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Design of Consensus Controllers for Multi-rate Sampled-data Strict-Feedback Multi-agent Systems

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Abstract: Design of state feedback consensus controllers is considered for multi-rate sampled-data strict-feedback multi-agent systems. First state feedback consensus controllers are designed for continuous-time strict-feedback multi-agent systems. Then the Euler model is used to estimate the states at input sampling times between successive measurement sampling times. It is shown that emulation controllers of designed continuous-time state feedback controllers with estimated states achieve consensus for multi-rate sampled-data strict-feedback multi-agent systems. The proposed design method is also applied to consensus control of multi-rate sampled-data fully-actuated ships.

Keywords: Consensus control, Sampled-data systems, Strict-feedback multi-agent systems, Ship control, Nonlinear multi-rate sampled-data stabilization.

1. INTRODUCTION

Consensus and formation control has been widely studied from linear multi-agent systems to nonlinear multi-agent systems, due to a large variety of applications (Arcak (2007), Casbeer et al. (2008), Moreau (2005), Muntz et al. (2011), Olfati-Saber et al. (2007), Ren and Beard (2008), Wang and Slotine (2006)). Due to a rapid development of computer technologies, modern control systems usually use digital computers as discrete-time controllers with samplers and zero-order holds to control continuous-time plants. Such a control system is called a sampled-data system. For linear single-rate sampled-data multi-agent systems, consensus control has been naturally extended (Cao and Ren (2010), Hayakawa et al. (2006), Liu et al. (2010), Qin et al. (2011), Yu et al. (2011)). Here a single-rate means that sampling periods of the input and the measurement channels are equal. Recently, for nonlinear single-rate sampled-data input-affine strict-feedback multi-agent systems that naturally appear in the consensus and formation control of vehicles such as automobiles and ships, a design of output feedback consensus controllers has been discussed (Katayama (2014)). Due to hardware restrictions, the input sampling period and the measurement sampling period are usually different and such a sampled-data system is called a multi-rate sampled-data system (Chen and Qiu (1994), Liu et al. (2008), Polushin and Marquez (2004)). In this paper we consider the design of state feedback controllers that achieve consensus for multi-rate sampled-data strict-feedback multi-agent systems.

Consider nonlinear input-affine strict-feedback multi-agent systems

$$\dot{x}_i = \Phi(x_i) + \Gamma(x_i)z_i,$$

$$\dot{z}_i = f_i(x_i, z_i) + g_i(x_i, z_i)u_i, \ i \in \mathbf{n} := \{1, .., n\} \quad (1)$$

where n is a number of agents, x_i, $z_i \in \mathbf{R}^m$ and $u_i \in \mathbf{R}^m$ are the states and the control input of the ith agent. In consensus and formation control problems of vehicles, x_i and z_i usually express the position and the velocity of the ith agent, respectively and functions Φ, Γ are same for all vehicles. For the multi-agent systems (1), we assume

A1: Φ, Γ, f_i, g_i are smooth functions over the compact domain of interest, $\Phi(0) = 0$ and $f_i(0, 0) = 0$.

A2: The $m \times m$ matrices Γ and g_i are nonsingular and their inverses are smooth over the compact of interest.

Let $T > 0$ be an input sampling period and assume that u_i, $i \in \mathbf{n}$ are realized through a zero-order hold, i.e.,

$$u_i(t) = u_i(kT) =: u_i(k), \ i \in \mathbf{n} \quad (2)$$

for any $t \in [kT, (k+1)T)$ and $k \in \mathbf{N}_0 = \{0, 1, 2, ...\}$. Let T_m be a fixed measurement sampling period that satisfies $T_m = qT$ for some integer q and assume that the sampled observations

$$y_i(j) = x_i(jT_m) = x_i(jqT), \ i \in \mathbf{n} \quad (3)$$

are available for control. Then we consider the design of state feedback controllers that achieve consensus for the multi-agent systems (1)-(3) by choosing a sufficiently small input sampling period.

First we design continuous-time state feedback controllers that achieve consensus for the multi-agent systems (1). Then we use the sampled observations (3) and the Euler models of the multi-agent systems (1) and (2) to estimate the states at input sampling times between successive measurement sampling times. We combine an emulation of the designed continuous-time state feedback controllers and the estimated states to construct multi-rate sampled-data controllers. Finally we apply the nonlinear multi-rate sampled-data stabilization theory summarized in Section

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2 to consensus problems to show that the obtained controllers achieve consensus for the multi-agent systems (1)-(3). We also apply the designed method to consensus control of multi-rate sampled-data fully-actuated ships. We give a comparison to a single-rate sampled-data consensus controllers to show the efficiency of the proposed design method.

Notation: Let $\mathbf{C}^- = \{\lambda = \alpha + i\beta | \alpha < 0\}$ and $\mathbf{1}_n = [1 \ \cdots \ 1]^T \in \mathbf{R}^n$. Let $\|\cdot\|$ be a norm of vectors and matrices, $\sigma(M)$ a set of eigenvalues of a square matrix M, and $\otimes$ a Kronecker product of matrices. A function α is of class $\mathcal{K}$ ($\alpha \in \mathcal{K}$) if it is continuous, zero at zero and strictly increasing. It is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. A function β: $\mathbf{R}_{\geq 0} \times \mathbf{R}_{\geq 0} \to \mathbf{R}_{\geq 0}$ is of class $\mathcal{KL}$ if for any fixed $t \geq 0$, the function $\beta(\cdot, t)$ is of class $\mathcal{K}$ and for each fixed $s \geq 0$ the function $\beta(s, \cdot)$ is decreasing to zero as its argument tends to infinity (Khalil (2002)).

2. PRELIMINARY RESULTS

Consider

$$\dot{x} = f(x, u), \quad x(0) = x_0 \tag{4}$$

where $x \in \mathbf{R}^{n_x}$ is the state, $u \in \mathbf{R}^{n_u}$ is the control input, and $f(\cdot, \cdot)$ is locally Lipschitz and saisfies $f(0, 0) = 0$. Let $T > 0$ be an input sampling period and assume

$$u(t) = u(kT) =: u(k) \tag{5}$$

for any $t \in [kT, (k + 1)T)$ and $k \in \mathbf{N}_0$. Let a fixed measurement sampling period $T_m > T$ be such that $T_m = qT$ for some integer q and assume that

$$y(j) = x(jT_m) = x(jqT) \tag{6}$$

is available for control. Then the difference equations corresponding to the exact model and the Euler model of the system (4)-(6) are given by

$$\begin{aligned} x_e(k+1) &= F_T^e(x_e(k), u(k)) \\ &= x_e(k) + \int_{kT}^{(k+1)T} f(x(s), u(k)) ds, \\ y(j) &= x_e(jq) \end{aligned}$$

and

$$\begin{aligned} x_a(k+1) &= F_T^a(x_a(k), u(k)) \\ &= x_a(k) + T f(x_a(k), u(k)), \\ y_a(j) &= x_a(jq), \end{aligned}$$

respectively. Note that $x_e(k) = x(kT)$ for the exact model. Since the exact model is not computable analytically, the Euler model is used to design controllers. Moreover, the Euler model is one-step consistent with the exact model, i.e., for each compact set $\Omega \subset \mathbf{R}^{n_x} \times \mathbf{R}^{n_u}$, there exist $\rho \in \mathcal{K}_\infty$ and $T^* > 0$ such that

$$\|F_T^e(\eta, u) - F_T^a(\eta, u)\| \leq T\rho(T)$$

for any $(\eta, u) \in \Omega$ and $T \in (0, T^*)$ (Laila et al. (2005), Nesic, Teel, and Kokotovic (1999)).

We assume the following (Liu et al. (2008), Polushin and Marquez (2004)).

B1: There exists a locally Lipschitz state feedback controller $u = u^F(x)$ that makes the equilibrium point $x = 0$ of the continuous-time closed-loop system $\dot{x} = f(x, u^F(x))$ globally asymptotically stable.
B2: The input sampling period can be assigned arbitrarily.

Let $x_e(j)$ be the state of the exact model. To use an emulation of a continuous-time state feedback controller, we first estimate the state at input sampling times $t = kT$. For $k = jq$ and $j \in \mathbf{N}_0$, we set

$$x_c(k) = x_e(k) \tag{7}$$

and we construct $x_c(k)$ for $k = jq + 1, .., (j + 1)q - 1$ by

$$x_c(k) = F_T^a(x_c(k-1), u^F(x_c(k-1))) \tag{8}$$

with initialization $x_c(jq) = x_e(jq)$. Then we use the estimated state $x_c(k)$ to construct the control input

$$u(t) = u^F(x_c(k)) \tag{9}$$

for any $t \in [kT, (k+1)T)$ and $k \in \mathbf{N}_0$. Consider the closed-loop exact model of the sampled-data system (4) and (7)-(9)

$$x_e(k+1) = F_T^e(x_e(k), u^F(x_c(k))). \tag{10}$$

Then we have the following result (Liu et al. (2008), Polushin and Marquez (2004), Nesic, Teel, and Sontag (1999)).

Theorem 1. Assume **B1** and **B2**. Then
1) For any positive real numbers $D > d$ there exist $T^* > 0$ and $\beta \in \mathcal{KL}$ such that solutions of the closed-loop exact model (10) satisfy

$$\max\{\|x_e(k)\|, \|x_c(k)\|\} \leq \beta(\|x(0)\|, kT) + d \tag{11}$$

for any $x_e(0) \in \mathbf{R}^{n_x}$ with $\|x_e(0)\| \leq D$, $T \in (0, T^*)$, and $k \in \mathbf{N}_0$.
2) For any positive real numbers $D > d$ there exist $T^* > 0$ and $\beta \in \mathcal{KL}$ such that solutions of the closed-loop sampled-data system (4) and (7)-(9) satisfy

$$\|x(t)\| \leq \beta(\|x(0)\|, t) + d \tag{12}$$

for any $x(0) \in \mathbf{R}^{n_x}$ with $\|x(0)\| \leq D$, $T \in (0, T^*)$, and $t \geq 0$.

Remark 2. For sufficiently large $D > 0$ and sufficiently small $d > 0$, we must choose sufficiently small $T^* > 0$ to achieve (11) and (12). In this case, for a fixed measurement sampling period T_m, we find an input sampling period $T \in (0, T^*)$ such that $T_m = qT$ for some large positive integer q.

3. MULTI-RATE SAMPLED-DATA CONSENSUS CONTROL

For multi-agent systems (1), let

$$r_i = \Phi(x_i) + \Gamma(x_i) z_i, \ i \in \mathbf{n}. \tag{13}$$

Then we have

$$\begin{aligned} \dot{x}_i &= r_i, \\ \dot{r}_i &= \kappa_i(x_i, r_i) + \Gamma(x_i) g_i(x_i, \psi(x_i, r_i)) u_i \end{aligned} \tag{14}$$

where