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# Network organization as a dynamical system ⋆

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**Abstract:** Real-world networks are not static, but continuously change to meet the evolving needs of society. To manage and control such dynamic networks, we studied a simple model of co-evolving network dynamics, combining the dynamics of random walkers and the dynamics of weighted connections that are regulated by the traffic of the walkers. Under suitable conditions, the density of the walkers and the link weights converged to stationary power-law distributions at the macroscopic level. However, they continued to change with time at the microscopic level, even though the dynamics of the proposed model is completely deterministic. We numerically analyzed the equilibrium states from perspective of the dynamical system and found that the system has multi-stability including chaotic states.



*Keywords:* Networks, Complex systems, Chaos, Adaptive network, Power-law distributions

## 1. INTRODUCTION

The term *network* is commonly used in broad research fields, including physics, mathematics, biology, computer science, engineering, and sociology. The word is usually used to represent complex relationships of the interactions observed in these research fields. The structure of the interactions is simply described by an adjacency matrix $a_{ij}$, in which $a_{ij}$ =1 or 0 depending on whether the interaction between elements $i$ and $j$ exists or not, respectively. This abstract representation has provided a general framework to analyze network features observed in the real-world networks (Watts and Strogatz, 1998; Albert and Barabási, 2002; Newman, 2003). Moreover, many types of dynamical processes on the networks have been studied to understand the effects of the network statistical structures (Barrat et al., 2008; Boccaletti et al., 2006).

Today, the dynamic aspect of complex networks is a growing interest for many disciplines (Gross and Blasius, 2008; Sayama et al., 2013). Real-world complex networks are continuously changing with time, in response to alterations in the states of the network. For example, consider networks for the transport of people and products between and within cities. As cities develop or decay, traffic networks are frequently reformed to meet current needs, either by the construction of new roads or the closure of existing roads. Network re-formation causes changes in traffic patterns and influences further expansion or contraction in cities. Thus, the re-formation of road networks is interdependent with the traffic dynamics. Another example is that web browsing traffic is guided by links on the web. Heavy traffic at one site will lead to the creation of additional links to that site. A similar process occurs in biological systems. In a neural network, the nerve cells in the brain connect to other cells via synapses, whose connections drastically changes in response to neuronal activities (Hebb, 1949; Bi and Poo, 1998). Another example is the autocatalytic gene regulatory network, in which the rates of interactions dynamically changes depending on the the amount of the resultant reaction products of the gene regulatory system. To appreciate how networks will change and to know how to manage them, we need to understand the dynamics of the state-dependent networks.

In this paper, we study a model of state-dependent network dynamics (Aoki and Aoyagi, 2012; Aoki et al., 2015), which combines the dynamics of random walkers on a weighted network with the dynamics of the link weights driven by a resource carried by the walkers. In Section 2, we describe the model that we use in this paper. In Section 3, we demonstrate that the power-law distributions of the resource and link weights emerge spontaneously, under feasible conditions. In this situation, the amount of resource at individual nodes and their link weights continue to change at the microscopic level. We investigate the behavior of this dynamical network, looking into the growth and decay process of the resource-rich nodes. Finally, Section 4 summarizes our findings.

## 2. MODEL

Diffusion is a fundamental process that is related to many physical and social phenomena of real-world networks, such as traffics, transports, human mobility, information disseminations and epidemic spreading (Boccaletti et al., 2006; Barrat et al., 2008; Vespignani, 2012). Therefore, we

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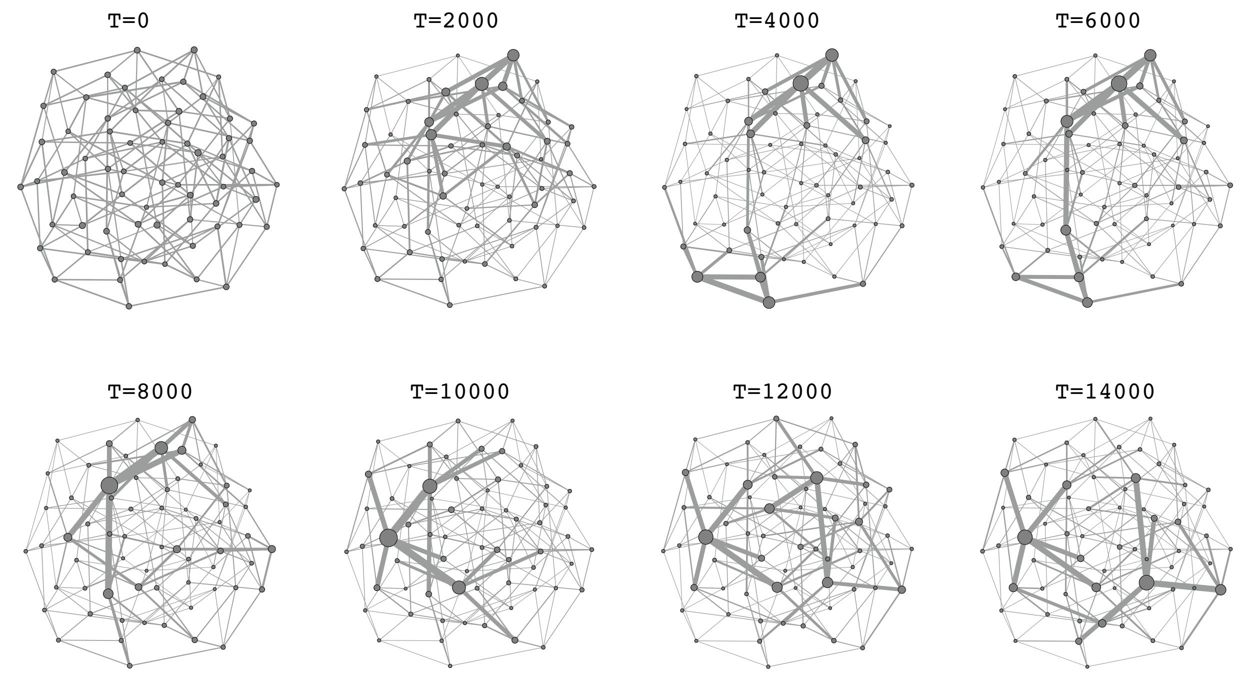


Fig. 2. Time development of the quantity of the resource at nodes and the weights of the connections, organized through the co-evolving dynamics defined by equations (3) and (4). In the graph, the size of the circle represents the quantity of the resource at the node, and the width of the link represents the weight of the link. The initial values of the resource and the weight are generated with mean $\mu = 1$ and standard deviation $\sigma = 0.1$. The underlying topology is given by a regular random graph of size $N = 64$ and degree $k = 5$). The other parameter values are $\epsilon = 0.01$, $\kappa = 0.05$, and $D = 0.35$.

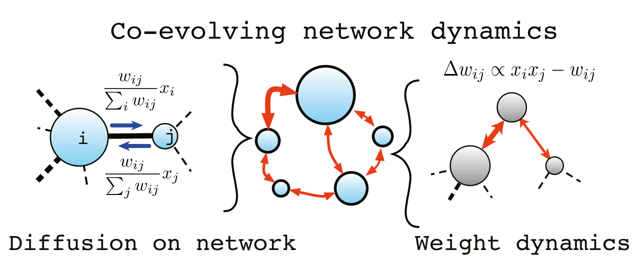


Fig. 1. Schematic illustration of co-evolving network dynamics, in which the link weights of the network and dynamical states of the nodes influence each other. In our model, the quantity of resource $x_i$ of the $i$th node is transported diffusively to the connected nodes through the weighted links $w_{ij}$, while the weights of the links evolve in a resource-dependent manner (i.e., the law of mass action).

consider a diffusion process on a weighted network as a paradigm for dynamical processes on networks. The state of each node at time $t$ is represented by the current amount of the diffusive resource at the node, $x_i(t)$. In real-life situations, the resource may be molecules, cells, people, money, data packets, and so on. The network topology with $N$ nodes is given by an adjacency matrix $a_{ij}$. The link weight from $j$-th to $i$-th node for each existing link is denoted by $w_{ij}$. We assume that the resource diffuses over a weighted network carried by many random walkers. Then, the diffusion process is described by

$$\frac{dx_i(t)}{dt} = F(x_i(t)) + D \sum_{j \in \mathcal{N}_i} \left[ T_{ij}(t) x_j(t) - T_{ji}(t) x_i(t) \right], \tag{1}$$

where $D$ is a diffusion scale parameter that controls the strength of diffusion over the whole system. The diffusion matrix $T_{ij}(t)$ is given by $w_{ij}(t)/s_j(t)$ and $s_j(t)$ ($= \sum_i w_{ij}(t)$) is the strength of the node $j$. $\mathcal{N}_i$ is the set of nodes connected to $i$-th node. The second and third terms are the inward and outward currents of the resource, respectively. The first term $F(x)$ describes a reaction process at the node, which represents the intrinsic dynamics of the resource except for the diffusion process. We assume a simple dissipation process with an equilibrium state, given by

$$F(x) = -\kappa(x - 1),$$

where $\kappa$ is a decay constant. The total amount of resource over the network, $\sum_i x_i(t)$, always converges to $N$, according to the equation, $\frac{d \sum_i x_i(t)}{dt} = -\kappa(\sum_i x_i(t) - N)$. To maintain the total amount of resource, the initial amounts of the resource at the nodes are constrained to satisfy the condition, $\sum_i x_i(0) = N$.

A significant feature of the network is that the diffusion matrix $T_{ij}(t)$ changes with time depending on the amounts of resource. For the sake of simplicity, we assume that the topology of the network $a_{ij}$ is constant and that the link weights $w_{ij}(t)$ for existing links ($a_{ij}$ =1) are time-dependent. Thus, we consider the dynamics of the link weights (i.e. interaction strength) to be a function of the resource.