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On a Model for Laser Cutting involving Surface Tension

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Abstract: A model for laser cutting describing the movements of the two free boundaries of the melt will be presented. The model is an asymptotic approximation of a free boundary problem and involves the surface tension of the melt. The stationary solution will be given and investigated for linear stability. The results are illustrated with data from laser cutting experiments.

Keywords: Modelling, Stability analysis, Laser cutting, Asymptotic approximation, Free boundary problem

1. INTRODUCTION

Laser material processing such as joining, cutting, ablating is widely used in industry. Here we focus on laser cutting which has been grown to one of the leading cutting applications. An essential problem are ripple structures at the cutting surface which can be induced by fluctuations in the melt flow. Using laser wavelengths of 1 μm, this effect is even more distinct than for 10 μm lasers. The underlying processes in ripple formation are still not thoroughly understood.

Modeling, model analysis and numerical simulation of laser cutting have been applied by several authors, e.g., Schulz et al. (2009); Poprawe et al. (2010); Nießen (2005); Hirano and Fabbro (2012); Schulz et al. (1999); Otto et al. (2012). Schulz (2003) presented a model for laser cutting as a free boundary problem on basis of the Navier-Stokes equations with moving phase transitions between the gaseous phase (the cutting phase), the liquid phase (the molten material) and the solid phase (the workpiece). For a simplified situation neglecting the surface tension and considering constant process parameters, a model for the description of the movement of the two free boundaries of the melt has been derived in Vossen and Schüttler (2012). An analysis has shown that its stationary solution is linearly unstable. Hence, small perturbations into the system can lead to (unwanted) ripple structures along the cutting front. Since in the application, however, there are several process parameter sets with a smooth cutting front, we are interested in identifying stabilization effects of the process. Model extensions to varying process parameters have been developed, e.g., in Vossen et al. (2015); Vossen and Hermanns (2014), with a focus on stabilization by the process parameters affecting from the outside of the process. Here, we will derive a model including the surface tension of the melt and aim for finding stabilization effects in the process itself.

The organization of the paper is as follows. In Section 2, we present the physical background and basic notations in the process as well as the model considered in this paper. Section 3 deals with the derivation of the model which is based on an asymptotic expansion approximation of a free boundary value problem of the melt. The stationary solution of the model will be investigated by linear stability analysis in Section 4. Finally, we will illustrate the results with some realistic laser cutting parameters in Section 5.

2. MODEL FORMULATION

2.1 Physical background and notations

Let us first describe the process from the technical and physical point of view. Hereby, all quantities are firstly given in physical units denoted with a tick. During their

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first appearance in the paper, the corresponding unit is given in squared brackets. After scaling, the model will be given in corresponding dimensionless coordinates, where all quantities are denoted without a tick. In a typical laser cutting machine, the laser is driven over a fixed workpiece. The arising melt is driven out by cutting gas to separate the workpiece.

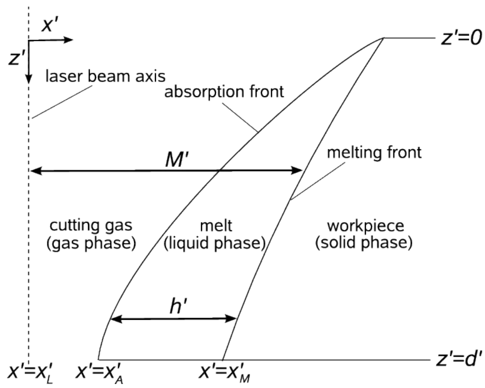

Fig. 1. Schematic 2D view of laser cutting

According to Vossen et al. (2015), the process will be modelled in the 2D cutting plane which is spanned by the cutting direction x' [m] and the laser beam propagation direction z' [m] (cf., Fig. 1). The time coordinate is t' [s], $t' \geq 0$. The workpiece (of width d' [m]) domain with upper face at $z' = 0$ and lower face at $z' = d'$ is (idealized) unbounded in the x'-direction. The process effects in the lateral direction are omitted. The model focusses on the investigations of the melt, especially of its interacting moving boundaries. The gaseous phase (cutting gas) and the solid phase (workpiece) are considered as inputs for the model through boundary conditions.

The following subsection presents the model in scaled quantities. A detailed derivation of the model will be given in Section 3.

2.2 The model

The model describes the movements of the absorption and the melting front determined by the distance $M = M(z,t)$ between the melting front and the laser beam axis as well as the melt width $h = h(z,t)$:

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial z}\left(h^2 + \frac{\sigma}{3}\frac{\partial^3(M-h)}{\partial z^3}h^3\right) = v_{\mathrm{p}}, \tag{1}$$

$$\frac{\partial M}{\partial t} = v_{\mathrm{p}} - 1 \tag{2}$$

with the melting front normal velocity

$$v_{\mathrm{p}} = \nu\mathcal{F}(\mu) - \frac{1}{H_{\mathrm{m}}} \tag{3}$$

coupling the two equations due to

$$\mathcal{F}(\mu) = \frac{4\mu^2\iota}{2\mu^2 + 2\mu\iota + \iota^2}, \quad \mu = -\alpha\frac{\partial(M-h)}{\partial z}. \tag{4}$$

Hereby, $\mathcal{F}$ is the Fresnel absorption function which depends on the dimensionless laser wavelength parameter ι and the cosine μ of the angle of incidence of the laser onto the absorption front. Furthermore, the model involves α as the ratio of the melt film and the workpiece thickness, ν describing the ratio between energy flux density provided by the laser and energy flux density required for melting the workpiece, H_{m} as the melting enthalpy, and, as a new component, the surface tension σ of the melt.

Remark 1. Note that for $\sigma = 0$, we obtain the first order model in Vossen et al. (2015). For $\sigma \neq 0$, the model structure is different since the highest occurring spatial derivative order for h and M is 4.

Remark 2. The boundary conditions for h, M,

$$h(0,t) = 0, \quad M(0,t) = w_0, \tag{5}$$

describe that the melt film at $z = 0$ has the width zero and its distance to the laser beam axis is given by the scaled laser beam radius w_0, representing a suitable approximation in many applications. For $\sigma = 0$, these conditions are sufficient to obtain a well-posed (first-order) problem; cf., Vossen et al. (2015). The fourth-order system for $\sigma \neq 0$ is expected to involve more boundary conditions of higher order whose exact formulation is still an open question. In this paper, we will show that the stationary solution for $\sigma = 0$ is also a stationary solution for the case $\sigma \neq 0$ and investigate this solution w.r.t. stability.

3. MODEL DERIVATION

3.1 Modelling as a free boundary value problem

The process will firstly be described in the liquid phase, the melt. This domain is bounded by the three boundaries Γ'_{A} (the absorption front), Γ'_{M} (the melting front) and Γ'_{B} (the lower boundary of the workpiece), defined by

$$\Gamma'_{\mathrm{A}} = \{(x',z') \in \mathbb{R}^2 : x' = x'_{\mathrm{A}},\, z' \in [0,d']\}, \tag{6}$$

$$\Gamma'_{\mathrm{M}} = \{(x',z') \in \mathbb{R}^2 : x' = x'_{\mathrm{M}},\, z' \in [0,d']\}, \tag{7}$$

$$\Gamma'_{\mathrm{B}} = \{(x',z') \in \mathbb{R}^2 : x'_{\mathrm{A}} \leq x' \leq x'_{\mathrm{M}},\, z' = d'\} \tag{8}$$

with the positions x'_{L} of the laser and

$$x'_{\mathrm{A}} = x'_{\mathrm{L}} + M' - h', \quad x'_{\mathrm{M}} = x'_{\mathrm{L}} + M' \tag{9}$$

of the absorption and the melting front, respectively. The melt thickness h' [m] and the distance M' [m] between the laser axis and the melting front satisfy the conditions

$$h'(0,t') = 0, \quad M'(0,t') = w'_0 \tag{10}$$

at the upper boundary of the workpiece where w'_0 describes the laser beam radius.

We consider the Navier-Stokes equations in the liquid phase with the workpiece as reference system:

$$\operatorname{div}'(v') = 0, \tag{11}$$

$$\rho'\left(\frac{\partial v'}{\partial t'} + \langle v', \nabla'\rangle v'\right) = \eta'\Delta' v' - \nabla' p', \tag{12}$$

$$\rho' c'_{\mathrm{p}}\left(\frac{\partial T'}{\partial t'} + \langle v', \nabla' T'\rangle\right) = \lambda'\Delta' T' \tag{13}$$

where $\langle\cdot,\cdot,\rangle$ is the Euklidean scalar product. The unknowns are the melt velocity v' $[\frac{\mathrm{m}}{\mathrm{s}}]$, $v' = (v'_x, v'_z)$, in x' and z' direction, the pressure p' [N/m^2], and the temperature T' [K]. Occuring constants are the density ρ' [kg/m^3], the dynamic viscosity η' [kg/(m s)], the specific heat capacity c'_{p} [J/(kg K)], and the thermal conductivity λ' [W/(K m)].

As the first two boundary conditions at the absorption front Γ'_{A}, we have an equilibrium of forces given by

$$\langle n'_{\mathrm{A}}, S' n'_{\mathrm{A}}\rangle = -p'_{\mathrm{g}} + \sigma'\kappa'_{\mathrm{A}}, \tag{14}$$

$$\langle t'_{\mathrm{A}}, S' n'_{\mathrm{A}}\rangle = -\tau'_{\mathrm{g}} \tag{15}$$