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# Comparison of logistic functions for modeling wind turbine power curves

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ABSTRACT

In recent years logistic functions have been used to model wind turbine power curves. Generally speaking, it can be said that the results provided by the logistic functions are good enough to choose them over other options considering its continuity and adaptability. However, there are some logistic functions that have never been used to model wind turbine power curves although their use can be adequate. Comparing all logistic functions can help definitely to decide which are the best options.

In this paper, the most known logistic functions are presented and tested to model wind turbine power curves, included those already used. Moreover, a comparison is made among them, after which two logistic functions are eventually recommended and some other are definitively discarded.



## 1. Introduction

A wind turbine (WT) power curve relates the wind speed through the area swept by the blades of a WT and the electric power provided in their output terminals. Usually, manufacturers provide the power curve of a WT by means of a graph or as a set of pairs of points given as a table. That is an empirical approximation of the power curve, but it should be taken into account that some aspects as weather, terrain conditions and, mainly, aging, affect to the power curve. As times go by, aging changes the behaviour of the wind turbine inevitably. Referring to a new WT or to a not new one, for a long time there have been attempts to model the power curve by means of mathematical functions [1], in order to implement them in an application program or to derive more complex relationships. Formerly, linear [2], quadratic [3] or cubic [4] models were the options more usually chosen to model power curves. They can be generally considered as good options because they are very simple and it is very easy to work with them. However, the results they provide are far from a good approximation, at least, in the area of the rated wind speed. Other alternatives are the splines [5] which give very good results but include a lot of parameters and are piecewise defined, which is an obstacle when they are combined with other expressions in order to derive more complex ones. Lately, logistic functions have been used to model WT power curves because they provide good results, need few parameters and they are easy to deal with because they are continuous functions. The applicability of continuous functions is mainly to combine the power curve with other expressions, such as the wind speed distributions, in order to obtain combinations of them, such as wind power distributions.

A logistic function, also called 's' shape function or sigmoid function appears in models of population growth and spread of epidemic diseases, using four [6,7] or five parameters [8]. In these types of behaviours the function relates the size of a set with respect to the time. In all of them, the growth is exponential at the beginning, then some kind of competition appears among the members of the set so the growth decreases and finally the size of the set reaches its limit. At a glance, it does not seem to have anything to do with WT power curves, however the shape of the curve is exactly the same as in those cases. More in depth, both behaviours coincide, considering the wind speed equivalent to the time and also in the limit, which in the case of the WT is the rated power. In fact, some kind of analogy can be established between the population growth and the power curve, just considering that the wind speed is increased gradually to obtain the power curve, so it is equivalent to the time, and the size of the population represents the output power.

As it has been said, over the last years, some logistic functions have been successfully applied to model WT power curves, using powers [9–12], square roots [13], exponentials [14–16], and simplification of them [17–19]. However, there are other functions of the same type that have not been applied to that assignment as

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the Gompertz [20,21], the Bass [22], and others [21,23,24]. Therefore, checking if these functions are valid or not and comparing all of them seem to be necessary and this is the proposal of this paper. Besides, some functions, used or not, may be derived from other, so those curves derived from other are discarded in order to avoid duplications. And, as all logistic functions have some kind of relationship among them, each relationship is explained.

Another aspect to be taken into account regarding functions to model WT power curves is the number of parameters. This number can rise up to six in the case of logistic functions. However, when dealing with fewer than three, the adaptability of the function to model the WT power curve is almost null. Therefore, the number of parameters considered in this survey ranges from three to six.

In order to obtain the values of the parameters of the logistic functions, there are two types of processes: deterministic and optimization approaches. In the first case, the results obtained are worse than in the second one but the process to obtain the parameters is simpler: it consists of just performing some operations using the WT parameters. This is true in the case of three or four parameters [17] but in case of a higher number, the process becomes more complicated. Therefore, here and in order to compare all the logistic functions with the same indicators, optimization processes are used [9].

As it has been said, the comparison is made regardless the number of parameters of the logistic function, but a low number of them is preferred, so a weighted comparison is performed, too.

Therefore, the focus of this paper is to find a logistic function with the minimum number of parameters and the best performance when modeling WT power curves. As two objectives are pursued, there may be one or two logistic functions proposed, according to the intended use.

This paper is organized as follows: in Section 2, the logistic functions and related information to them are presented. In Section 3, the results of applying the logistic functions to several WTs are provided. Finally, Section 4 states the conclusions.

## 2. Logistic functions

As it was mentioned, logistic functions have been applied to model the WT power curves over the last years in an effective way because they are continuous functions and the errors involved are very low.

The functions are classified depending on the number of parameters, from three to six. Some of the functions are related among them, and the relationships are explained just after one of the functions involved.

In some cases there is available information about the technical meaning of the parameters and can be provided, but not always.

### 2.1. 3-Parameter logistic function

It is generally known that the most used 3-parameter logistic function is the 3PLE, shown in Eq. (1) [17].

$$P(v) = \frac{\alpha}{1 + \exp\left(-\beta \cdot (v - v_0)\right)} \tag{1}$$

where P is the output power, v is the wind speed, $\alpha$ is the curve's maximum value, $v_0$ is the value of the midpoint and $\beta$ is the slope of the curve. Notice that the term $\exp\left(-\beta \cdot (v - v_0)\right)$ can be converted into $Q \cdot \exp\left(-\beta \cdot v\right)$ where $Q = \exp\left(\beta \cdot v_0\right)$ providing a different expression for the same relationship. Eq. (1) has been used in Ref. [17] to model the WT power curve based on the parameters of the WT, as it can be seen in Eq. (2).

$$P(v) = \frac{P_r}{1 + \exp\left(4s\left(v_{ip} - v\right)/P_r\right)} \tag{2}$$

where $P_r$ is the rated power of the WT, $v_{ip}$ its wind speed inflection point and s is the slope at the inflection point. In fact, Eqs. (1) and (2) coincide when equalling $\alpha = P_r$, $v_0 = v_{ip}$ and $\beta = 4s/P_r$. The only difference is that in Eq. (2) the parameters can be obtained directly from the data provided by the manufacturer of the WT while in other case, they have to be calculated by an optimization procedure.

Mbamalu et al. introduced Eq. (3) in Ref. [13] saying it is a typical symmetrical "s" curve with horizontal asymptotes at both A and C, and proposing it as a hypothetical mathematical model for WT power curves.

$$P(v) = \frac{A \cdot v}{\sqrt{B - v^2}} + C \tag{3}$$

There is no relationship between Eqs. (1) and (3) but it seems to be a function with the same shape and it has been used to model a WT power curve.

It is also well known the Gompertz (GPTZ) function [20], which is shown in Eq. (4) and it was not applied to WT power curves, yes.

$$P(v) = D \cdot \exp(-G \cdot \exp(-K \cdot v)) \tag{4}$$

where D is an asymptote, $G > 0$ sets the displacement along the horizontal axis (translates the graph to the left or right) and $K > 0$ sets the vertical scaling. Besides, in Ref. [20], it is proposed some kind of equivalence between the parameters of the Gompertz function and the logistic function in Eq. (1): $D = \alpha$, $G = \exp\left(\beta \cdot v_0\right)$, $K = \beta$.

Another model is the Bass one [22], which can be seen in Eq. (5).

$$P(v) = S\frac{1 - \exp(-(P + Q)v)}{1 + \frac{P}{Q}\exp(-(P + Q)v)} \tag{5}$$

where S is the maximum value but the other two parameters have no specific meaning, at least when applied to WT power curves. This model has been used to know the degree of diffusion of wind power.

Finally, there is another 3-parameter model [15] with the shape shown in Eq. (6).

$$P(v) = \frac{k \cdot y_0 \cdot \exp(r \cdot v)}{k + y_0 \cdot \exp(r \cdot v)} \tag{6}$$

where r is the rate of increase, k is the maximum value and $y_0$ has no specific meaning.

Eq. (6) can be easily converted into Eq. (1) just making the following changes ($\alpha = k$, $\beta = r$ and $v_0 = \log\left(k/y_0\right)/r$).

### 2.2. 4-Parameter logistic function

The 4-parameter logistic function most applied to power curve modeling is the one shown in Eq. (7) [1] named here 4PLEE.

$$P(v) = L\frac{1 + m \cdot \exp\left(-v/\tau\right)}{1 + n \cdot \exp\left(-v/\tau\right)} \tag{7}$$

which can be related with the Bass one $\left(L = S, m = -1, n = P/Q, \tau = 1/(P + Q)\right)$ and with the 3PLE $\left(L = \alpha, m = 0, \tau = 1/\beta, n = \exp\left(v_0/\tau\right)\right)$.