (4) will drive the conductor temperature from T_{cin} to T_{max} in the time range from 0 to t_f (final time), with time variable or constant climatic parameters, can be formulated as an optimal control problem expressed as:

$$\min(J) = \min\left(\left(T_c(t_f) - T_{\max}\right)^2\right) \tag{8}$$

subjected to:

$$\frac{dT_C}{dt} = f(T_C(t), I(t), t) = \frac{1}{m \cdot c_p} \cdot \left(I^2(t) \cdot R_{ac}(T_C(t)) + P_s(t) - P_c(T_C(t), t) - P_r(T_C(t), t)\right)$$

$$T_c(0) = T_{cin}$$

$$I \geq 0$$

$$T_c(t) \leq T_{\max}$$

In Eq. (8) the objective function is formulated as the quadratic difference between the conductor temperature at final time ($T_c(t_f)$) and T_{max}. The imposed constraints on the optimal control problem are introduced from the differential Eq. (4), the initial value of conductor temperature, from the fact that the conductor current cannot be negative, and from the fact that the conductor temperature in any time interval cannot exceed T_{max}.

In the optimal control theory terms the conductor current can be considered as the control variable, and the conductor temperature as the state variable. The goal is to find the current I that will minimize the defined objective function and at the same time satisfy all the defined constraints. In the next section the principle of solving the problem defined by Eq. (8) with the application of direct collocation method will be shown.