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Multiple Delayed Feedback Control of Discrete-time Quasi-periodic Orbits ⋆

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Abstract: The multiple delayed feedback control (MDFC) is proposed for stabilizing discrete-time quasi-periodic orbits (QPOs). There is an inevitable difference between the current and past states in QPOs. In the MDFC, we estimate the current state by the interpolation method using the multiple past states. We apply the MDFC to the sampled-data control of a continuous-time periodic orbit, because the sampled time series is quasi-periodic if the sampling duration is rationally independent of the period of the periodic orbit.

Keywords: quasi-periodic orbit, delayed feedback control, interpolation, sampled-data control, chaotic system

1. INTRODUCTION

The delayed feedback control (DFC) has been proposed by Pyragas to stabilize unstable periodic orbits in chaotic systems (Pyragas (1992)). The discrete-time version of the DFC is defined by

$$x_{n+1} = F(x_n) + Ku_n, \tag{1}$$
$$u_n = x_{n-d} - x_n, \tag{2}$$

where $x_n \in \mathbb{R}^N$ is the state, F is the function of the system, K is the feedback coefficient, u_n is the feedback input, and d is the delay corresponding to the period of the unstable periodic orbit. If the stabilization is achieved, the feedback input u_n vanishes. This is an advantage of the DFC, because the control can be achieved by a small feedback input.

If we consider the DFC of unstable quasi-periodic orbits (QPOs), there is an inevitable time-delay mismatch because there is no delay d such that $x_{n-d} = x_n$. However, we can choose recurrence time d such that the difference between the current and past states is always small:

$$|x_{n-d} - x_n| < \epsilon, \tag{3}$$

for a small $\epsilon > 0$. We can stabilize the QPO by using recurrence time d as the delay of the DFC (Ichinose and Komuro (2014)).

Novičenko and Pyragas (2012) have shown that the DFC having a small time-delay mismatch can be evaluated by using the phase reduction method. They constructed a phase response curve of the DFC with the exact delay and evaluated the difference of period in the mismatch system. We have applied their idea of the phase reduction method to the DFC of QPOs and shown that the mismatch of the control orbit is consistent with the time-delay mismatch (Ichinose and Komuro (2014)). However, the problem that the feedback input cannot vanish remains unsolved. As a result, there always exists a difference between the orbit of the DFC and the unstable QPO.

In this work, we propose the method of multiple delayed feedback control (MDFC) (Ahlborn and Parlitz (2004)) in which the current state is interpolated by multiple past states. The MDFC can give a smaller feedback input and a closer control orbit to the unstable QPO than those of the single DFC. We apply the MDFC to the sampled-data control of unstable periodic orbits in continuous-time systems. We assume that sampling duration τ is rationally independent of period T of the periodic orbit, *i.e.*, $k_1\tau + k_2T \neq 0$ for all $k_1, k_2 \in \mathbb{Z}\backslash\{0\}$. Then, the sampled time series is quasi-periodic. We show that the MDFC can be achieved by relatively long sampling duration.

2. MULTIPLE DELAYED FEEDBACK CONTROL

We assume that the control-free system $x_{n+1} = F(x_n)$ has an unstable QPO defined on an invariant closed curve. The rotation number ω is an important invariant of QPOs, because a QPO is topologically conjugate to the irrational rotation (MacKay (1988)):

$$\theta_{n+1} = \theta_n + \omega, \tag{4}$$

where $\theta_n \in \mathbb{S}$ is the phase of the circle $\mathbb{S} = \mathbb{R}/\mathbb{Z}$. Let $\tilde{x}_n$ be the unstable QPO in the system. By the topological conjugacy, the QPO is associated with the irrational rotation via the homeomorphism ψ:

$$\tilde{x}_n = \psi(\theta_n). \tag{5}$$

We consider the single DFC (1) and (2) of the unstable QPO. We assume that the stabilization of the QPO is

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achieved, *i.e.*, $x_n = \tilde{x}_n$. The phase of the past state x_{n-d} is defined by

$$\theta_{n-d} = \theta_n - d\omega. \tag{6}$$

If ω is irrational, there exists no delay d such that $d\omega \in \mathbb{Z}$. Therefore, $\theta_{n-d} \neq \theta_n$ always holds and the feedback input never vanish:

$$u_n = x_{n-d} - x_n = \psi(\theta_n - d\omega) - \psi(\theta_n) \neq 0, \tag{7}$$

even if the achievement of the control is assumed. Our idea is that we construct the interpolation of ψ by using multiple past states and approximately determine $\psi(\theta_n)$.

We use the Lagrange polynomial as the interpolation method. Let (η_i, ν_i) $(i = 1, 2, \ldots, M)$ be given data of the pairs such that $\nu_i = \psi(\eta_i)$. The Lagrange polynomial L is defined as follows:

$$L(\theta) = \sum_{i=1}^{M} \nu_i f_i(\theta), \tag{8}$$

where f_i is the coefficient defined by

$$f_i(\theta) = \prod_{j=1, j\neq i}^{M} \frac{\theta - \eta_j}{\eta_i - \eta_j}. \tag{9}$$

By a certain set of (η_i, ν_i), the Lagrange polynomial can estimate the homeomorphism ψ: $\psi(\theta) \approx L(\theta)$.

We here estimate $\psi(\theta_n)$ by using multiple past states. We assume that a set of multiple delays $D = \{d_1, d_2, \ldots, d_M\}$ is given. Then, η_i and ν_i are assigned as follows:

$$\eta_i = \theta_{n-d_i} = \theta_n - d_i\omega, \quad \nu_i = \tilde{x}_{n-d_i}. \tag{10}$$

We can rewrite the coefficient f_i at θ_n by

$$f_i(\theta_n) = \prod_{j=1, j\neq i}^{M} \frac{[\theta_n - \theta_{n-d_j}]}{[\theta_{n-d_i} - \theta_{n-d_j}]} = \prod_{j=1, j\neq i}^{M} \frac{[d_j\omega]}{[d_j\omega - d_i\omega]}, \tag{11}$$

where $[\cdot]$ is the operator translating $\mathbb{S}$ to $\mathbb{R}$:

$$[\theta] = \begin{cases} \theta - \lfloor\theta\rfloor & \text{if } \theta - \lfloor\theta\rfloor < 0.5 \\ \theta - \lfloor\theta\rfloor - 1 & \text{otherwise} \end{cases}, \tag{12}$$

where $\lfloor\cdot\rfloor$ is the floor function. It should be noted that $f_i(\theta_n)$ is independent of n. Therefore, the Lagrange polynomial estimating $\psi(\theta_n)$ can be shown by the weighted sum of the past states:

$$\psi(\theta_n) \approx L(\theta_n) = \sum_{i=1}^{M} c_i \tilde{x}_{n-d_i}, \tag{13}$$

where c_i is the constant coefficient:

$$c_i = \prod_{j=1, j\neq i}^{M} \frac{[d_j\omega]}{[d_j\omega - d_i\omega]}. \tag{14}$$

Using the Lagrange polynomial, we define the feedback input u_n of the MDFC:

$$u_n = \sum_{i=1}^{M} c_i x_{n-d_i} - x_n. \tag{15}$$

3. SAMPLED-DATA CONTROL OF CONTINUOUS-TIME PERIODIC ORBITS

We consider the continuous-time system defined by

$$\frac{dx}{dt} = G(x), \tag{16}$$

and obtain sampled data with the sampling duration τ. Then, the sampled-data system can be shown as the discrete-time system:

$$x(t_{n+1}) = F(x(t_n)) = x(t_n) + \int_{t_n}^{t_{n+1}} G(x)dt, \tag{17}$$

where $t_{n+1} - t_n = \tau$ for all n. We assume that the continuous-time system has an unstable periodic orbit with period T. The sampled time series of the continuous-time periodic orbit is quasi-periodic, if τ and T are rationally independent of each other. In this case, we can immediately determine the rotation number of the QPO:

$$\omega = \frac{\tau}{T}. \tag{18}$$

The rational independence between τ and T implies that ω is irrational. Therefore, we can apply the MDFC to this sampled-data system:

$$x(t_{n+1}) = F(x(t_n)) + u(t_n), \tag{19}$$

$$u(t_n) = \sum_{i=1}^{M} c_i x(t_{n-d_i}) - x(t_n). \tag{20}$$

Note that the feedback input is an impulsive signal in the continuous-time system:

$$\frac{dx}{dt} = G(x) + \delta(t - t_n)u(t_n), \quad t_n \leq t < t_{n+1}, \tag{21}$$

where δ is the delta function.

We apply the MDFC to the Rössler system:

$$\frac{dx}{dt} = -y - z, \tag{22}$$

$$\frac{dy}{dt} = x + ay, \tag{23}$$

$$\frac{dz}{dt} = b + z(x - c). \tag{24}$$

We fix the parameters at $(a, b, c) = (0.2, 0.2, 5.7)$ where the system is chaotic. We estimate an unstable periodic orbit by using the continuous-time version of the original DFC. We choose an unstable periodic orbit with period $T = 11.759$. We fix the sampling duration at $\tau = 0.5$. Then, there are 24 sampling points a period on average. The rotation number is $\omega \approx 0.0425$. The feedback input is given only to the state y:

$$K = \begin{pmatrix} 0 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{25}$$

In Fig. 1, we compare the control orbit of the single DFC and the unstable periodic orbit. In this case, there is only a delay and we choose $d = 24$ that is equivalent to the average sampling points for a period. We show these