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Huygens' Synchronization: Experiments, Modeling, and Local Stability Analysis

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Abstract: The synchronization of two metronomes on a freely moving platform is examined. Existence and local stability of the synchronized solutions is studied when the platform parameters are subject to change, i.e., a numerical continuation and bifurcation software tool is used to reveal local stability of the in- and/or anti-phase synchronized state as function of system parameters. It is demonstrated that the platform parameter values have a significant influence on the stability of the anti-phase synchronized solution.

Keywords: Synchronization, Stability analysis, Nonlinear systems, Mechanical systems

1. INTRODUCTION

Synchronization is a well-known phenomenon in physics, biology, chemistry, and engineering (Pikovsky et al., 2003; Strogatz, 2003). A few examples are synchronous flashing of fire flies (Buck, 1988), synchronous contraction of the left heart ventricle (Maeda, 2004), unconscious synchronized behavior of monkeys (Nagasaka et al., 2013), and the biological clock in our brain (Aton and Herzog, 2005). This internal time-keeper consists of several thousand nerve cells, and each of them can keep time. Altogether, these clocks form a perfectly synchronized network with an accuracy up to a few of minutes per day, especially since each day the clocks are resynchronized by external stimuli, such as daylight.

Although synchronization is currently attracting significant interest, it has been already observed for many ages. One of the first reports on synchronization came from the Dutch scientist Huygens (1967). He observed in February of 1665—during a brief illness when he was confined to his room—that two pendulum clocks hanging on a common support converge independently of the initial state to a final state that now is called anti-phase synchronization, i.e., the pendulums swing in opposite motion with the same frequency. Inspired by this observation, Huygens conducted further experiments; however, to the best of our knowledge, he has not explicitly mentioned in-phase synchronization.

More than three centuries later, Bennett et al. (2002) experimentally reproduced the observations of Huygens by using real pendulum clocks and examined the influence of the coupling strength by changing the mass ratio between the pendulums and coupling bar. The anti-phase behavior as described by Huygens was observed as well 'beating death' behavior, i.e., when one of both clocks cease to run. Likewise, Czolczynski et al. (2011) repeated Huygens experiments with high-precision pendulum clocks and observed, depending on the initial conditions, both in- and anti-phase synchronization.

Another experimental approach used by several authors is to replace the pendulum clocks with mechanical metronomes. These simple inexpensive instruments, which produce regular rhythmic ticks to help musicians maintain a steady tempo as they play, have all the properties of a self-sustained oscillator and are, therefore, commonly used in synchronization experiments. For instance, Pantaleone (2002) used two metronomes on a very basic experimental setup consisting of a light wooden board rolling on two empty soda/beer cans. He points out that independent of the initial conditions in-phase synchronization was obtained. Anti-phase synchronization was only achieved by adding damping to the platform. Inspired by the work of Pantaleone (2002), Oud et al. (2006) conducted further research with a more sophisticated setup containing metronomes. Both in- and anti-phase solutions were produced; however, robustness of the in-phase solution remained problematic. More recently, Wu et al. (2012) noticed a problematic occurrence of the anti-phase solution and numerically as well experimentally investigated the relationship between platform damping and anti-phase synchronization. Their analysis shows that an increase in platform damping gives rise to a stable anti-phase solution.

The previously mentioned studies focused on the influence of coupling parameters, whereas one has to consider that an essential part of the synchronized behavior is due to the pendulum clock or metronome, and since there is a broad variation among these, e.g., in pendulum mass, pendulum length, and type of escapement mechanism, it is not possible to directly compare experimental results of different setups. With this in mind, Chakrabarty et al. (2014) examined the influence of metronome parameters on the final synchronized state and show that by varying the damping or frequency of one of the two metronomes, a transition from in- to anti-phase is possible.

Other authors have contributed to the understanding of the Huygens' synchronization phenomenon from a theoretical point of view. For instance, Jovanovic and Koshkin (2012)

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studied the effect of frame damping on coupled harmonic oscillators with a van der Pol escapement. As analysis strategy, they used the Poincaré method, which assumes small oscillations angles, and as a result, it is not valid for metronomes. Similarly, Peña Ramirez et al. (2014) studied, by using the Poincaré method, the effect of coupling bar stiffness on the final synchronized state. Both by an analytical and numerical analysis, it is demonstrated that the coupling bar stiffness has a large effect on the in-phase synchronized state.

Despite that these and other authors indicate both numerically and experimentally stable in- and anti-phase solutions by changing parameters of the setup and/or metronomes, no one, to the best of our knowledge, has performed a numerical continuation and bifurcation analysis to reveal local stability of the in- and/or anti-phase synchronized state as function of system parameters. Some authors used simulation-based bifurcation diagrams, see e.g., Kapitaniak et al. (2012); however, limitations with this method are the dependence on the initial conditions and inability to find the unstable solutions. Another approach used by some authors is an analytical study, see e.g., Kuznetsov et al. (2007); however, shortcoming with this method is that the findings are generally very conservative.

This paper, which is inspired by the experimental work of Ikeguchi (2014), addresses synchronization of two off-the-shelf metronomes on a freely moving platform, and it examines the local stability of the synchronized solutions when only a subset of the system parameters—the platform parameters—is subject to change. The paper is organized as follows: Section 2 describes the experimental setup and measurement method. In Section 3, a model of the experimental setup is derived and identified. Experimental results are presented in Section 4. In Section 5, these experimental observations are compared with simulation results. Section 6 provides a local stability analysis of the in- and anti-phase solution when the platform parameters are subject to change. Finally, Section 7 concludes this paper with a short discussion of the results.

2. EXPERIMENTAL SETUP

Figure 1 depicts the experimental setup that was used in this study. It consists of a rectangular platform made of lightweight foam that is suspended at the corners by four thin cables. On the platform, two metronomes were placed next to each other. To detect the phase of both metronomes and displacement of the platform, a video camera setup was used. This setup allows contactless measurements and requires no additional instrumentation on the metronomes and platform other than placing markers and covering the metal objects with glossy surface since they can show grayish reflections. To increase contrast in the image, white markers were used on a blacked out pendulum surface and metronome casing. As high-speed video camera, a Casio Exilim EX-F1 was used at a recording speed of 300 fps. Post-processing of the videos was performed in MATLAB, and it involved determining the centers of the markers and mapping these image coordinates back to world coordinates.

The metronomes used in the experiments were Nikko Lupina Orange 311 metronomes. The frequency range setting of these metronomes varies from *largo* 40 bpm to *prestissimo* 208 bpm, and it can be adjusted by sliding a small mass, called the bob, along the pendulum. In this paper, the mass was set to the lowest possible setting and therefore maximum natural frequency. To compensate for friction and, thus, dissipated energy during oscillation cycles, an escapement mechanism is present inside the metronome. This mechanism, which consists of a coiled spring, axles, gears, and a toothed wheel, drives the pendulum every back and forth swing with a short pulse. During this pulse, a tooth 'escapes' and impacts the pendulum, which makes the typical ticking sound of a metronome, and since these escapements happen at a constant interval—twice per cycle—the metronome oscillates with a constant frequency.

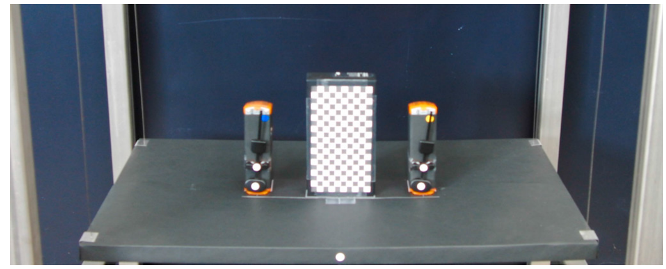

Figure 1. Experimental setup with metronomes placed next to each other. The checkerboard pattern is used to determine the camera position relative to the setup.

3. MODELING OF THE EXPERIMENTAL SETUP

3.1 Metronome

Various synchronization experiments have shown that the synchronization phenomenon with metronomes are robust, see e.g., Pantaleone (2002) and Ikeguchi (2014); hence, it is not expected that a marginal difference between model and metronome results in different synchronization phenomenon. For this reason, the metronome is described in simplified representation as a viscous damped pendulum with length ℓ and point mass m. The escapement mechanism is modeled as a pulsating function. This results in the following equation of motion: (see Kapitaniak et al., 2012, pp. 12-13)

$$m\ell^2\ddot{\theta} + d\dot{\theta} + mg\ell\sin\theta = u(\theta,\dot{\theta}), \tag{1}$$

with the escapement function defined as:

$$u(\theta,\dot{\theta}) = \begin{cases} \tau, & \text{if } \theta_s \le |\theta| \le \theta_e \wedge \dot{\theta} > 0; \\ -\tau, & \text{if } \theta_s \le |\theta| \le \theta_e \wedge \dot{\theta} < 0; \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where d is the damping due to the pendulum hinge friction, g denotes the gravitational acceleration, τ is the driving torque of the escapement mechanism, and θ_s, θ_e denote the angles between which the escapement mechanism operates.

Escapement function (2) is, however, discontinuous and results in a non-smooth model. Since both the parameter estimation procedure, and continuation and bifurcation software tool require a smooth model, the escapement function is approximated by a smooth function. To do so, we propose the following escapement function

$$u(\theta,\dot{\theta}) = \tau s\left(\tanh((s\theta-\theta_s)/\varepsilon) - \tanh((s\theta-\theta_e)/\varepsilon)\right)/2, \tag{3}$$