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Adjoint assisted geometry design of a feedback controlled missile ⋆

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Abstract: A novel optimisation framework using an adjoint cost sensitivity calculation, and integrating computer simulations of fluid dynamics, rigid body dynamics and control is proposed. A generic tail-fin steered missile under closed-loop control is used to show that the framework is able to generate a detailed geometrical tail-fin design and tune control performance parameters that are directly related to the range and manoeuvrability of the missile. It is shown that this new methodology is able to reduce the aerodynamic drag by 2% and the tracking error by about 3% relative to the original design.

Keywords: Aerospace; Parametric optimization; Large scale optimization problems.

1. INTRODUCTION

The use of computer simulations as part of the design process is becoming increasingly common in complex and multi-disciplinary engineering products. For missile design, multi-point geometry optimisation ((Anderson et al. (2000)) and trajectory and geometry optimisation (Tekinalp and Bingol (2004) and Yang et al. (2012)) have been reported in the literature. These previous studies utilised low fidelity semi-empirical aerodynamic models such as Missile DATCOM (Vukelich et al. (1988)) rather than modern computational fluid dynamics (CFD) models to generate the aerodynamic data. An obvious criticism of these methods is the accuracy of the semi-empirical aerodynamic models. Moreover, these models implicitly place limitations on both the fidelity and novelty of the shapes that can be generated by the optimiser.

The field of aerodynamic shape optimisation was pioneered by Lighthill (1945) who utilised analytical inverse field methods to determine an optimal shape for a known pressure distribution. More recently, the adjoint method has gained popularity due to the computational efficiency of the method. Jameson (1988) was the first to demonstrate the capability of the adjoint method in calculating the sensitivity of an aerodynamic functional with respect to the geometry design variables with just two simulations. A *primal* simulation is used to capture the behaviour of the physical system and an *adjoint* simulation is used to calculate the gradient of a cost function with respect to all of the design variables. In comparison, calculating the gradient of a cost function using finite differences would require at least $\mathcal{N}+1$ simulations, where $\mathcal{N}$ is the number of design variables. The gradient can then be used within a gradient-based optimiser to find a local minimum. Jameson's work was initially concerned with just optimisation of geometry at a single steady-state, but has since been extended for multi-point optimisation (Reuther et al. (1997)), rotorcraft blade optimisation (Economon et al. (2012)) and aerofoil optimisation with a predefined pitching motion (Economon et al. (2013)). Adjoint methods are limited in that the solution can become entrapped in local minima. The alternative is to use global optimisation methods to overcome this, but global methods require much higher computational resources especially as the number of design variables increases. In Lee et al. (2013), a global extremum seeking method is proposed that utilises semi-converged CFD evaluations to reduce the computational load.

In order to optimise the performance of an aircraft undertaking a set of manoeuvres, one may be tempted to simultaneously compute both the fluid dynamics and the feedback-controlled rigid-body dynamics. Such a task is, however, computationally expensive. Moreover, a complication is introduced by the fact that the achieved aircraft trajectory depends upon the controller, which is typically tuned to the aircraft geometry. In order to address these issues, this paper considers a high velocity regime where the aerodynamic forces acting on the aircraft are adequately described by steady flow. Note that it is common practice in missile modelling to consider only steady flow and to neglect aerodynamic rate effects (see Menon and Ohlmeyer (2001) and Siouris (2004)). A set of CFD simulations is performed in order to map the aerodynamic forces acting on the aircraft for a variety of flow regimes. These maps are then used in a rigid-body dynamic simulation of the aircraft undertaking a set of commanded manoeuvres. An optimiser tunes both the controller gains and the aircraft geometry in order to maximise the aircraft performance, which is expressed in terms of both drag and the ability

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of the aircraft to track its commanded trajectory. Further to this, cost function gradients are calculated through a novel combination of an adjoint approach for the CFD and a finite-differencing approach for the (computationally cheap) rigid-body dynamics.

2. PROBLEM DESCRIPTION

Consider an aircraft subject to the dynamics,

$$\dot{x} = f\left(x, r(t), X_d, F(\chi(x), X_i)\right), \tag{1}$$

where:

x is the system state which may, for example, include states related to the rigid-body dynamics of the aircraft, states related to the control surface dynamics, and states used within a feedback controller for motion control.

$r(t)$ is a reference signal, defined over the time interval $[0, T]$, that describes a "mission" over which the performance of the aircraft is to be evaluated.

X_d is a vector of "direct" design parameters which may, for example, include vehicle mass, position of centre of gravity, actuator characteristics, controller tuning parameters and geometrical parameters whose effects can be adequately described by available data/models (without undertaking CFD simulations).

F is a vector of aerodynamic forces and moments that cannot readily be written it terms of a known algebraic or ordinary differential equation. Instead F is to be found by performing a CFD simulation for the steady flow about the aircraft (or a feature thereof).

X_i is a vector of design parameters related to the geometry of the aircraft. These design parameters are referred to as "indirect" since they only enter the system dynamics via F.

$\chi(x)$ is the "pose" or configuration of the aircraft with respect to the incident flow. The mapping from x to χ is typically a simple truncation transformation. For example, χ may be described by aircraft states such air speed, the angle of attack, slip angle, roll angle, and control surface deflections. χ cannot contain quantities such as linear accelerations and angular velocities since their effect on aerodynamic forces cannot easily be captured using *steady* CFD simulations. As a result of this, the proposed approach is most applicable to high velocity aircraft where "rate" effects on aerodynamic forces are typically negligible.

The performance of the aircraft for the "mission" can be written as a cost function,

$$J_r := \int_0^T v\left(x, r(t), X_d, F(\chi(x), X_i)\right) \mathrm{d}t. \tag{2}$$

The design objective is to seek the design parameters $X := (X_d, X_i)$ which optimise the aircraft's performance, in other words,

$$\arg\min_X J_r, \tag{3}$$

subject to $(X_d, X_i) \in \mathcal{D}_d \times \mathcal{D}_i$ where $\mathcal{D}_i$ and $\mathcal{D}_d$ are compact sets.

2.1 Discrete approximation for aerodynamic force maps

Consider a given aircraft geometry so that X_i is fixed. F then maps the aerodynamic forces in terms of the aircraft pose. It is reasonable to constrain the pose to a compact set, $\chi \in \mathcal{G}$. Nonetheless, even under these conditions, the domain of F is continuous. Recalling that F can only be "discovered" through CFD simulations, then a single CFD simulation for a given value of χ reveals only one point on the mapping, $F(\cdot, X_i)$. It follows that only a finite number of points on the mapping can be found using CFD simulations, and an interpolation scheme must then be used to approximate F for those values of χ that are not tested. Suppose CFD simulations are performed for n values of χ, denoted by $\chi_1, \ldots, \chi_n$. Then the interpolated mapping $\hat{F}_k(\chi, X_i)$ should have the property that, for any $\epsilon > 0$, there is a n^* such that for all $n > n^*$, $\chi \in \mathcal{G}$ and $X_i \in \mathcal{D}_i$,

$$\left\| \hat{F}_k\left(\chi, X_i\right) - F\left(\chi, X_i, \right) \right\| < \epsilon. \tag{4}$$

3. OPTIMISATION FRAMEWORK

3.1 The Adjoint Method

The adjoint method is a means of calculating the gradient of the cost function with respect to the design variables. The derivation of the adjoint equations following from Nadarajah and Jameson (2000) and Economon et al. (2012) are reproduced here.

Consider a cost function J, that is a function of the flow-field quantities, U, and geometric design variables (indirect design variables) X_i.

$$J = J\left(U, X_i\right). \tag{5}$$

In aerodynamic studies, the cost functions of interest are predominantly some function of the pressure over the surface boundary S of the aircraft. Let the class of these functionals be written as,

$$J = \int_S \boldsymbol{d} \cdot \left(p \boldsymbol{n_S}\right) ds, \tag{6}$$

where, $\boldsymbol{d}$ is a force projection vector, p is the pressure and $\boldsymbol{n_S}$ is the local normal vector on the surface.

By calculus of variations a change in X_i results in a change in the cost,

$$\delta J = \frac{\partial J}{\partial U}\delta U + \frac{\partial J}{\partial X_i}\delta X_i. \tag{7}$$

It is expensive to compute variations in the flow-field quantities, δU, that is, each variation will require an additional CFD simulation. The aim of the adjoint approach is to eliminate this term in (7). Suppose that the governing equations of the flow are introduced in the form of an equality constraint,

$$R\left(U, X_i\right) = 0. \tag{8}$$

For example, $R\left(U, X_i\right)$ could be the conservative form of the compressible Euler equations. The variation in (8) is,

$$\delta R = \left[\frac{\partial R}{\partial U}\right]\delta U + \left[\frac{\partial R}{\partial X_i}\right]\delta X_i = 0. \tag{9}$$

Equation (7) can be combined with (9) via a Lagrange Multiplier, ψ, which gives,

$$\begin{aligned} \delta J &= \frac{\partial J}{\partial U}\delta U + \frac{\partial J}{\partial X_i}\delta X_i - \psi\left(\left[\frac{\partial R}{\partial U}\right]\delta U + \left[\frac{\partial R}{\partial X_i}\right]\delta X_i\right) \\ &= \left\{\frac{\partial J}{\partial U} - \psi\left[\frac{\partial R}{\partial U}\right]\right\}\delta U + \left\{\frac{\partial J}{\partial X_i} - \psi\left[\frac{\partial R}{\partial X_i}\right]\right\}\delta X_i. \end{aligned} \tag{10}$$