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Identification of ARARX models in presence of additive noise

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Abstract: The identification of dynamic processes can be performed by means of different classes of models relying on different stochastic environments to describe the misfit between the model and process observations. This paper introduces a new class of models by considering additive error terms on the observations of the input and output of ARARX models and proposes a three–step identification procedure for their identification. ARARX + noise models extend the traditional ARARX or ARMAX ones and can be seen as errors–in–variables models where both measurement errors and process disturbances are taken into account. The results of Monte Carlo simulations show the good performance of the proposed identification procedure.

Keywords: System identification, errors–in–variables models, ARARX models, linear systems

1. INTRODUCTION

The modeling of a dynamic process on the basis of observed sequences i.e. its identification, can rely on many families of possible models, describing different stochastic environments, as well as on different selection criteria within a specified class of models. The choice of model families and criteria is often based more on the planned use of the model then on the adherence of the associated stochastic contexts to real ones because real processes are in general more complex than the representations used for their description.

Equation error models constitute a very useful category of models because of their applicability in prediction and control (Söderström and Stoica, 1989; Ljung, 1999); the description of the misfit between model and observations only by means of an error term on the output is, however, restrictive.

Errors–in–Variables (EIV) models are a class of models based on the assumption that the process behind the data can be described by means of a linear model whose observations are corrupted by additive errors, see (Söderström, 2007) and the references therein. These models are often more realistic because all measures are considered as affected by errors.

This paper considers a new family of models that derives from the integration of EIV models and ARARX ones. Inside the class of equation error models, ARARX are very peculiar since they can be considered as an extension of ARX models and can approximate, at any desired degree, the family of ARMAX models (Guidorzi, 2003; Söderström and Stoica, 1989). This characteristic leads to the use of ARARX processes also in model reduction (Söderström et al., 1991; Tjärnström and Ljung, 2003).

ARARX + noise models consider additive error terms on the observations of the input and output of an ARARX model. In this way, it is possible to obtain representations that take into account both measurement errors and process disturbances. This feature is particularly suitable for fault detection and filtering purposes.

This paper proposes a three–step identification procedure for identifying ARARX + noise models. The first step concerns the identification of an auxiliary high–order ARX model and is based on the results reported in (Diversi et al., 2007). The second and third steps take advantage of the properties of polynomials with common factors and consist in simple least–squares algorithms. The proposed method has been tested by means of Monte Carlo simulations and compared with an instrumental variable approach.

The organization of the paper is as follows. Section 2 contains a description of the considered stochastic context and the statement of the identification problem. Section 3 describes the steps to be performed in the identification procedure. Section 4 concerns the identification of an auxiliary high–order ARX model while the complete ARARX + noise identification procedure is described in Section 5. In Section 6 the ARARX + noise identification problem is solved by using an instrumental variable approach. Section 7 reports some numerical results while short concluding remarks are finally given in Section 8.

2. CONTEXT AND STATEMENT OF THE PROBLEM

Consider a linear, single input single output, discrete time ARARX model described by the equation

$$A(q^{-1})\bar{y}(t) = B(q^{-1})u_0(t) + \frac{e(t)}{D(q^{-1})}, \tag{1}$$

where $A(q^{-1})$, $B(q^{-1})$ and $D(q^{-1})$ are polynomials in the backward shift operator q^{-1}

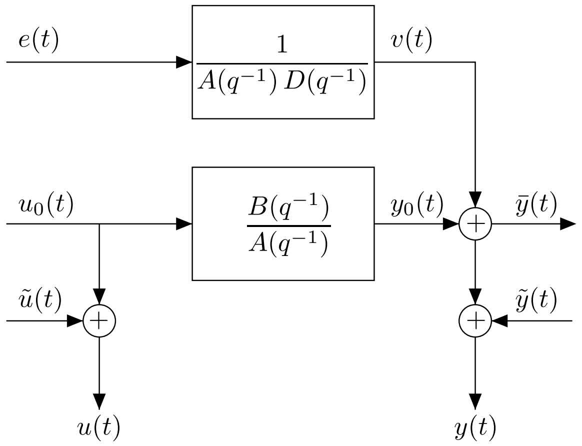

Fig. 1. ARARX model with noisy input and output.

$$A(q^{-1}) = 1 + a_1\, q^{-1} + \cdots + a_n\, q^{-n} \qquad (2)$$

$$B(q^{-1}) = b_0 + b_1\, q^{-1} + \cdots + b_n\, q^{-n} \qquad (3)$$

$$D(q^{-1}) = 1 + d_1\, q^{-1} + \cdots + d_{n_d}\, q^{-n_d}. \qquad (4)$$

The ARARX structure admits the following interpretations:

- An equation error model, with input $u_0(t)$ and output $\bar{y}(t)$,whose equation error is given by the autoregressive process $e(t)/D(q^{-1})$ (see (1)).
- A "true" system $B(q^{-1})/A(q^{-1})$, with input $u_0(t)$ and output $y_0(t)$, whose output is affected by the additive colored noise $v(t) = e(t)/(A(q^{-1})\,D(q^{-1}))$ (see Fig. 1). In this case, $\bar{y}(t)$ denotes the observed output: $\bar{y}(t) = y_0(t) + v(t)$.

Remark 1. Since a moving average process driven by a white noise $\eta(t) = C(q^{-1})\,e(t)$ can be approximated by an autoregessive process of suitable high order, $\eta(t) \approx \frac{1}{D(q^{-1})}\,e(t)$, $v(t)$ can be seen as an approximation of a generic ARMA model $\frac{C(q^{-1})}{A(q^{-1})}\,e(t)$. As a consequence, an ARARX model can approximate an ARMAX structure.

In this paper, we will assume that $u_0(t)$ and $\bar{y}(t)$ are corrupted by the additive noises $\tilde{u}(t)$ and $\tilde{y}(t)$ so that the available signals $u(t), y(t)$ are given by

$$u(t) = u_0(t) + \tilde{u}(t) \qquad (5)$$

$$y(t) = \bar{y}(t) + \tilde{y}(t) = y_0(t) + v(t) + \tilde{y}(t). \qquad (6)$$

Model (1)–(6) can thus be seen as an errors–in–variables model where the noise–free input $u_0(t)$ is affected by the measurement error $\tilde{u}(t)$ while the noise–free output $y_0(t)$ is affected by two noise contributions, a measurement error $\tilde{y}(t)$ and a process disturbance $v(t)$ whose sum could also be considered as a single colored noise generated by an ARMA process. The separation of the output disturbance into a white noise and a colored one considered in this paper is however useful in the solution of specific problems like, for instance, diagnosis.

Remark 2. Note that the EIV model (1)–(6) can approximate the extended–noise Kalman filter context described in (Diversi et al., 2005), where input, output and state noises are present.

The following assumptions are introduced.

A1. $A(z)$ and $D(z)$ have all zeros outside the unit circle.
A2. $A(z)$ and $B(z)$ do not share any common factor.
A3. The orders n and n_d are assumed as *a priori* known.
A4. The noise–free input $u_0(t)$ is a zero–mean ergodic random signal and is persistently exciting of a suitably high order.
A5. $e(t)$, $\tilde{u}(t)$ and $\tilde{y}(t)$ are zero–mean ergodic white processes with unknown variances σ_e^{2*}, $\tilde{\sigma}_u^{2*}$ and $\tilde{\sigma}_y^{2*}$ respectively. These processes are mutually uncorrelated and uncorrelated with the noise–free input $u_0(t)$.

The problem under investigation is the following.

Problem 1. Given a set of noisy input–output observations $u(1), \ldots, u(N), y(1), \ldots, y(N)$, determine an estimate of the coefficients a_k ($k = 1, \ldots, n$), b_k ($k = 0, \ldots, n$), d_k ($k = 1, \ldots, n_d$) and of the variances σ_e^{2*}, $\tilde{\sigma}_u^{2*}$, $\tilde{\sigma}_y^{2*}$.

3. A THREE–STEP IDENTIFICATION PROCEDURE

By defining the polynomials of degree $\bar{n} = n + n_d$

$$\bar{A}(q^{-1}) = A(q^{-1})\,D(q^{-1}) \qquad (7)$$

$$\bar{B}(q^{-1}) = B(q^{-1})\,D(q^{-1}), \qquad (8)$$

with coefficients

$$\bar{A}(q^{-1}) = 1 + \alpha_1\, q^{-1} + \cdots + \alpha_{\bar{n}}\, q^{-\bar{n}} \qquad (9)$$

$$\bar{B}(q^{-1}) = \beta_0 + \beta_1\, q^{-1} + \cdots + \beta_{\bar{n}}\, q^{-\bar{n}}, \qquad (10)$$

it is possible to rewrite (1) as

$$\bar{A}(q^{-1})\,\bar{y}(t) = \bar{B}(q^{-1})\,u_0(t) + e(t). \qquad (11)$$

so that model (1)–(6) can be seen also as an ARX process with noisy input and output, whose identification has been treated in (Diversi et al., 2007).

On the basis of the above consideration, we will solve Problem 1 by means of the following steps.

Procedure 1.

(1) Estimation of the high–order ARX model (11) and of the variances σ_e^{2*}, $\tilde{\sigma}_u^{2*}$, $\tilde{\sigma}_y^{2*}$.
(2) Estimation of $A(q^{-1})$ and $B(q^{-1})$ by using the estimates of $\bar{A}(q^{-1}), \bar{B}(q^{-1})$.
(3) Estimation of $D(q^{-1})$ from the estimates obtained in steps (1) and (2).

Let us introduce the regressor vectors

$$\varphi_0(t) = [\,-y_0(t)\ \ldots\ -y_0(t-n)\ u_0(t)\ \ldots\ u_0(t-n)\,]^T \qquad (12)$$

$$\varphi(t) = [\,-y(t)\ \ldots\ -y(t-n)\ u(t)\ \ldots\ u(t-n)\,]^T \qquad (13)$$

$$\tilde{\varphi}(t) = [\,-\tilde{y}(t)\ \ldots\ -\tilde{y}(t-n)\ \tilde{u}(t)\ \ldots\ \tilde{u}(t-n)\,]^T \qquad (14)$$

$$\varphi_v(t) = [\,-v(t)\ \ldots\ -v(t-n)\ \underbrace{0\ \ldots\ 0}_{n+1}\,]^T, \qquad (15)$$

and the parameter vector

$$\theta_0 = \left[1\ a_1\ \cdots\ a_n\ b_0\ \cdots\ b_n\right]^T = \left[1\ \theta^{*T}\right]^T. \qquad (16)$$

From $A(q^{-1})\,y_0(t) = B(q^{-1})u_0(t)$ and (5)–(6) it is possible to rewrite model (1)–(6) as follows

$$\varphi_0^T(t)\,\theta_0 = 0, \qquad (17)$$

$$\varphi(t) = \varphi_0(t) + \tilde{\varphi}(t) + \varphi_v(t). \qquad (18)$$

Similarly, define the regressor vectors