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Nonlinear observer and controller design for sensorless operation of a continuously rotating energy harvester ⋆

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Abstract: This paper presents the design of a nonlinear observer and a nonlinear feedback controller for sensorless operation of a continuously rotating energy harvester. A mathematical model of the harvester with its power electronic interface is discussed. This model is used to design an observer that estimates the mechanical quantities from the measured electrical quantities. The gains of the observer depend on the solution of a modified Riccati equation. The estimated mechanical quantities are used in a control law that sustains power generation across a range of source rotation speeds. The proposed scheme is assessed through simulations and experiments.

Keywords: Nonlinear observer, sensorless, energy harvester, nonlinear controller, Riccatti equation

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1. INTRODUCTION

Energy harvesting devices have become a viable method of powering sensor networks and low-power electronics using ambient energy sources such as heat, solar, radio or vibration. Harvesters offer several advantages over conventional battery power sources like extended lifetime of operation, and clean energy generation (Paradiso and Starner, 2005). They have a wide range of applications from wearable biomedical sensors for health monitoring to condition monitoring of machinery. These applications makes energy harvesting an attractive research area given the growth of worlds population and its ever increasing demand for energy (Barker et al., 2013).

Eliminating sensors and transducers in the harvesters, thus enabling sensorless operation, reduces operational cost while increasing the ruggedness and reliability. Sensorless operation is used for on-line tool fault detection and in reporting faults in modern manufacturing industry (Franco-Gasca et al., 2006). Measuring mechanical quantities or operating states in energy harvesters still remains a challenging task due to the unavailability of access points. The existing approaches to sensorless operation are based on saliency, Kalman filter and model reference techniques (Rashed and Stronach, 2004). These techniques are computationally intensive, require special construction for estimation or require proper initialisation (Degner and Lorenz, 1998). This paper presents, as an example application, sensorless operation of a continuously rotating harvester, *i.e.* a rotational energy harvester.

The harvester is based on balancing the torque generated by the gravitational force and the motor torque acting on a suspended mass, see Toh et al. (2008). The sensorless control scheme discussed in Nunna et al. (2013) improves the efficiency of the harvester given in Toh et al. (2008) by estimating the mechanical quantities from the measured electrical quantities and using these in a nonlinear control law. The optimization problem *i.e.* generating maximum energy for a given source rotation speed is transformed into a stabilization problem; maintaining the angle of the suspended mass at an angle of $\pi/2\,\mathrm{rad}$ to the vertical axis allows maximal power extraction.

This paper generalises the observer design in Nunna et al. (2013) by eliminating the requirement of any prior knowledge on the source rotation speed and extends it to the case in which the source rotation speed varies as a function of time. The observer gains depend on the solution of a modified algebraic Riccati equation. This makes it an attractive alternative to the design discussed in Nunna et al. (2013). The control design presented here is a variation of the one in Nunna et al. (2013) and maintains the angular position of the suspended mass at an angle of $\pi/2\,\mathrm{rad}$ to the vertical axis for any source rotation.

The rest of the paper is organized as follows. Section 2 discusses the mathematical model of the harvester and its validation. The basics of the observer design technique are given in Section 3 together with their application to the harvester and the description of the control design procedure. Section 4 gives simulation results and Section 5 provides experimental evidence. The paper is concluded in Section 6 with suggestions for future work and some comments.

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2. DEVELOPMENT AND VALIDATION OF THE HARVESTER'S MATHEMATICAL MODEL

In this section, for completeness, we recall the mathematical model for the rotational energy harvester, and describe its validations steps which have been presented in Nunna et al. (2013)

The rotational energy harvester consists of a DC generator with its stator coupled to a continuously rotating source and a semicircular mass m attached to the rotor at a distance l from the axis of rotation (see Figure 1). When power is drawn from the generator the torque between the stator and rotor (motor torque) is counteracted by the torque generated by the gravitational force acting on the offset mass (gravitational torque). The difference between these two torques creates a difference in the angular speeds of the stator and the rotor that can be tapped off as power. The excess generated power is stored in an energy reservoir in the form of a supercapacitor, see Toh et al. (2008). To ensure optimal power transfer from the harvester to the load, the load resistance, R_L, should be closely matched to the harvester's armature resistance, R_a. Since the input impedance of a boost converter R_{in} can be controlled by varying its duty cycle, δ (see (Toh et al., 2008))

$$R_{in} = R_L(1-\delta)^2\,, \tag{1}$$

it is used as a power electronic interface circuit between the harvester and the load.

The experimental set-up for the harvester and the interface electronics is illustrated in Figure 2. In this set-up the source rotation is connected to a gear box with a conversion ratio of 1 : 4.4 to generate a higher voltage at low source rotation speeds. The values for the various mechanical constants and the circuit components used in the experimental model are given in Table 1. For a more detailed explanation of the construction and the choice of the circuit components, see Toh et al. (2008) and references therein.

Table 1. Component values for the experimental set-up.

k_E	8.6436 rad/sV
k_T	0.0610 mNm/A
g	9.8 m/s^2
m	100 g
l	0.03 m
L	680 μH
C	4.53 mF
r_m	0.04 m
R_a	11.2 Ω
Microprocessor	PIC18F1320
Maxon motor	118733

From the free body diagram of the offset mass in Figure 1, the torque balance on the mass attached to the rotor of the harvester is given by

$$J\dot{\omega} = \Gamma_M - mgl\sin\theta\,, \tag{2}$$

where J is the moment of inertia of the semicircular mass calculated as $\frac{2mr_m^2}{5}$, ω is the angular velocity, and θ is the deflection angle of the mass measured from the vertical axis. The motor torque Γ_M is calculated as

$$\Gamma_M = -k_T I_{in}\,,$$

where k_T is the torque constant of the motor, and I_{in} is the current drawn by the DC generator. The negative sign indicates that the current flows out of the generator and into the boost converter. The voltage generated by the harvester, E_g, when it is attached to a continuously rotating source at a speed ω_s is calculated as

$$k_E(\omega_s - \omega)\,, \tag{3}$$

where k_E is the "motor constant" of the DC generator.

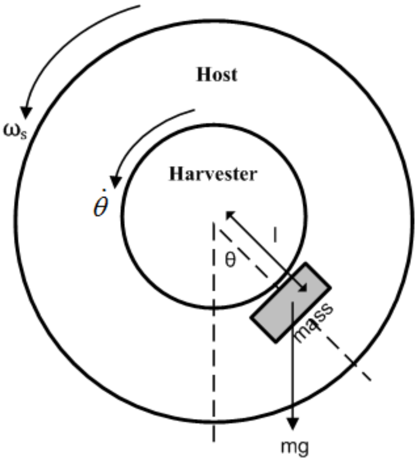

Fig. 1. A schematic diagram of the rotational energy harvester.

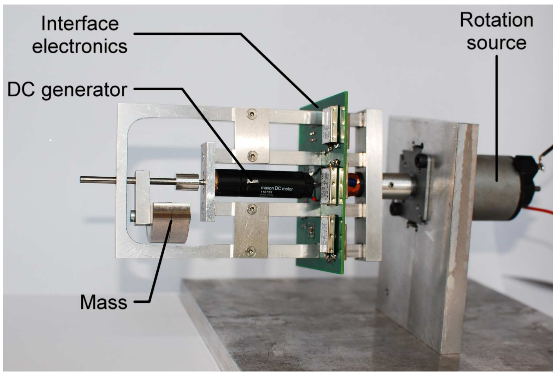

Fig. 2. Experimental set-up of the rotational energy harvester.

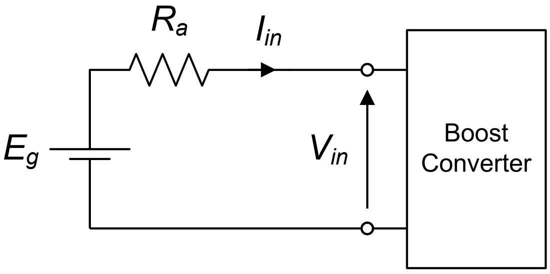

Fig. 3. Equivalent circuit diagram to calculate I_{in}.

Application of Kirchoff's voltage law to the circuit diagram in Figure 3 and use of the relations $I_{in} = \frac{V_{in}}{R_{in}}$ and (1) yield

$$V_{in} = \frac{k_E(\omega_s - \omega)(1-\delta)^2 R_L}{(R_a + (1-\delta)^2 R_L)}\,. \tag{4}$$