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Voltage Control Using Limited Communication ⋆

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Abstract:
In electricity distribution networks, the increasing penetration of renewable energy generation necessitates faster and more sophisticated voltage controls. Unfortunately, recent research shows that local voltage control fails in achieving the desired regulation, unless there is some communication between the controllers. However, the communication infrastructure for distribution systems are less reliable and less ubiquitous as compared to that for the bulk transmission system. In this paper, we design distributed voltage control that use limited communication. That is, only neighboring buses need to communicate few bits between each other before each control step. We investigate how these controllers can achieve the desired asymptotic behavior of the voltage regulation and we provide upper bounds on the number of bits that are needed to ensure a predefined accuracy of the regulation. Finally, we illustrate the results by numerical simulations.

Keywords: Distributed Optimization, Smart Grid, Voltage Regulation, Reactive Power, Limited Communication

1. INTRODUCTION

There is an increasing penetration of distributed energy resources such as renewable energy in distribution networks. Unfortunately, such a penetration causes faster voltage fluctuations than today's distribution networks can handle, see Carvalho et al. (2008). Therefore, to avoid overloading the distribution networks, the integration of renewable energy resources must be accompanied by faster and more sophisticated voltage regulation.

These challenges have motivated recent research on voltage control, where fast voltage fluctuations are regulated through real-time reactive power injections to ensure that the voltage is maintained within an acceptable range. Such fast voltage control can be implemented in the emerging power devices such as inverters. The research efforts have focused on two main directions: local and distributed control strategies. In the local voltage control, control devices at each bus update the reactive power injections using only locally available information, such as local voltage measurements, see Farivar et al. (2013); Li et al. (2014); Zhu and Liu (2016) and references therein. On the other hand, in distributed voltage control schemes, control devices at each bus determine the reactive power injection with additional information communicated from its neighboring buses in the distribution network, see Zhang et al. (2015); Šulc et al. (2014); Bolognani and Zampieri (2013); Bolognani et al. (2015). Local control strategies have the obvious advantage over distributed ones in that they do not rely on communication. However, even though local control strategies perform well in some cases, they may fail to ensure that the voltage is maintained within the accepted range under certain scenarios, as proved by the impossibility result in Cavraro et al. (2016). Therefore, some communication among the local controllers is always needed to guarantee the performance of voltage regulation.

However, the communication capabilities of today's distribution networks generally suffer from very low data rates, Yan et al. (2013); Galli et al. (2011). To compensate for this deficiency, power system operators and industries are currently investing heavily in integrating the distribution networks with a sophisticated communication infrastructure. However, even with the promising capabilities of the future low latency networks, fast real-time control applications, like voltage control, rely on short packages that carry coarsely quantized information, Durisi et al. (2016). Therefore, it is important to develop voltage control with very limited communication for early integration of renewable resources using today's grid limited communication capabilities and also for sustainable developments of the future smart grid.

In this paper, we study a distributed voltage control where only few bits of communication between neighboring buses are needed. In particular, the voltage control device on each bus determines the reactive power injection based on its local voltage measurement and current reactive power injection, in addition to a few bits of information communicated from its physical neighbors. We show that the algorithm can regulate the voltages to an acceptable range, for any predefined accuracy, in a finite number

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of iterations. We also provide an upper bound on the number of communicated bits that are needed to ensure a predefined accuracy of the desired voltage level. Moreover, we prove that this control strategy asymptotically achieves the desired regulation by varying the parameters of the controller with time. Lastly, we also illustrate the results by numerical simulations.

An extended version is found in Magnusson et al. (2017).

1.1 Notation

Vectors and matrices are represented by boldface lower and upper case letters, respectively. The imaginary unit is denoted by $\boldsymbol{i}$, i.e., $\boldsymbol{i}=\sqrt{-1}$. The set of real, complex, and natural numbers are denoted by $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{N}$, respectively. The set of real n vectors and $n{\times}m$ matrices are denoted by $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$, respectively. Otherwise, we use calligraphy letters to represent represent sets. We let $\mathcal{S}^{n-1}{=}\{\boldsymbol{x}{\in}\mathbb{R}^n \big| 1{=}||\boldsymbol{x}||\}$ denote the unit sphere. The superscript $(\cdot)^{\mathrm{T}}$ stands for transpose. $\mathrm{diag}(\boldsymbol{A}_1,\ldots,\boldsymbol{A}_n)$ denotes the diagonal block matrix with $\boldsymbol{A}_1,\ldots,\boldsymbol{A}_n$ on the diagonal. $||\cdot||$ denotes the 2-norm.

2. SYSTEM MODEL AND PROBLEM FORMULATION

2.1 System Model: Linearized Power Distribution Network

Consider a radial power distribution network with $N+1$ buses represented by the set $\mathcal{N}_0=\{0\}\cup\mathcal{N}$, where $\mathcal{N}=\{1,\ldots,N\}$. Bus 0 is a feeder bus and the buses in $\mathcal{N}$ are branch buses. Let $\mathcal{E}\subseteq\mathcal{N}_0\times\mathcal{N}_0$ denote the set of directed flow lines, so if $(i,j)\in\mathcal{E}$ then i is the parent of j. For each i, let $s_i=p_i+\boldsymbol{i}q_i\in\mathbb{C}$, $V_i\in\mathbb{C}$, and $v_i\in\mathbb{R}_+$ denote the complex power injection, complex voltage, and squared voltage magnitude, respectively, at Bus i. For each $(i.j)\in\mathcal{E}$, let $S_{ij}=P_{ij}+\boldsymbol{i}Q_{ij}\in\mathbb{C}$ and $z_{ij}=r_{ij}+\boldsymbol{i}x_{ij}\in\mathbb{C}$ denote the complex power flow and impedance in the line from Bus i to Bus j. To model the relationship between the variables, we use the linearized branch flow model from Baran and Wu (1989), which gives a good approximation in radial distribution networks.[1]

$$-p_i = P_{\sigma_i i} - \sum_{k:(i,k)\in\mathcal{E}} P_{ik}, \quad i\in\mathcal{N}, \tag{1a}$$

$$-q_i = Q_{\sigma_i i} - \sum_{k:(i,k)\in\mathcal{E}} Q_{ik}, \quad i\in\mathcal{N}, \tag{1b}$$

$$v_j - v_i = -2r_{ij}P_{ij} - 2x_{ij}Q_{ij}, \quad (i,j)\in\mathcal{N}, \tag{1c}$$

where σ_i is the parent of bus $i\in\mathcal{N}$, i.e., the unique $\sigma_i\in\mathcal{N}_0$ with $(\sigma_i,i)\in\mathcal{E}$. By rearranging Equation (1) we get that

$$\boldsymbol{v} = \boldsymbol{A}\boldsymbol{q} + \boldsymbol{B}\boldsymbol{p} + \boldsymbol{1}v_0, \tag{2}$$

where $\boldsymbol{v}=[v_1,\ldots,v_N]^{\mathrm{T}}$, $\boldsymbol{q}=[q_1,\ldots,q_N]^{\mathrm{T}}$, $\boldsymbol{p}=[p_1,\ldots,p_N]^{\mathrm{T}}$,

$$\boldsymbol{A}_{ij} = 2\sum_{(h,k)\in\mathcal{P}_i\cap\mathcal{P}_j} x_{hk}, \quad \text{and} \quad \boldsymbol{B}_{ij} = 2\sum_{(h,k)\in\mathcal{P}_i\cap\mathcal{P}_j} r_{hk},$$

where $\mathcal{P}_i\subseteq\mathcal{E}$ is the set of edges in the path from Bus 0 to Bus i. We use the following result in the algorithm development.

[1] The results also directly apply to the linearized power flow model in Cavraro et al. (2016).

Lemma 1. $\boldsymbol{A}$ is a positive definite matrix whose inverse has the following structure

$$a_{ij}{:=}[\boldsymbol{A}^{-1}]_{ij}{=}\begin{cases}\frac{1}{2}\left(x_{\sigma_i i}^{-1}{+}\sum_{k:(i,k)\in\mathcal{E}} x_{ik}^{-1}\right) & \text{if } i{=}j,\\ -\frac{1}{2}x_{ij}^{-1} & \text{if } (i,j){\in}\mathcal{E} \\ & \text{or } (j,i){\in}\mathcal{E},\\ 0 & \text{otherwise.}\end{cases} \tag{3}$$

Proof. It is proved in (Farivar et al., 2013, Lemma 1) that $\boldsymbol{A}$ is positive definite. Direct calculations show that $\boldsymbol{A}\boldsymbol{A}^{-1}=\boldsymbol{I}$. □

We now introduce the Voltage Regulation Problem.

2.2 Voltage Regulation Problem

Suppose that the real power injection $\boldsymbol{p}$ at each bus has been decided. We also write the reactive power injection $\boldsymbol{q}$ as two parts, i.e., $\boldsymbol{q}=\boldsymbol{q}^{\mathrm{V}}+\boldsymbol{q}^{\mathrm{U}}$, where $\boldsymbol{q}^{\mathrm{V}}$ is the adjustable reactive power that can be used for voltage regulation and $\boldsymbol{q}^{\mathrm{U}}$ denotes other reactive power injection that cannot be changed by the voltage control devices. Then the goal of the voltage regulation problem is to find feasible voltages $\boldsymbol{v}$ and reactive powers $\boldsymbol{q}^{\mathrm{V}}$ so that the physical relationship (2) holds and that $\boldsymbol{v}$ and $\boldsymbol{q}^{\mathrm{V}}$ are inside some feasible operation range $[\boldsymbol{v}^{\min},\boldsymbol{v}^{\max}]$ and $[\boldsymbol{q}^{\min},\boldsymbol{q}^{\max}]$. Formally, the voltage regulation problem is to find the reactive power injection $\boldsymbol{q}^{\mathrm{V}}$ so that,

$$\boldsymbol{v}(\boldsymbol{q}^{\mathrm{V}}) = \boldsymbol{A}\boldsymbol{q}^{\mathrm{V}} + \boldsymbol{d}, \tag{4a}$$

$$\boldsymbol{v}^{\min} \leq \boldsymbol{v}(\boldsymbol{q}^{\mathrm{V}}) \leq \boldsymbol{v}^{\max} \tag{4b}$$

$$\boldsymbol{q}^{\min} \leq \boldsymbol{q}^{\mathrm{V}} \leq \boldsymbol{q}^{\max} \tag{4c}$$

where $\boldsymbol{d}=\boldsymbol{A}\boldsymbol{q}^{\mathrm{U}}+\boldsymbol{B}\boldsymbol{p}+\boldsymbol{1}v_0$. In the rest of the paper we drop the superscript V from $\boldsymbol{q}^{\mathrm{V}}$ for sake of notational ease, without causing any confusion. We also assume, without loss of generality, that every bus in $\mathcal{N}$ can adjust its reactive power.

In this paper, we study distributed control laws for finding feasible reactive power injections and voltages that satisfy Equation (4). In particular, each bus updates its reactive power injection by following a local control law that depends only on information available at the bus and limited information communicated from neighboring buses. Formally, each bus $i\in\mathcal{N}$ updates its reactive power injection according to the following rule

$$\boldsymbol{q}_i(t+1) = K_i(\text{Local\_Information}_i(t), \bar{\boldsymbol{b}}_i(t)),$$

where t is the iteration index and K_i is the local control law at bus i. The function K_i depends on the local information, which we denote by $\text{Local\_Information}_i(t)$, at Bus i at iteration t. Formally,

$$\text{Local\_Information}_i(t) = (\boldsymbol{q}_i(0),\ldots,\boldsymbol{q}_i(t),\boldsymbol{v}_i(0),\ldots,\boldsymbol{v}_i(t)),$$

and $\bar{\boldsymbol{b}}_i(t)$, the information available from neighboring buses of Bus i at time t is given by

$$\bar{\boldsymbol{b}}_i(t) = ((\boldsymbol{b}_j(0))_{j\in\mathcal{N}_i},\ldots,(\boldsymbol{b}_j(t))_{j\in\mathcal{N}_i}),$$

where $\mathcal{N}_i=\{j\in\mathcal{N}|(i,j)\in\mathcal{E} \text{ or } (i,j)\in\mathcal{E}\}$ and $\boldsymbol{b}_j(t)$ is the information that Bus j communicates to its neighbors at iteration t. In Section 3 we provide the explicit control algorithm, where $\boldsymbol{b}_j(t)$ is communicated using 2 bits per iteration. Before that, first we need to provide some related background in the following subsection.