Proceedings of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

$\mathcal{H}_\infty$-Controller Design Methods Applied to One Joint of a Flexible Industrial Manipulator ⋆

Patrik Axelsson, * **Anders Helmersson,** * **Mikael Norrlöf** *,**

* *Division of Automatic Control, Department of Electrical Engineering, Linköping University, SE-581 83 Linköping, Sweden (e-mail: {axelsson, andersh, mino}@isy.liu.se).*
** *ABB AB – Robotics SE-721 68 Västerås, Sweden*

Abstract: Control of a flexible joint of an industrial manipulator using $\mathcal{H}_\infty$ design methods is presented. The considered design methods are i) mixed-$\mathcal{H}_\infty$ design, and ii) $\mathcal{H}_\infty$ loop shaping design. Two different controller configurations are examined: one uses only the actuator position, while the other uses the actuator position and the acceleration of the end-effector. The four resulting controllers are compared to a standard PID controller where only the actuator position is measured. The choices of the weighting functions are discussed in details. For the loop shaping design method, the acceleration measurement is required to improve the performance compared to the PID controller. For the mixed-$\mathcal{H}_\infty$ method it is enough to have only the actuator position to get an improved performance. Model order reduction of the controllers is briefly discussed, which is important for implementation of the controllers in the robot control system.

Keywords: Robotics, Flexible, H-infinity control, Accelerometers

1. INTRODUCTION

The requirements for controllers in modern industrial manipulators are that they should provide high performance, at the same time, robustness to model uncertainty. In the typical standard control configuration the actuator positions are the only measurements used in the higher level control loop. At a lower level, in the drive system, the currents and voltages in the motor are measured to provide torque control for the motors. In this contribution different $\mathcal{H}_\infty$-controller design schemes are compared when using two different sensor configurations. First, the standard case where only the position of the actuator rotation is used, and second a configuration where, in addition, the acceleration of the tool tip is measured. Two different $\mathcal{H}_\infty$ methods are investigated: i) loop shaping [McFarlane and Glover, 1992], and ii) multi-$\mathcal{H}_\infty$ design [`mixedHinfsyn`, 2013, Zavari et al., 2012].

Motivated by the conclusions from Sage et al. [1999] regarding the area of robust control applied to industrial manipulators, this contribution includes:

- results presented using realistic models,
- a comparison with a standard PID control structure,
- model reduction of the controllers to get a result that more easily can be implemented in practice.

The model used in this contribution represents one joint of a typical modern industrial robot [Moberg et al., 2009]. It is a physical model consisting of four masses, which should be compared to the typical two-mass model used in many previous contributions, see Sage et al. [1999] and the references therein. The joint model represents the first joint of a serial 6-DOF industrial manipulator, where the remaining five axes have been configured to minimise the couplings to the first axis. To handle changes in the configuration of the remaining axes, gain scheduling techniques can be used based on the results in this paper.

An important part of the design is the choice of the weighting functions, which is an essential task to get a satisfactory performance. The work of choosing the weights is difficult, tedious and time consuming. This can be be the reasons for why $\mathcal{H}_\infty$ methods are not used that often in practice even though the performance and robustness can be increased. In particular, the use of two measurements for control of one variable requires special treatment. The development of the weighting functions for the four controllers are discussed in details, and provides a significant part of the contributions in the paper.

Controller synthesis using $\mathcal{H}_\infty$ methods has been proposed in Song et al. [1992], Stout and Sawan [1992], where the complete non-linear robot model first is linearised using exact linearisation, second an $\mathcal{H}_\infty$ controller is designed using the linearised model. The remaining non-linearities due to model errors are seen as uncertainties and/or disturbances. In both papers, the model is rigid and the $\mathcal{H}_\infty$ controller, using only joint positions, is designed using the mixed-sensitivity method. In Sage et al. [1997] $\mathcal{H}_\infty$ loop shaping with measurements of the actuator positions is applied to a robot. The authors use a flexible joint model which has been linearised. The linearised model makes it possible to use decentralised control, hence $\mathcal{H}_\infty$ loop shaping is applied to n SISO-systems instead of the complete MIMO-system.

⋆ This work was supported by the Vinnova Excellence Center LINK-SIC and by the Excellence Center at Linköping-Lund in Information Technology, ELLIIT.

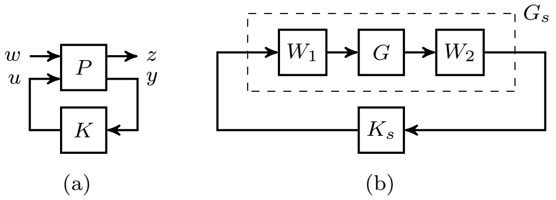

Fig. 1. System description for general $\mathcal{H}_\infty$ synthesis (a) and loop shaping (b).

Explicit use of acceleration measurements for control in robotic applications has been reported in, for example, de Jager [1993], Dumetz et al. [2006], Kosuge et al. [1989], Readman and Bélanger [1991] and Xu and Han [2000]. In Dumetz et al. [2006], a control law using motor position and acceleration of the load in the feedback loop is proposed for a Cartesian robot [1]. The robot is assumed to be flexible and modelled as a two-mass system, where the masses are connected by a linear spring-damper pair. Another control law of a Cartesian robot using acceleration measurements is presented in de Jager [1993]. The model is a rigid joint model and the evaluation is made both in simulation and experiments.

In Kosuge et al. [1989] a 2-*degree-of-freedom* (DOF) manipulator is controlled using acceleration measurements of the end-effector. The model is assumed to be rigid and it is exactly linearised. The joint angular acceleration used in the non-linear feedback loop is calculated using the inverse kinematic acceleration model and the measured acceleration. The use of direct measurements of the angular acceleration in the feedback loop is presented in Readman and Bélanger [1991] for both rigid and flexible joint models. A more recent work is presented in Xu and Han [2000], where a 3-DOF manipulator is controlled using only measurements of the end-effector acceleration.

The theory for synthesis of $\mathcal{H}_\infty$ controllers is presented in Section 2. The model describing the robot joint is explained in Section 3. In Section 4, the requirements of the system as well as the design of four controllers are described, and in Section 5 the simulation results are shown. Finally, Section 6 discuss low order controller synthesis and Section 7 concludes the work.

2. CONTROLLER DESIGN METHODS

In this section, a brief introduction to mixed-$\mathcal{H}_\infty$ design [`mixedHinfsyn`, 2013, Zavari et al., 2012] and $\mathcal{H}_\infty$ loop shaping [McFarlane and Glover, 1992] will be presented.

2.1 Mixed-$\mathcal{H}_\infty$ Controller Design

A common design method is to construct the system $P(s)$ in

$$\begin{pmatrix} z \\ y \end{pmatrix} = \begin{pmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{pmatrix} \begin{pmatrix} w \\ u \end{pmatrix} = P(s) \begin{pmatrix} w \\ u \end{pmatrix} \tag{1}$$

by augmenting the original system $y = G(s)u$ with the weights $W_u(s)$, $W_S(s)$, and $W_T(s)$, such that the system $z = F_l(P,K)w$, depicted in Figure 1(a), can be written as

[1] For a Cartesian robot the joint acceleration is measured directly by an accelerometer, while for a serial type robot there is a non-linear mapping depending on the states.

$$F_l(P,K) = \begin{pmatrix} W_u(s)G_{wu}(s) \\ -W_T(s)T(s) \\ W_S(s)S(s) \end{pmatrix}, \tag{2}$$

where $S(s) = (I + G(s)K(s))^{-1}$ is the sensitivity function, $T(s) = I - S(s)$ is the complementary sensitivity function, and $G_{wu}(s) = -K(s)(I + G(s)K(s))^{-1}$ is the transfer function from w to u. The $\mathcal{H}_\infty$-controller is then obtained by minimising the $\mathcal{H}_\infty$-norm of the system $F_l(P,K)$, i.e., minimise γ such that $\|F_l(P,K)\|_\infty < \gamma$. Using (2) gives

$$|W_u(i\omega)G_{wu}(i\omega)| < \gamma,\ \forall\omega, \tag{3a}$$
$$|W_T(i\omega)T(i\omega)| < \gamma,\ \forall\omega, \tag{3b}$$
$$|W_S(i\omega)S(i\omega)| < \gamma,\ \forall\omega. \tag{3c}$$

The transfer functions $G_{wu}(s)$, $S(s)$, and $T(s)$ can now be shaped to satisfy the requirements by choosing the weights $W_u(s)$, $W_S(s)$, and $W_T(s)$. The aim is to get a value of γ close to 1, which in general is a hard to achieve and it requires insight in the deign method as well as the system dynamics. For more details about the design method, see e.g. Skogestad and Postletwaite [2005], Zhou et al. [1996].

The mixed-$\mathcal{H}_\infty$ controller design [`mixedHinfsyn`, 2013, Zavari et al., 2012] is a modification of the standard $\mathcal{H}_\infty$ design method. Instead of choosing the weights in (2) such that the norm of all weighted transfer functions satisfies (3), the modified method divides the problem into design constraints and design objectives. The controller can now be found as the solution to

$$\underset{K(s)}{\text{Minimise}} \quad \gamma \tag{4a}$$
$$\text{subject to} \quad \|W_P(s)S(s)\|_\infty < \gamma \tag{4b}$$
$$\|M_S(s)S(s)\|_\infty < 1 \tag{4c}$$
$$\|W_u(s)G_{wu}(s)\|_\infty < 1 \tag{4d}$$
$$\|W_T(s)T(s)\|_\infty < 1 \tag{4e}$$

where γ not necessarily has to be close to 1. Here, the weight $W_S(s)$ has been replaced by the weights $M_S(s)$ and $W_P(s)$. The method can be generalised to other control structures and in its general form it is formulated as a multi-objective optimisation problem. More details about the general form and how to solve the optimisation problem are presented in `mixedHinfsyn` [2013], Zavari et al. [2012].

2.2 Loop Shaping using $\mathcal{H}_\infty$ Synthesis

For loop shaping [McFarlane and Glover, 1992], the system $G(s)$ is pre- and post-multiplied with weights $W_1(s)$ and $W_2(s)$, see Figure 1(b), such that the shaped system $G_s(s) = W_2(s)G(s)W_1(s)$ has the desired properties. The controller $K_s(s)$ is then obtained using the method described in Glover and McFarlane [1989] applied on the system $G_s(s)$, giving the controller $K_s(s)$. Finally, the controller $K(s)$ is given by

$$K(s) = W_1(s)K_s(s)W_2(s). \tag{5}$$

Note that the structure in Figure 1(b) for loop shaping can be rewritten as a standard $\mathcal{H}_\infty$ problem according to Figure 1(a), see Zhou et al. [1996] for details. It will be used in Section 6 for synthesis of low order controllers.

The MATLAB function `ncfsyn`, included in the Robust Control Toolbox, is used in this paper for synthesis of $\mathcal{H}_\infty$ controllers using loop shaping.