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The influence of insulating and conductive ellipsoidal objects on the impedance and permittivity of media

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ABSTRACT

For ellipsoidal objects, the complex conductivity of the suspension depends on the objects' axis ratio and orientation. It can be described by analytical equations that were derived by combining the influential radius approach with the mixing equation of Maxwell and Wagner. Here, we consider conductive or insulating homogeneous spheroids, with their symmetry axes being oriented in parallel, in perpendicular or at random with respect to the external field. Considerations show that the field-induced orientations of both nonconductive and conductive objects will result in a reduction of the suspension's impedance and an increased dissipation of electrical energy.

1. Introduction

Impedance characterization is a common task in physics, chemistry, colloid sciences and biology [1–9]. The complex (marked by lower dash) specific impedance $\underline{\varrho}$ in Ωm or its reciprocal parameter, the effective complex conductivity (admittance) $\underline{\sigma}$ in S/m of homogeneous gaseous, liquid or solid media, is changed in the presence of objects or inclusions [10,11]. Whereas the conductivity of media is reduced by objects made of insulating material, such as gas, oil or plastics, their conductivity is increased in the presence of highly conductive objects, such as metallic particles. For a given volume fraction of the objects, the efficiency at which the objects decrease or increase the impedances of suspensions depends on the objects' shape and orientation with respect to the external electric field. It can be conceived that reorientation will induce the highest impedance alterations for disk- or needle-shaped objects, which are close to their limiting shapes. Accordingly, the limiting shapes suggest criteria for the efficient use of the objects' material to alter the impedance of suspensions. The limiting shape cases and the limiting ratios of the relative polarizabilities of the external medium and the objects may be of technological relevance, e.g., for the characterization of emulsions [12,13], in electrorheological applications [14–20], for particles or droplets in gas or air streams (see air/fuel ratio), for particles in waste gases or gas bubbles in liquids [21,22], or for fluids in sand [23,24].

Maxwell [10] was the first to derive an expression for the static resistance of a dilute suspension of monodisperse shelled spheres. Wagner [11] introduced complex specific conductivities into Maxwell's theory and simplified the expression by using a Taylor-series development around zero object concentrations. The obtained mixing equation has been expanded to the impedance of suspensions of single- and multi-shell ellipsoidal objects [8,25–29]. Currently, the term “Maxwell-Wagner dispersion” is used in a more general way for geometrically structured dielectric models.

In the 1920s, chemists used the spherical geometry to consider the influence that external electric fields have on the behavior of molecules in solutions. The molecules were assumed to occupy an otherwise empty spherical cavity corresponding to their gyroscopic radii. Later, depolarizing factors were introduced to describe the degree of the local field amplification inside spherical and ellipsoidal cavities [30]. After explicit expressions were derived for the depolarizing factors of spheroidal objects [31], expressions for the general ellipsoidal shape were found [32–34]. These expressions have later been applied to model the polarization of ellipsoidal single- and multi-shell objects suspensions [35,36].

Our influential radius approach provides a more electro-technical view at the depolarizing factors because it describes the local field amplification inside an ellipsoidal cavity in a direct manner and allows the electric and geometric problems to be separated for objects of the general ellipsoidal shape. Influential radii have been successfully applied in the modeling of AC-electrokinetic effects [37–39], the induced transmembrane potential of biological cells [37,40,41] and the impedance of suspensions [28,39].

In this manuscript, the influential radius approach is used to consider the impedance of suspensions of oriented and randomly oriented

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objects of the general ellipsoidal shape. For the polarization of spheroidal objects (i.e., ellipsoids of rotation), closed analytical solutions can be derived [40]. These solutions can be approximated by simplified equations [42] to avoid the use of complex equations for the depolarizing factors. The obtained equations permit the easy derivation of solutions for the limiting shapes of flat oblate spheroids (disks), spheres, or long prolate spheroids (needles). The limiting impedance cases for insulating and highly conductive spheroids of oblate and prolate shapes are summarized in the two appendices.

2. The model

2.1. Object geometry, depolarizing factors and influential radii: separating the electrical and geometric models

For an ellipsoidal cavity or object, depolarizing factors (n^a, n^b, n^c) are defined along the three principal semiaxes: a, b, and c. The sum of the depolarizing factors is always unity [30–34]:

$$n^a + n^b + n^c = 1 \tag{1}$$

The local field is constant inside homogeneous objects confined by surfaces of the second degree. Inside vacuum cavities or homogeneous objects of very low polarizability (negligible electric susceptibility), an external field (or field component) E_0^a parallel with the semiaxis a of the object induces the local field (or local field component) [38,43]:

$$E_{loc}^a = \frac{1}{1-n^a}E_0^a \tag{2}$$

Please note that for simplicity, the vector notation was not used. Inside spherical cavities with $n^a = n^b = n^c = 1/3$, the local field E_{loc}^a along semiaxis a is increased by a maximum factor of $\frac{1}{1-n^a} = \frac{3}{2}$ with respect to the undisturbed homogeneous external field E_0^a. In the ellipsoidal case, the maximum potential of Ψ^a at the pole of semiaxis a is (Fig. 1)

$$MAX(\Psi^a) = MAX(aE_{loc}^a) = \frac{a}{1-n^a}E_0^a = a_{\inf}E_0^a \tag{3}$$

with E_{loc}^a, $a_{\inf}$, and $a_{\inf}^{rel} = \frac{a_{\inf}}{a} = \frac{1}{1-n^a}$ being the constant local field inside the homogeneous object, the influential radius, and the relative influential radius (field amplification factor) along semiaxes a, respectively [42]. Along each principal semiaxis, the influential radii are the distances from the object's respective symmetry planes to those undisturbed equipotential planes that are just touching the respective pole of a "vacuum" object of the identical shape [38]. The properties of such objects are approached for $|\underline{\sigma}_i| \ll |\underline{\sigma}_e|$, with $\underline{\sigma}_i = \sigma_i + j\omega\varepsilon_0\varepsilon_i$ and $\underline{\sigma}_e = \sigma_e + j\omega\varepsilon_0\varepsilon_e$ being the complex specific conductivities of the internal (subscript i) and external (subscript e) media.

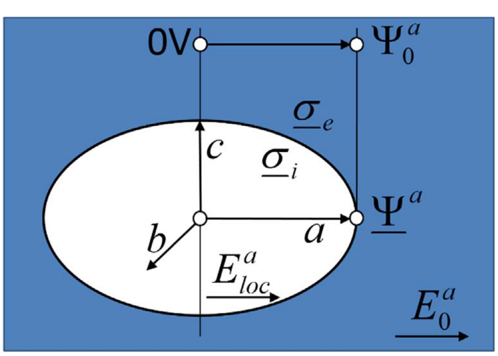

Fig. 1. Sketch illustrating the potentials induced by the external field component E_0^a at pole a of a homogeneous ellipsoidal object with the semiaxes a, b, and c. The potentials $\underline{\Psi}^a$ and Ψ_0^a are induced with respect to the reference potential of 0 V at the symmetry plane of the object in the presence and absence of the object. In the general case, the complex potential $\underline{\Psi}^a = a\underline{E}_{loc}^a$ is given by the effective constant (local) field $\underline{E}_{loc}^a$ inside Maxwell's equivalent body.

Fig. 1 illustrates the relations for the general AC case, in which the induced potentials and local fields may be out-of-phase with the external field. A reference potential of 0 V can be assumed at the symmetry plane of the object without limitation in generality. Please note that the maximum possible DC (or AC) potentials and local fields are in-phase with the inducing fields. For a more detailed consideration, please see Ref. [39].

From Eqs. (1) and (3), for the inverse relative influential radii follows [38]:

$$\frac{a}{a_{\inf}} + \frac{b}{b_{\inf}} + \frac{c}{c_{\inf}} = 2 \tag{4}$$

2.2. The Clausius-Mossotti factor

For a homogeneous ellipsoidal object of volume

$$V = \frac{4\pi}{3}abc, \tag{5}$$

the component $\underline{m}^a$ of the induced dipole moment along semiaxis a is proportional to the external field component E_0^a and the permittivity $\varepsilon_e\varepsilon_0$ of the external medium:

$$\underline{m}^a = \varepsilon_e\varepsilon_0 V \underline{f}_{CM}^a E_0^a \tag{6}$$

The complex Clausius-Mossotti factor,

$$\underline{f}_{CM}^a = \frac{\underline{\sigma}_i - \underline{\sigma}_e}{\underline{\sigma}_e + (\underline{\sigma}_i - \underline{\sigma}_e)n^a}, \tag{7}$$

describes the frequency dependence of $\underline{m}^a$ [34]. As an alternative to the (effective) media properties, "measuring parameters" can be used:

$$\underline{f}_{CM}^a = \frac{1}{n^a}\left(\frac{E_0^a - \underline{E}_{loc}^a}{E_0^a}\right) = \frac{1}{n^a}\left(\frac{\Psi_0^a - \underline{\Psi}^a}{\Psi_0^a}\right) \tag{8}$$

in the form of normalized differences of the constant local field $\underline{E}_{loc}^a$ in the presence and the undisturbed field E_0^a in the absence of the object [34] or of the potentials at pole a in the presence ($\underline{\Psi}^a$) and absence (Ψ_0^a) of the object [38].

Depending on the effective electrical properties of external medium and object, the actual potentials at the three poles can be calculated from simple voltage dividers between the maximum potentials of $a_{inf}E_0^a$, $b_{inf}E_0^b$, or $c_{inf}E_0^c$ and 0 V at the three symmetry planes of the object (Fig. 1). Each divider is formed by geometric elements with the electric bulk properties of the object (subscript i) and the external medium (subscript e). Along axis a, the two elements possess the complex impedances $\underline{Z}_i^a$ and $\underline{Z}_e^a$, which can be described by resistor-capacitor (RC) pairs for most of the media. The RC properties are determined by the geometry, the specific conductivity and the permittivity of the bulk media.

In the general case, the attenuation of the divider is frequency-dependent, generating the following potential

$$\underline{\Psi}^a = \frac{\underline{Z}_i^a}{\underline{Z}_i^a + \underline{Z}_e^a}a_{\inf}E_0^a \tag{9}$$

at pole a [37]. The impedances of the two elements are

$$\underline{Z}_i^a = \frac{a}{\underline{\sigma}_i}A \quad \text{and} \quad \underline{Z}_e^a = \frac{a_{\inf} - a}{\underline{\sigma}_e}A \tag{10}$$

Please note that homogeneous objects resemble the so-called Maxwell-equivalent bodies for objects with confocal shells [10] and frequency-dependent effective polarizabilities [11,38,44].

2.3. Limiting cases for the Clausius-Mossotti factor

The pole potentials exhibit two limiting cases. At pole a, these pole potentials are $\underline{\Psi}_{loc}^a = a_{\inf}E_0^a$ and $\underline{\Psi}^a = 0V$ for the limiting cases of $|\underline{Z}_i^a| \gg |\underline{Z}_e^a|$ (nonconductive, low polarizable or vacuum object) and