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Numerical study on transient response of droplet deformation in a steady electric field

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ABSTRACT

The transient response of droplet deformation in a steady electric field is investigated by the numerical simulation and the motion of interface is captured by level-set method. The numerical scheme is validated and found to be in good agreement with classic analytical solutions. The effects of electric field intensity, interfacial tension, oil viscosity and droplet size on the transient deformation process are systematically discussed. The numerical results show that electric field intensity can accelerate the deformation of the droplet, while interfacial tension and oil viscosity damp it. Furthermore, the relation between electric capillary number and dimensionless deformation time is obtained.

1. Introduction

A droplet suspended in another immiscible fluid will deform under an externally applied electric field. Applications of this phenomenon encompass electrospinning [1], electrospraying [2], electrostatic coalescence of droplets for emulsification purposes [3,4] and electrowetting-based droplet manipulation in microfluidic systems [5], to name a few.

For a better understanding of droplet deformation under the influence of electric field, a number of experimental and numerical researches have been conducted from different perspectives [6–9]. The mathematical description of this phenomenon has been established, and some aspects of the equilibrium and transient process under different types of electric field have been substantially investigated.

At first, Taylor [10] proposed an electrohydrodynamic model for the deformation of a conducting droplet suspended in another immiscible fluid under the influence of electric field based on the hypothesis of neutral droplet, quasi-static electric field, small deformation and absence of convection charge. The model, generally referred to as the "leaky dielectric model", can reasonably predict the deformation and has become a cornerstone of the theory of drop deformation under the external electric field. Nevertheless, Torza et al. [11] found that the discrepancy between experimental results and Taylor's analytical results may exist when the deformation degree is relatively large. Subsequently Ajayi [12] extended Taylor's linear theory to higher order and developed a new theoretical model with second order term. However, this model did not avoid the discrepancy. Torza et al. [11] generalized Taylor's model to AC electric field. The leaky dielectric model has been developing and new improved models are proposed over the decades [13,14].

With the rapid development of computer technology, a great number of researchers pay attention to numerical simulation of these problems due to its low cost and convenience. The pioneering work on the deformation of droplets subjected to electric field was carried out by Brazier-Smith [15,16]. He investigated the stability of conducting droplets and calculated the critical values of $E\,(R/T)^{0.5}$ required for the disintegration of a droplet. By means of the Galerkin finite-element method, Feng and Scott [17] investigated the nonlinear free-boundary problem of electrohydrodynamics of a neutrally buoyant droplet subjected to an electric field within the framework of the leaky dielectric model. Under conditions of creeping flow and vanishingly small droplet deformation, the results of finite-element computations were in excellent agreement with Taylor's asymptotic solutions. Lac and Homsy [18] studied the rules of deformation and stability of a suspended droplet in a

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steady electric field by synchronously solving the electric and the velocity fields with a boundary integral method. Shadloo et al. [19] studied the influence of the electric field intensity, permittivity, surface tension and initial droplet radius on the droplet deformation in detail by employing a smoothed particle hydrodynamics (SPH). Zhentao Wang et al. [20] investigated the deformation and internal flow of the droplet subjected to the electric field based on the volume of fluid (VOF) and leaky dielectric theory. They found that when RQ < 1, the internal electrohydrodynamic flow goes from the equator to poles. When RQ > 1, the internal electrohydrodynamic flow goes from the poles to the equator. Their results are well consistent with the theory of Taylor [11]. WeiFeng Huang et al. [21] simulated the deformation and instability of droplets in the electrostatic field by applying the lattice Boltzmann method (LBM). They obtained the critical permittivity ratio and the critical electric field intensity when the liquid droplet became unstable.

All the above mentioned researches are mainly confined to steady-state analysis and instability of the liquid droplet in the electric field. These results do not pertain to the transient process of droplet deformation. The transient response of the deformed droplet to the electric field is closely related to the interfacial properties, which could be used to characterize the properties and explore the main factors. However, the transient response of the droplet to an external electric field is rarely investigated. Berg et al. [22] developed an experimental method to study the oscillations of droplets with different rheological properties by means of applying various voltage waveforms. They modeled a damped oscillator for the droplet oscillation and investigated the influence of viscosity on the eigen-frequency and damping coefficient for different droplet sizes. Supeene et al. [23] stimulated the response of droplet deformation to a step change in the electric field for both perfect and leaky dielectric systems, investigating the influence of the fluid, interfacial and electrical properties on the dynamic process. Ye Yao et al. [24] numerically investigated the deformation of uncharged droplet under a steady non-uniform electric field by employing a three-dimensional spectral boundary element method and the leaky dielecric theory. They found that the droplet migration velocity is affected by the permittivity ratio and the resistivity ratio.

In this paper, the transient process of droplet deformation in a steady electric field is investigated by finite-element method and the motion of interface is captured by level-set method. The level-set method has several outstanding advantages [25]. The first is that the level-set method can simulate the change of topology, such as break, merge, and formation of sharp corner. The second advantage of level-set method is that intrinsic geometric properties of the moving interface are easily determined from the level function. For example, the curvation can be easily obtained from the divergence of the gradient of the unit normal vector to the interface. Furthermore, the level-set method is easy to be extended to the three-dimensional case on the algorithm. Afterwards the effects of key factors including electric field intensity, interfacial tension, oil viscosity and droplet size on the transient process are discussed in detail.

2. Problem description

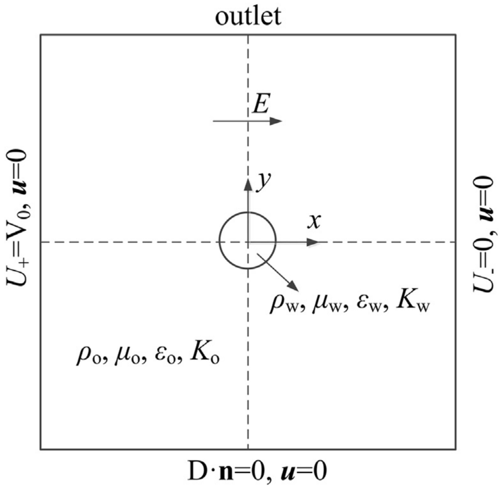

Fig. 1. Schematic diagram of computational model.

The schematic diagram of computational model is displayed in Fig. 1, a neutral water droplet with the density of ρ_w, the kinematic viscosity of μ_w, the relative dielectric constant of ε_w, and the conductivity of K_w is suspended in the oil phase with the density of ρ_o, the kinematic viscosity of μ_o, the relative dielectric constant of ε_o, and the conductivity of K_o. A square domain with the length of 5 mm is applied as the computational domain. The left positive voltage and right negative voltage are U_+ and U_- respectively, where U_- equals 0 V. The droplet will deform in the electrical field due to different physical properties of water and oil. Through coupling hydrodynamics and electrostatics, the effects of key factors including electric field intensity, interfacial tension, oil viscosity and droplet size on the deformation process are investigated in this paper.

3. Mathematical model

In order to study the dynamic problem of the deformation of the droplet in oil under an applied electric field, the Navier-Stokes equation should be solved numerically and the interface separating two immiscible fluids is supposed to be traced. The two-phase flow model coupling electrostatics and hydrodynamics is employed to investigate the complex physical problem. The effect of gravity is negligible because the droplet is micron-sized and the densities of oil and water are very close. In addition, the interfacial force and the electrical force are added to the Navier-Stokes as source terms. The mass conservation equation and the momentum conservation equation for incompressible fluid can be expressed as

$$\nabla\cdot\mathbf{u}=0, \tag{1}$$

and

$$\rho\frac{\partial\mathbf{u}}{\partial t}+\rho(\mathbf{u}\cdot\nabla)\mathbf{u}=\nabla\cdot\left[p\mathbf{I}+\mu\left(\nabla\mathbf{u}+(\nabla\mathbf{u})^T\right)\right]+\mathbf{F}_{st}+\mathbf{F}_{es}, \tag{2}$$

where ρ is the fluid density, μ is the fluid viscosity, $\mathbf{u}$ is the velocity vector, $\mathbf{I}$ is the identity matrix, p is the fluid pressure, $\mathbf{F}_{st}$ is the interfacial force of two immiscible fluids, $\mathbf{F}_{es}$ represents the electric force.

The level set method is employed to trace the boundary of deformed droplet. The interface thickness and mesh size are of the same order of magnitude, and the location of interface is determined by solving the transport equation of the level set function Φ. In general, the interface of two immiscible fluids is represented by the 0.5 contour of the level set function Φ, in water $\Phi=1$ and in oil $\Phi=0$. The level set function can be viewed as the volume fraction of water. The transport equation of the interface separating the two phases is given by

$$\frac{\partial\phi}{\partial t}+\mathbf{u}\cdot\nabla\phi=\gamma\nabla\cdot\left(\varepsilon_{ls}\nabla\phi-\phi(1-\phi)\frac{\nabla\phi}{|\nabla\phi|}\right), \tag{3}$$

where γ and ε_{ls} are the reinitialization parameters, γ determines