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Numerical investigation of electroconvection induced by strong unipolar injection between two rotating coaxial cylinders

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ABSTRACT

In this paper, the interaction between a Couette shear flow and an electroconvective motion induced by an unipolar injection between two-coaxial cylinders is numerically investigated. A flow is generated by two counter-rotating coaxial cylinders inducing a shear flow. Space charges are injected in the flow through a metallic electrode placed on the inner cylinder and brought to a given potential. Transient numerical simulations have been carried out to investigate the structure of the induced flow. The entire set of the coupled Navier-Stokes and EHD equations is solved using an efficient finite volume technique. The behaviour of the flow subjected to an applied voltage between the two electrodes is analyzed and time evolution of the charge density distributions is presented. The interaction between the convective motion induced by space charge injection and the mainstream flow, emphasizes the appearance of periodic counter-rotating electroconvective cells. The electroconvective cells are convected in the annular space by the azimuthal fluid velocity. From the stability point of view the bifurcation diagram is very similar to the one obtained in the case when $Re = 0$. We observe a threshold value T_c of the instability parameters T above which the electroconvective instability initiates. A non-linear criterion T_f under which the electro-convective motion is suppressed is also found. When increasing the Reynolds number the flow induced by the two rotating cylinders has a sweeping effect on the charge density distribution. Consequently the instability parameter T must be drastically increased to allow the electroconvective instability to develop. For $Re = 10$ a subcritical instability characterized by an hysteresis loop and therefore a linear and non-linear criteria, T_c and T_f respectively, are determined. While for $Re = 0$, $T_c = 122.42$ and $T_f = 86.5$, for $Re = 10$ we numerically found that $T_c = 802$ and $T_f = 722$. The magnitude of the linear and non-linear criteria are directly linked to the value of the Reynolds number.

1. Introduction

Considerable interest has been shown in recent years in fluid motion driven by Coulomb force which arises in many natural situations and industrial processes. The resulting flow which occurs when an electric field is applied across a dielectric liquid layer containing electric charges, have received much attention by the scientific engineering and industrial communities. In most of these situations the electric field is the cause of the movement of the flow itself by electroconvection. In this study we examine the effect of a circular Couette shear flow on a radially Coulomb driven electroconvection in a two-dimensional annular fluid. Numerical simulations are carried out to investigate electroconvective phenomena in a dielectric liquid confined between two counter-rotating coaxial cylinders. In an unsheared case (absence of rotational motion from the two cylinders) strong unipolar injection of ions either from the inner or outer cylinder leads to the development of electroconvective instability [1–3]. Similarly as in the case of two planar electrodes, the flow is characterized by the development of a subcritical bifurcation in the finite amplitude regime [4–6]. In this situation, a linear stability criterion T_c and a nonlinear one T_f that correspond to the onset and stop of the flow motion, respectively, are linked with an hysteresis loop [3]. When the cylinders are set into an angular motion, this annular geometry and the Couette shear induces the development of another instability: The Taylor-Couette instability. However the flow between concentric cylinders is only unstable for 3D geometry and 2D Couette flow is by itself stable [7]. In this geometry the shearing effect induced by the angular motion of the cylinders does not lead to the development of a second instability and thus no

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competition between electroconvective and Taylor-Couette instabilities could be expected. However several interesting phenomena and stabilizing effect are observed when pure electroconvection is superimposed with a Couette shear flow. The most important effect of the Couette flow is in fact to suppress the onset of electroconvection [8]. In Ref. [9] the authors have experimentally investigated the bifurcation that arises in an electrically-driven convection layer submitted to an imposed shear due to the rotation of two coaxial cylinders. They showed that this bifurcation could be supercritical or either subcritical, depending on the radii ratio of the two cylinders or on the Reynolds number based on the azimuthal velocity. They have successfully demonstrated that the flow could remain two-dimensional in this geometrical configuration. The purpose of this article is to investigate numerically how and under which conditions the shear flow interacts with the development of the subcritical electroconvective instability. In particular we shall determine how the linear and non-linear critical values T_c and T_f respectively are affected by the Reynolds number based on the azimuthal velocity: $Re = \frac{\rho_0 R_0 \omega_0 d}{\mu_0}$ where ω_0 is the angular velocity of the inner cylinder, and $d = R_1 - R_0$ where R_O et R_1 are respectively the radius of the inner and outer cylinders. ρ_0 and $\mu_0$are the fluid density and fluid dynamic viscosity respectively. In the following section we state the problem and its governing equations. We shall describe too the numerical method used in this study. The results are discussed in Section 3. Finally, a conclusion is summarized in section 4.

2. Problem formulation and numerical method

2.1. Governing equations

The system under consideration in this article is a dielectric liquid layer enclosed between two concentric cylinders of radius R_O and R_1 respectively (Fig. 1). The layer of a perfectly incompressible and insulating liquid is subjected to an electrical potential difference $\Delta V = V_0 - V_1$ which will induce a radial electric field $\vec{E}$. Under the action of this electric field and due to complex electrochemical processes at the emitter electrode electric charge injection into the bulk will occur. We consider the case of unipolar injection, which means that ions are injected from one electrode only. The emitter electrode will be the inner cylinder and an amount of charge q_O is injected into the bulk. The collector electrode is thus the outer cylinder which is grounded. The inner cylinder has an angular velocity $\omega_0$and the outer one, an angular velocity ω_1 See Fig. 1.

The complete formulation of a dielectric liquid subjected to electric field is governed by the following EHD equations [10]. We consider the limit case of homogeneous and autonomous unipolar injection, which means that the charge injection arises from one electrode and the density of injected charges is always constant and not related to the electric field [11].

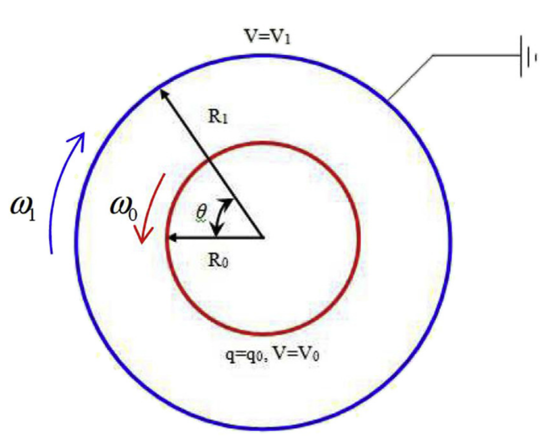

Fig. 1. Sketch of the physical domain.

The problem is formulated considering the usual hypotheses of a Newtonian and incompressible fluid of dynamic viscosity $\mu_0$and density ρ_0, governed by the Navier-Stokes and Electo-Hydro-Dynamic (EHD) equations as follows:

$$\nabla \cdot \vec{u} = 0 \quad (1)$$

$$\rho_0 \left(\frac{\partial \vec{u}}{\partial t} + (\vec{u}\,.\,\nabla)\vec{u} \right) = -\nabla \tilde{p} + \mu_0 \nabla^2 \vec{u} + q\vec{E} \quad (2)$$

$$\frac{\partial q}{\partial t} + \nabla \cdot (q(\vec{u} + K\vec{E})) = 0 \quad (3)$$

$$\nabla^2 V = -\frac{q}{\varepsilon} \quad (4)$$

$$\vec{E} = -\nabla V \quad (5)$$

where $\vec{u}$ is the fluid velocity, $\tilde{p}$is the modified pressure which includes the contribution from the electrostriction force term [11]. q is the volumic charge density, K and $\varepsilon$are respectively the ionic mobility and the permittivity of the liquid in consideration.

For a sake of universal description for such studies it is particularly convenient to work with dimensionless equations. In order to transform the last set of equations into a dimensionless form we introduce the following dimensionless quantities denoted with a star:

$$x_i^* = \frac{x_i}{d}\ \rho^* = \frac{\rho}{\rho_0}\ u_i^* = \frac{u_i}{u_0}\ q^* = \frac{q}{\varepsilon(V_0 - V_1)/d^2}\ \tilde{p}^* = \frac{\tilde{p}}{\rho_0 u_0^2}\ V^* = \frac{V}{(V_0 - V_1)}\ E_i^* = \frac{E_i}{(V_0 - V_1)/d}$$

This leads to the following set of dimensionless parameters:

$T = \frac{\varepsilon\,(V_0 - V_1)}{\mu_0 \rightleftarrows K_0}$ is the electrical Rayleigh number which accounts for the Coulomb and viscous forces.$C = \frac{q_0 d^2}{\varepsilon\,\Delta V}$ is a dimensionless measure of the injection strength.$M = \frac{1}{K_0}\left(\frac{\varepsilon}{\rho_0}\right)^{1/2}$ accounts for the electro-hydrodynamic properties of the liquid.

$Re = \frac{\rho_0 u_0 d}{\mu_0}$ is the classical Reynolds number.

Several choices for the characteristic velocity $u_0$to scale the velocity field are possible. In this study the most obvious choice seems to use the azimuthal velocity of the inner cylinder $u_0 = R_0\omega_0$. The Reynolds number is thus: $Re = \frac{\rho_0 \omega_0 R_0 d}{\mu_0}$

For convenience we shall, also define$R = \frac{T}{M^2}$ which is known as the electrical Reynolds number.

If we drop the star indices for a sake of clarity, the set of dimensionless equation becomes:

$$\nabla \cdot \vec{u} = 0 \quad (6)$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\,.\,\nabla)\vec{u} = -\nabla p + \frac{1}{Re}\nabla^2 \vec{u} + \frac{M^2 R^2}{Re^2} q\vec{E} \quad (7)$$

$$\frac{\partial q}{\partial t} + \nabla \cdot \left(q\left(\vec{u} + \frac{R}{Re}\vec{E} \right) \right) = 0 \quad (8)$$

$$\nabla^2 V = -q \quad (9)$$

$$\vec{E} = -\nabla V \quad (10)$$

2.2. Numerical method

The numerical procedure used to solve the entire set of coupled Navier-Stokes and EHD equations is similar to the one already addressed in previous papers [11,12], and thus will not be discussed further more here. The set of coupled equations (6)–(9) are integrated with a second order in space and in time finite volume method [13]. The boundary conditions are depicted in the Fig. 2. All these numerical