Proceedings of the 19th World Congress
The International Federation of Automatic Control
Cape Town, South Africa. August 24-29, 2014

Nonlinear model predictive missile control with a stabilising terminal constraint ⋆

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Abstract: In this paper, an MPC scheme for a missile pitch axis autopilot is proposed. The scheme uses a nonlinear prediction model to give it an ability to push the controlled missile very close to its operating limits, and is stabilised through the use of an ellipsoidal terminal constraint. Tracking performance and computational load of the scheme are compared to that with a linear prediction model and other types of terminal constraint. Specifically, the choice of ellipsoidal, polytopic, or no terminal constraint is discussed. The terminally constrained nonlinear MPC scheme achieves comparable solution times to that with a linear prediction model, whilst being more aggressive to give a superior tracking performance.

Keywords: Predictive control, nonlinear control, missiles, stability

1. INTRODUCTION

Model Predictive Control (MPC) has a number of attractive properties for the control of a guided airframe. It explicitly handles constraints and has the ability to directly take into account plant nonlinearities. These are crucial as MPC would be able to push a missile to operate near its physical limits, at high angles of attack where aerodynamics are highly nonlinear (Gros et al., 2012). The other components of MPC such as objective cost and prediction horizon can be formulated to achieve the most desirable control behaviour. This makes MPC suited to address current challenges in missile autopilot design, which often revolve around the inability to account for nonlinearities, changes in missile behaviour during flight, and different missile configurations (Jackson, 2010).

Despite the aforementioned advantages, applications of MPC for guiding missiles are rare. This is primarily due to the high computational demand of MPC which can be problematic for applications in areas where plants posses fast, nonlinear dynamics, such as in the case of a missile (Hu and Chen, 2007). This makes the trade-off between theoretical advantages and implementability of MPC for missile control an important discussion.

To address the issue regarding computational burden, typical approaches include simplification of the problem formulation through model linearisation, relaxation of constraints, or bounding by uncertain linear models. Although reducing computational cost, these approximations can potentially have detrimental effects on the closed loop performance of the controller, such as a steady-state error due to model-plant mismatch or even instability.

The prediction model dictates the complexity of the pertaining optimisation problem in MPC. A nonlinear prediction model is associated with a higher computational load than a linear model due to the nonconvex nature of the resulting optimisation problem for most real-world applications. Early methods in solving MPC with a nonlinear prediction model utilised the direct multiple-shooting (Bock and Plitt, 1984) or collocation (Biegler, 1984). To achieve computational times needed in missile applications, a fast algorithm involving a sequential convex programming (Tran-Dinh and Diehl, 2010) to solve the nonlinear MPC scheme is followed in this paper. For benchmarking, the prediction model is simplified by a typical linearisation.

In this paper, a terminal constraint is used to guarantee stability of the MPC. In the early derivations of MPC stability, the terminal constraint was used to impose the terminal state to coincide at an invariant point, the origin (e.g. Mayne and Michalska, 1990). This is rather restrictive and in subsequent developments the notion of an invariant region (rather than an invariant point) was introduced (Michalska and Mayne, 1993). Chen et al. (1998; 2001) extended the theory by approximating the invariant region as an ellipsoid. An alternative approach in approximating such a region is to use a polytope (Cannon et al., 2003), which could better approximate the invariant region. These approaches derive closed-loop MPC stability with a linear differential inclusion (LDI) approximation of the nonlinear plant model, which simplifies calculation of the terminal region at the expense of being conservative. This paper compares the ellipsoid and polytopic terminal

⋆ This research was supported under the Australian's Research Council's Linkage Projects funding scheme (Project LP11020025).

region for missile autopilot. Further, these two approaches are compared to a computationally simpler approach with a relaxed constraint, i.e. the use of no terminal region.

The comparisons of the prediction models and terminal constraints are on the basis of how quickly the controller can drive the missile to track a given acceleration command and its computational load. This is to establish an understanding of the quality of nonlinear MPC with an ellipsoidal terminal constraint in terms of tracking performance and implementability of achieved solution times.

2. PLANT MODEL

2.1 Missile pitch-axis dynamics

The missile autopilot control in the pitch-axis as depicted in Fig. 1 is governed by the equations

$$\dot{\alpha} = q + \cos(\alpha) F_z(\alpha,\delta)/(mV) \quad (1a)$$
$$\dot{q} = L(\alpha,\delta)/I_y \quad (1b)$$
$$a = F_z(\alpha,\delta)/(mg) \quad (1c)$$

where the aerodynamic lift force F_z and pitching moment L are modelled by (Nichols et al., 1993)

$$F_z = 0.7M^2P_0S\left[C_{Z\alpha,1}(2 - M/3)\alpha + C_{Z\alpha,2}\alpha\,|\alpha| + C_{Z\alpha,3}\alpha^3 + C_{Z\delta}\delta\right] \quad (2a)$$
$$L = 0.7M^2P_0Sd\left[C_{L\alpha,1}(8M/3 - 7)\alpha + C_{L\alpha,2}\alpha\,|\alpha| + C_{L\alpha,3}\alpha^3 + C_{L\delta}\delta\right]. \quad (2b)$$

α is the angle of attack and $q = \dot{\varphi}$ is the pitch rate of the missile. The output of the system a is the normal acceleration of the missile in multiples of gravitational acceleration g. Missile speed $V = MV_s$, where V_s is the speed of sound, is treated as constant at Mach number $M = 2.5$. The actuation of the fin deflection δ is modelled as a second order system:

$$\ddot{\delta} = -\omega_a^2\delta - 2\zeta\omega_a^2\dot{\delta} + \omega_a^2\delta_c. \quad (3)$$

Aerodynamic coefficients $C_{Z\alpha,1}, C_{Z\alpha,2}$ etc., other flight condition and missile frame parameters, along with constants related to (3) used are the same as given in Nichols et al. (1993). Note that the symbols $C_{Z\alpha,1}, C_{Z\alpha,2}$ etc. follow the standard nomenclature, and are different to that used by Nichols et al. (1993).

The dynamics of the missile can be put in a concise form:

$$\dot{\tilde{x}} = \tilde{f}(\tilde{x}, u) \quad (4a)$$
$$y = h(\tilde{x}) \quad (4b)$$

with state variables $\tilde{x} = [\alpha,\, q,\, \delta,\, \dot{\delta},\, \delta_c]^\mathsf{T} \in \mathbb{R}^{n_x}$, control variables $u = \dot{\delta}_c \in \mathbb{R}^{n_u}$, and output $y = a$.

The missile operation is subject to a number of constraints. A constraint on α is associated with the fact that aerodynamic coefficients used are only valid for a range of α. Fin deflection δ and its rate $\dot{\delta}$ are subject to mechanical limits of the actuator. Although there is no physical restriction that limits q, a constraint is imposed (made large for it to never be active) to make compact constraint polytopes

$$X = \{\tilde{x} : -\bar{x} \le \tilde{x} \le \bar{x}\},\ U = \{u : -\bar{u} \le u \le \bar{u}\} \quad (5)$$

where $\le$ and $\ge$ denote element-wise inequalities. This describes a region within missile physical limitations and accuracy of aerodynamic coefficients. The states and input limits are $\bar{x} = [\bar{\alpha},\, \bar{q},\, \bar{\delta},\, \bar{\dot{\delta}},\, \bar{\delta}]^\mathsf{T}$, and $\bar{u} = \bar{\dot{\delta}}$ respectively. Due to these constraints, the missile is only capable of tracking a maximum acceleration of $a_{\max}$.

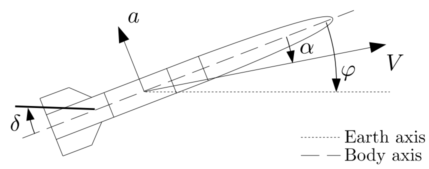

Fig. 1. Missile on the pitch-axis plane.

2.2 Tracking problem formulation

The autopilot control receives a commanded normal acceleration $a_\circ$ to track from the missile guidance law. Desired steady-state values of state variables $x_\circ$ which achieve $a_\circ$ are obtained from the solution of $0 = \tilde{f}(x_\circ, u_\circ)$ and $a_\circ = h(x_\circ)$. The system is injective therefore a commanded normal acceleration $a_\circ$ is associated with one unique steady state $x_\circ = [\alpha_\circ,\, q_\circ,\, \delta_\circ,\, 0,\, \delta_\circ]^\mathsf{T}$, $u_\circ = 0$.

If $a_\circ \neq 0$ then $x_\circ \neq 0$. The system can be formulated as an error system with an equilibrium at the origin $x = 0$:

$$x = \tilde{x} - x_\circ \quad (6a)$$
$$\dot{x} = \tilde{f}(\tilde{x}, u) = \tilde{f}(x + x_\circ, u) =: f(x, u) \quad (6b)$$
$$X = \{x : -\bar{x} \le x + x_\circ \le \bar{x}\}, U = \{u : -\bar{u} \le u \le \bar{u}\} \quad (6c)$$

3. MODEL PREDICTIVE CONTROL

The missile in continuous time t is controlled at each sampling time t_i, for $i = 1, 2\ldots$ separated by a sampling period T_s, i.e. $t_{i+1} = t_i + T_s$. At each sampling instant, with current state $x(t_i)$, the MPC scheme considered in this paper is to solve the optimisation problem

$$\min_{\substack{x_k\ k=1\ldots N+1 \\ u_k\ k=1\ldots N}} \sum_{k=1}^{N} \ell(x_k, u_k) + e(x_{N+1}) \quad (7a)$$
$$\text{s.t.} \quad x_1 = x(t_i) \quad (7b)$$
$$x_{k+1} = \Phi(x_k, u_k) \quad \forall k = 1\ldots N \quad (7c)$$
$$x_k \in X,\ u_k \in U \quad \forall k = 1\ldots N \quad (7d)$$
$$x_{N+1} \in \mathcal{X}_f \quad (7e)$$

$$\text{where} \quad \Phi(x_k, u_k) = x_k + \int_{t_{k+i-1}}^{t_{k+i}} f(x(\tau), u_k)\, d\tau. \quad (8)$$

The subscript k is used to discretise the continuous variables x and u into N discrete prediction variables to be solved by computational means. Here, N characterises the prediction horizon of the MPC scheme. The prediction model $\Phi(x_k, u_k)$ takes an initial state x_k and integrates the tracking error model (6b) over one sampling period with zero-order hold input u_k to obtain a predicted state x_{k+1}. The solution of (7a–e) are the optimal state x_k^* and control sequence u_k^*, $\forall k = 1\ldots N$, the first of which, u_1^*, is applied as feedback control to the plant.

3.1 Cost function

The cost function (7a) (with stage cost $\ell(\cdot)$ and terminal cost e$(\cdot)$) is a performance measure of the missile within the prediction horizon, indicating how far the states are to the desired values. A typical quadratic cost function:

$$\ell(x, u) = \|x\|_Q^2 + \|u\|_R^2, \quad e(x) = \|x\|_P^2 \quad (9)$$