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Markov chain modeling for very-short-term wind power forecasting

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ABSTRACT

A Wind power forecasting method based on the use of discrete time Markov chain models is developed starting from real wind power time series data. It allows to directly obtain in an easy way an estimate of the wind power distributions on a very short-term horizon, without requiring restrictive assumptions on wind power probability distribution. First and Second Order Markov Chain Model are analytically described. Finally, the application of the proposed method is illustrated with reference to a set of real data.

1. Introduction

Wind power forecasting methods can be divided into two main groups: physical and statistical methods [1,2]. The former [3–6] are based on physical considerations to provide estimates of future wind power output starting from meteorological predictions. The latter [7–10] consist of emulating the relationship between historical values of wind power, historical and forecasted values of meteorological variables and future wind power output, whose parameters have to be estimated from data, without making any assumption on the physics of the phenomenon under study.

Both approaches are used to provide wind power forecasts on very short-term (up to 30 min ahead), short-term (from 30 min up to 6 h ahead), medium-term (from 6 h to 1 day ahead) and long-term (from 1 day up to 1 week ahead) [11–13]. Very short-term forecasting models are usually statistically-based [1].

In the case of short or longer term forecasts, statistical methods need Numerical Weather Predictions to provide an acceptable forecast accuracy. On the contrary, for a very short-term, pure statistical methods, including the sole autoregressive part, exhibit good performances [2]. Combinations of physical and statistical approaches and combinations of different time-scale models (short-term and medium-term) are referred to as hybrid approach [12–17].

The main limitation of many of the abovementioned models consists in the fact that their use only enables to perform point forecast of the random variable of interest (i.e. the wind power generated in a future time), whereas they do not allow to formulate its probability distribution. In fact, decision making processes in electrical power systems management [18] and electricity market trading strategies [19,20], generally, require more information than a point forecast.

Models which allow formulating the probability distribution functions of the wind power are proposed in [21–28]. Unfortunately, also these models presents some limitations. Indeed, they adopt generalist methods, that are either too complex to be applied in practice or based on assumptions that are usually far to be verified in the application (e.g. residuals are independent and identically distributed Gaussian random variables). In addition, all these models are difficult to calibrate on the basis of the kind of data that are commonly available in practical settings.

The models proposed in this paper fall in the category of pure statistical methods. They have been formulated, starting from an initial idea presented in [29], on the basis of the Markov Chain (MC) theory, a kind of approach that have been already used in relevant literature for the generation of synthetic wind speed and wind power time series [30–33].

These Markov models are based on few non restrictive hypotheses and can be calibrated and applied on the basis of set of data that are usually available in practice. Indeed, only past values of wind power are required for their use. With respect to the models presented in [29], here, the First Order Markov Chain model (FOMC) is strongly reformulated while the Second Order Markov Chain Model (SOMC) is comprehensively formulated by introducing the concepts of auxiliary transition matrices and auxiliary state vec-

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tor probabilities which allow treating SOMC (or even higher order models) as an ordinary FOMC from a mathematical point of view. Moreover, interval prediction, which was not considered in [29], is addressed also discussing in major details point prediction both for SOMC and FOMC. Finally, a more extensive literature review is presented.

From the applicative point of view, the main characteristic of the proposed models is that they allow to estimate the probability distribution of wind power over future time horizon, deriving from it point forecasts and other figures. This gives to the analysts all the information they need to perform risk analyses and economic performances evaluation, which are required in electrical power systems management. Of course the proposed statistical model can be included in a hybrid model that uses numerical weather predictions.

In what follows, two models are presented, one based on the use of First Order Markov Chain, and the other on the use of Second Order Markov Chain. Then, the probability estimation procedure is described. In the last section, the application of the proposed method is briefly illustrated with reference to a case-study.

2. Proposed method

In order to formulate the proposed models, the time axis is divided into contiguous and equispaced intervals of length $\Delta t = 10$ min. Moreover, the state variable is discretized defining a finite set of (representative) values $\{s_1, s_2, \ldots, s_N\}$, where N is a calibration parameter. Finally, the average power generated by the wind farm over the time interval $[t_{h-1}, t_h]$, where $t_h = h \cdot \Delta t$, is considered as state variable of the process, $S_P(t_h)$. So stated, let $\left\{S_P(t_h), h = 0, 1, 2, \ldots\right\}$ denote a discrete time Markov Chain that describes the evolution of the state variable over the time.

In order to define the set $\{s_1, s_2, \ldots, s_N\}$ it is to consider that, very often wind farm output equals zero, because the individual turbines deliver no output outside the so-called cut-in and cut-out wind speed interval. Moreover, very frequently, the output equals the nominal wind farm power, P_n, because the turbines deliver their nominal power when the nominal wind speed is reached, and cut-out conditions do not apply.

For this reason, the minimum and maximum values, s_1 and s_N, of the state variable are set to 0 and P_n, respectively. The remaining values $s_2, s_3, \ldots, s_{N-1}$ are set to the centers of the N-2 classes of equal length defined on the interval$]0, P_n[$.

2.1. First Order Markov Chain

A FOMC satisfies the following equality:

$$\begin{aligned} &Pr\left\{S_P(t_{h+1}) = s_j \left| S_P(t_h) = s_{i_h}\right., S_P(t_{h-1}) = s_{i_{h-1}}, \ldots, S_P(t_1) = s_{i_1}\right\} \\ &= Pr\left\{S_P(t_{h+1}) = s_j \left| S_P(t_h) = s_{i_h}\right.\right\}, \\ &\text{for each} \quad j, i_1, i_2, \ldots, i_h \in \left\{1, \ldots, N\right\} \end{aligned} \tag{1}$$

Eq. (1) states that, in a FOMC, the probability that $S_P(t_{h+1})$ at t_{h+1} is s_j, given the state of the process at t_h, does not depend on the previous history of the process.

Hence, in order to completely define the process it is necessary to formulate the one-step transition matrix $\mathbf{P}(t_h)$, whose generic element, $p_{ij}(t_h)$, represents the probability that the state of process at t_{h+1} is s_j, given that the state at t_h is s_i:

$$p_{ij}(t_h) = Pr\left\{S_P(t_{h+1}) = s_j \left| S_P(t_h) = s_i\right.\right\}. \tag{2}$$

Since, in general, the evolution over the time of power generated from a wind farm cannot be modeled via an homogeneous Markov process (i.e. a process with a stationary transition matrix, $\mathbf{P}(t_h)$), it is necessary to define the one step transition matrix for each h.

	1	**2**	**…**	**N-1**	**N**
1	$\hat{p}_{1,1}$	$\hat{p}_{1,2}$	…	$\hat{p}_{1,N-1}$	$\hat{p}_{1,N}$
2	$\hat{p}_{2,1}$	$\hat{p}_{2,2}$	…	$\hat{p}_{2,N-1}$	$\hat{p}_{2,N}$
⋮	⋮	⋮	⋮	⋮	⋮
N-1	$\hat{p}_{N-1,1}$	$\hat{p}_{N-1,1}$		$\hat{p}_{N-1,N-1}$	$\hat{p}_{N-1,N}$
N	$\hat{p}_{N,1}$	$\hat{p}_{N,2}$	…	$\hat{p}_{N,N-1}$	$\hat{p}_{N,N}$

Fig. 1. First order one-step transition matrix.

In order to obtain an estimate, $\hat{\mathbf{P}}(t_h)$, of the transition matrix at time step t_h, the (most recent) data, collected in the time window, $[t_{h-\mathrm{ws}}, t_h]$, that slides with t_h, can be used, where the sliding window size, ws, is a calibration parameter.

In particular, an estimate for $p_{ij}(t_h)$ can be (easily) obtained as:

$$\hat{p}_{ij}(t_h) = \frac{n_{ij}(t_h)}{\sum\limits_{j} n_{ij}(t_h)} \quad \forall i, j, \quad \text{with} \quad \sum_{j=1}^{N} \hat{p}_{ij}(t_h) = 1 \quad \forall i, \tag{3}$$

where $n_{ij}(t_h)$ indicates the number of transitions from state s_i to state s_j observed in the sequence of wind power data contained in the sliding window $[t_{h-ws}, t_h]$. Estimates (3) are the maximum likelihood estimates of the transition probabilities [30].

If for a given i it is $n_{ij}(t_h) = 0\, \forall j = 1, 2, \ldots, N$, then it is assumed:

$$\hat{p}_{ij}(t_h) = \begin{cases} 1 & j = i, \\ 0 & \forall j \neq i. \end{cases} \tag{4}$$

Estimates of the transition probabilities at time t_{h+1} can be easily obtained updating those performed at time t_h, by means of recursive algorithms.

For N states, the first order transition matrix is an $N \times N$ matrix. According to the representation reported in Fig. 1, each row of the matrix corresponds to the current state of the process, while each column corresponds to one of the N possible states at next time step. The elements of each row of the matrix sum up to 1, since this sum corresponds to the probability of a transition from a current state to any possible state (i.e. $\mathbf{P}(t_h)$ is a stochastic matrix).

2.2. Second Order Markov Chain

For a SOMC it results:

$$\begin{aligned} &Pr\left\{S_P(t_{h+1}) = s_j \left| S_P(t_h) = s_{i_h}\right., S_P(t_{h-1}) = s_{i_{h-1}}, \ldots, S_P(t_1) = s_{i_1}\right\} \\ &= Pr\left\{S_P(t_{h+1}) = s_j \left| S_P(t_h) = s_{i_h}\right., S_P(t_{h-1}) = s_{i_{h-1}}\right\} \\ &\text{for each} \quad j, i_1, i_2, \ldots, i_h \in \left\{1, \ldots, N\right\} \end{aligned} \tag{5}$$

Eq. (5) states that, in a SOMC, the probability that the process is in the state s_j, at t_{h+1}, given the state of the process at t_h and t_{h-1} does not depend on the previous history.

This implies that a SOMC can be modeled as a FOMC introducing composite states $\left\{11, 12, \ldots, 1N, 21, \ldots, 2N, \ldots, N1, \ldots, NN\right\}$ [34]. Hence, in order to completely define the SOMC it is necessary to formulate the auxiliary one-step, $N^2 \times N^2$, transition matrix, $\mathbf{P}(t_h, t_{h-1})$, of this “auxiliary” FOMC, where the term “one-step” refers to the number of steps elapsed from the current epoch, t_h, to the subsequent epoch, t_{h+1}.