ARTICLE IN PRESS

ISA Transactions xxx (2018) 1–10

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Decentralized state estimation for a large-scale spatially interconnected system

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ARTICLE INFO

Article history:
Received 26 September 2017
Revised 22 December 2017
Accepted 1 January 2018
Available online XXX

Keywords:
State estimation
Decentralized
Networked system
Large-scale system
Kalman filter

ABSTRACT

A decentralized state estimator is derived for the spatially interconnected systems composed of many subsystems with arbitrary connection relations. An optimization problem on the basis of linear matrix inequality (LMI) is constructed for the computations of improved subsystem parameter matrices. Several computationally effective approaches are derived which efficiently utilize the block-diagonal characteristic of system parameter matrices and the sparseness of subsystem connection matrix. Moreover, this decentralized state estimator is proved to converge to a stable system and obtain a bounded covariance matrix of estimation errors under certain conditions. Numerical simulations show that the obtained decentralized state estimator is attractive in the synthesis of a large-scale networked system.

1. Introduction

State estimation is one of the fundamental issues in control theory and system engineering. With the comprehensive application of smart sensors and actuators, the called spatially interconnected systems [1,2] have obtained extensive interests recently. It consists of a great number of spatially dispersed subsystems interacting with each other, for example, automated highway systems [3], power smart grids [4]. Particularly, we consider an array of closely packed microcantilevers (about 4000) employed in an atomic force microscope application [5]. The system consists of many microcantilevers connected to the same base, each forming a micro-capacitor, with the second rigid plate located underneath the cantilever. The cantilever can move flexibly in the vertical axis, however be rigid along the horizontal axis. The vertical displacement of each cantilever can be controlled by applied voltage across the plates. Although each cantilever is independently actuated, its dynamics are affected by other cantilevers owing to the mechanical and electrical couplings. See Sarwar and Voulgaris [5] for more details. State feedback control with state estimator is attractive in this case, however, direct utilization of the analysis and synthesis approaches based on the rearranged lumped systems may usually encounter implementation prohibitions [1,2,6] or need too many costs. Under these situations, state estimations using only local system output measurements are generally much more appreciated [7–9]. Specially, the design of decentralized schemes are motivated where the information exchange between the interconnected subsystems of large-scale systems is not required, which is more feasible and more economical in practical engineering applications [10,11].

In Hashemipour et al. [12], parallel structures for state estimation using the Kalman filter are proposed for multisensor network. Decentralized reduced order estimators are proposed for large-scale dynamical system in Saif and Guan [13] treating the interconnection effects among the subsystems as totally unknown inputs to each subsystems, but the number of the subsystem interaction inputs and the locally measured outputs is restricted. In Mallory and Miller [14], a decentralized state estimator based on model reduction theory is provided for a heavily coupled large-scale system, while the decentralized state estimation in Caro et al. [15] is based on the Lagrangian relaxation and the previous decomposition approaches. Yang et al. [16] provided optimal allocation strategies for the agents subject to communication constraints when decentralized state estimation is considered based on the local measurement data and the received data from the other agents. In Farina et al. [17], three novel moving-horizon estimation approaches are derived for the systems decomposed into coupled subsystems with non-overlapping states. A discrete-time overlapping decentralized state estimator of large-scale systems is proposed in Stanković et al. [18] based on a combi-

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https://doi.org/10.1016/j.isatra.2018.01.007
0019-0578/

Please cite this article in press as: Liu H, Yu H. Decentralized state estimation for a large-scale spatially interconnected system, ISA Transactions (2018), https://doi.org/10.1016/j.isatra.2018.01.007

nation of the local Kalman filter and a dynamic consensus strategy, and extended in Stanković et al. [19] assuming intermittent observations and communication faults. In Bauer et al. [20], decentralized observer-based output-feedback controllers is designed where the controllers, sensors and actuators are connected via a common communication network subject to time-varying transmission intervals and delays. A decentralized algorithm is proposed for the power system in Singh and Pal [21] to estimate the real-time states using unscented Kalman filtering, while Behrooz and Boozarjomehry [22] proposed a framework for distributed and decentralized state estimation in high-pressure and long-distance gas transmission networks. These investigations have greatly advanced studies on decentralized state estimations for a large-scale networked system. However, some assumptions of these methods such as system structure constraints, subsystem dynamic characteristic, etc., sometimes cannot be easily satisfied in practical engineering problems. Moreover, online computations of decentralization design also restrict their engineering applications.

In this paper, we investigate the decentralized state estimation for a large-scale spatially interconnected system in which the dynamics of each subsystem and the interactions among the subsystems (that is, the system structure) are explicitly expressed [2]. Connections among the subsystems are arbitrary and time-varying, and every subsystem can have different dynamics. An LMI based optimization problem is constructed for the computations of improved subsystem parameter matrices. A computationally effective condition is derived for the design of the decentralized state estimator, which depends only on the block-diagonal system parameters and the sparse subsystem connection matrix. Another condition dependent only on individual subsystem is then provided, which is attractive for a large-scale networked system. Furthermore, the conditions guaranteeing the stability of the estimator and the boundedness of estimation errors are also investigated. Different to the distributed one-step state predictor in Zhou [2] for which communications among the subsystems are required, especially a collaboration unit is utilized to provide optimal update gains for each individual subsystem, in this paper a completely decentralized state estimator is derived for which the subsystem communications are not needed.

This paper is outlined as follows. In Section 2, the problem formulation and some preliminaries are presented, while several computation conditions for the decentralized state estimator are developed in Section 3. The conditions guaranteeing the stability of the estimator and the boundedness of estimation errors are presented in Section 4. In Section 5, numerical examples are provided to illustrate the effectiveness of the derived approaches. Finally, some concluding remarks are given in Section 6 on some characteristics of the suggested methods.

We adopt the following symbols and notations. $\mathcal{F}_u(*,\#)$ stands for the upper linear fractional transformation. $\mathbf{col}\{X_i|_{i=1}^{L}\}$ represents a vector/matrix stacked by $X_i|_{i=1}^{L}$ with its i-th row block being X_i, while $\mathbf{diag}\{X_i|_{i=1}^{L}\}$ the block-diagonal matrix with its i-th diagonal block being X_i. $\{X_{ij}|_{i=1,j=1}^{i=M,j=N}\}$ denotes a matrix with $M \times N$ blocks and its i-th row j-th column block matrix being X_{ij}. 0_m and $0_{m\times n}$ denote respectively the m dimensional zero column vector and the $m \times n$ dimensional zero matrix, while the subscript with respect to dimensions is omitted under the premise of no ambiguity. Similarly, I_m, identity matrix with $m \times m$ dimension, is abbreviated as I. The superscript T and H are used to denote respectively the transpose and conjugate transpose of a matrix/vector and X^TWX is sometimes abbreviated as $(*)^TWX$ or $X^TW(*)$, especially when the term X has a complicated expression. $\mathbf{R}^\sharp$ means real column vectors set with appropriate dimensions. For symmetrical matrices with compatible dimensions $A, B \in \mathcal{H}$, $A > (\leq, <, \geq) B$ means $A - B$ is positive definite (negative semi-definite, negative definite, positive semi-definite).

2. Problem description and some preliminaries

Consider a spatially interconnected system $\mathbf{\Sigma}$ which is composed of N linear time-varying subsystems and the state space description of its i-th subsystem $\mathbf{\Sigma}_i$ is represented as follows,

$$\begin{bmatrix} x(t+1,i) \\ z(t,i) \\ y(t,i) \end{bmatrix} = \begin{bmatrix} A_{\mathbf{TT}}(t,i) & A_{\mathbf{TS}}(t,i) & B_{\mathbf{T}}(t,i) & 0 \\ A_{\mathbf{ST}}(t,i) & A_{\mathbf{SS}}(t,i) & 0 & 0 \\ C_{\mathbf{T}}(t,i) & C_{\mathbf{S}}(t,i) & 0 & D_{\mathbf{T}}(t,i) \end{bmatrix} \begin{bmatrix} x(t,i) \\ v(t,i) \\ d(t,i) \\ w(t,i) \end{bmatrix}, \tag{1}$$

where i denotes the index number of a subsystem, $i = 1, 2, \ldots, N$, and t the temporal variable. These subsystems are connected by

$$v(t) = \Phi(t) z(t), \tag{2}$$

in which $z(t) = \mathbf{col}\{z(t,i)|_{i=1}^{N}\}$ and $v(t) = \mathbf{col}\{v(t,i)|_{i=1}^{N}\}$. $z(t,i)$ and $v(t,i)$ respectively denote the output vector to other subsystems and input vector from others, which are called internal output and input vectors. $x(t,i)$ represents the state vector of the i-th subsystem $\mathbf{\Sigma}_i$ at time t, while $d(t,i), w(t,i)$ and $y(t,i)$ respectively process disturbance vector, measurement error vector and external output vector. To simplify mathematical expressions, it is assumed without loss of generality that both the process disturbances and measurement errors are independent of each other, as well as spatially and temporally white. Furthermore, the mathematical expectations of $d(t,i)$ and $w(t,i)$ are assumed to be zero and the covariance matrices be identity matrix respectively.

The dimensions of the vectors $x(t,i), v(t,i), z(t,i), d(t,i), w(t,i)$ and $y(t,i)$, are respectively assumed $m_{\mathbf{x}i}(t), m_{\mathbf{v}i}(t), m_{\mathbf{z}i}(t), m_{\mathbf{d}i}(t), m_{\mathbf{w}i}(t)$ and $m_{\mathbf{y}i}(t)$. From Zhou [2], it is without loss of generality that every row of the matrix $\Phi(t)$ has only one nonzero element which is equal to one. Moreover, the elements of the corresponding diagonal blocks with compatible dimensions of $\Phi(t)$ are all zeros. These assumptions can be realized through some transformations and manipulations for an arbitrary connection relations among the subsystems. From the assumptions we know that $\Phi^T(t)\Phi(t) = \Sigma^2(t)$, in which $\Sigma^2(t) = \mathbf{diag}\{\Sigma_j^2(t)|_{j=1}^{N}\}, \Sigma_j^2(t) = \mathbf{diag}\{m(t,i)|_{i=M_{\mathbf{z},j-1}(t)+1}^{M_{\mathbf{z},j}(t)}\}, M_{\mathbf{z},j}(t) = \Sigma_{k=1}^{j} m_{\mathbf{z}k}(t), j = 1, \ldots, N$, and $m(t,i)$ denotes the number of subsystems affected directly by the i-th element of the vector $z(t)$.

To make algebraic derivations more compact, the following vectors or matrices are defined. $A_{*\#}(t) = \mathbf{diag}\{A_{*\#}(t,i)|_{i=1}^{N}\}$, $B_{\mathbf{T}}(t) = \mathbf{diag}\{B_{\mathbf{T}}(t,i)|_{i=1}^{N}\}$, $C_*(t) = \mathbf{diag}\{C_*(t,i)|_{i=1}^{N}\}, D_{\mathbf{T}}(t) = \mathbf{diag}\{D_{\mathbf{T}}(t,i)|_{i=1}^{N}\}$, where $*, \# = \mathbf{T},\mathbf{S}$, and $f(t) = \mathbf{col}\{f(t,i)|^{N}\}, f = x, y, u$, Let $R(t,i) = D(t,i)D^T(t,i)$, then $R(t) = \mathbf{diag}\{R(t,i)|_{i=1}^{N}\}$. From straightforward algebraic manipulations, the dynamic system $\mathbf{\Sigma}$ can be equivalently described by the following state space representation,

$$\begin{bmatrix} x(t+1) \\ y(t) \end{bmatrix} = \begin{bmatrix} A(t) & B_{\mathbf{T}}(t) & 0 \\ C(t) & 0 & D_{\mathbf{T}}(t) \end{bmatrix} \begin{bmatrix} x(t) \\ d(t) \\ w(t) \end{bmatrix}, \tag{3}$$

in which

$$A(t) = \mathcal{F}_u\left(\begin{bmatrix} A_{\mathbf{SS}}(t) & A_{\mathbf{ST}}(t) \\ A_{\mathbf{TS}}(t) & A_{\mathbf{TT}}(t) \end{bmatrix}, \Phi(t) \right),$$

$$C(t) = \mathcal{F}_u\left(\begin{bmatrix} A_{\mathbf{SS}}(t) & A_{\mathbf{ST}}(t) \\ C_{\mathbf{S}}(t) & C_{\mathbf{T}}(t) \end{bmatrix}, \Phi(t) \right). \tag{4}$$