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Robust Attitude Control with Improved Transient Performance ⋆

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Abstract: This paper aims to present a robust attitude control strategy with guaranteed transient performance. Firstly, a Lyapunov-based control law is designed to achieve high-performance attitude control in the absence of disturbance and parameter variation. The proposed control law uses small feedback gains to suppress the control torque at large attitude error, and increases those gains with the convergence of attitude error to accelerate the system response. The overshooting phenomenon is also avoided by imposing a restriction on the parameter selection. Then, the integral sliding mode control technique is employed to improve the robustness, where the Lyapunov-based control law is used as the equivalent control part. Theoretical analysis and simulation results verify the effectiveness of the proposed strategy.

1. INTRODUCTION

Controlling the rotational motion of rigid spacecraft is a challenging issue. The difficulty lies in the highly nonlinear and coupled governing equations, as well as the undesired torque caused by disturbance and parametric uncertainty (Schaub and Junkins [2009]). Therefore, for achieving desired control performance, nonlinear control techniques with strong robustness should be utilized.

Since the first systematic study in (Meyer [1971]), Lyapunov-based control technique has been extensively investigated in the attitude control literature (Wie et al. [1989],Wen and Kreutz-Delgado [1991],Suk et al. [2001],Schlanbusch et al. [2010]). By finding some energy-like Lyapunov functions, the associated attitude controllers are constructed by two parts, the attitude variable feedback terms and the nonlinearity compensation terms (Wie et al. [1989],Wen and Kreutz-Delgado [1991]). The closed-loop dynamics can be approximated using a simple damped harmonic oscillator model, which makes the controller very convenient to validate, tune and implement. Nonetheless, only a boundedness conclusion can be obtained in the presence of disturbance and parametric uncertainty (Schlanbusch et al. [2010]). As a result, the control accuracy is unacceptable for space missions such as rendezvous and docking, where a highly accurate pointing or slewing is required. Moreover, there is a tradeoff between accelerating system response and suppressing the peak control torque, which will degrade the control performance if unsuitable parameters are selected.

In order to address those shortcomings, various strategies have been adopted. On the one hand, the robustness issue has been considered in many research works. In (Lizarralde and Wen [1996]) and (Tsiotras [1998]), the inertia matrix is not required in the attitude controller design by exploiting the passivity properties of the attitude control system. Hence, the control precision will not be affected by the uncertain inertia matrix and the robustness is therefore enhanced. However, such a conclusion only holds for the case of attitude reorientation. With respect to attitude tracking, an exact knowledge of inertia matrix is still required. In (Akella [2001]), a simple adaptive law was designed to estimate the slow varying inertia matrix. However, the disturbance torque is not taken into account. Integrating with disturbance observer is another effective approach of improving the robustness of Lyapunov-based attitude controller (Yamashita et al. [2004],Sun and Li [2013],Sun and Li [2011]). Nonetheless, the control accuracy depends directly on the disturbance observer and a rigorous stability analysis under the composite controller is generally absent due to the challenging separation principle issue.

On the other hand, in order to ensure high performance, the backstepping method has been applied to attitude control (Krstić and Tsiotras [1999],Kim and Kim [2003],I.Ali et al. [2010]). Compared with conventional Lyapunov-based control, in backstepping, required specifications can be considered during the design procedure, instead of a careful parameter tuning after the controller design. In (Krstić and Tsiotras [1999]), an inverse optimal attitude control law, which is optimal with respect to a meaningful cost function, was proposed. By virtue of the backstepping design, the task of solving Hamilton-Jacobi equation has been avoided. Aiming to address the tradeoff problem between excessive control torque and the sluggish motion, a nonlinear virtual control law (also termed as tracking function) was employed in (Kim and Kim [2003]). Similar strategy was developed in (I.Ali et al. [2010]) to handle the input saturation problem. As is well known, in the backstepping based attitude controller design, desired system response is characterized by the virtual control and is realized by the tracking of virtual control output by the actual control input. Nonetheless, such a tracking can only be achieved asymptotically or in finite time. In

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other words, the expected performance cannot be globally realized throughout the control action.

In this paper, the attitude control problem of rigid spacecraft is firstly addressed in the absence of disturbance and parameter variation. The related object is improving the transient performance, e.g., accelerating the attitude tracking evolution while avoiding excessive control torque. Unlike the backstepping strategy using a nonlinear virtual control law in (Kim and Kim [2003]), a simple Lyapunov-based attitude control law with state-dependent feedback gains is presented. By restricting the damping ratio at the equilibrium point, the overshooting phenomenon can be avoided. Furthermore, with respect to the robustness issue, the integral sliding mode (ISM) control technique is utilized to redesign the Lyapunov-based control law. As a result, a robust attitude controller with improved transient performance is developed. The effectiveness of the proposed strategy will be verified by theoretical analysis and numerical simulation.

2. MATHEMATICAL MODEL AND PROBLEM STATEMENT

Consider a thruster-controlled rigid spacecraft, whose governing equations are described by

$$\hat{\mathcal{J}}\dot{\boldsymbol{\omega}}_b + \boldsymbol{\omega}_b^\times \hat{\mathcal{J}}\boldsymbol{\omega}_b = \boldsymbol{T}_c + \boldsymbol{T}_d + \boldsymbol{T}_p \tag{1}$$

$$\dot{\boldsymbol{\sigma}}_b = \boldsymbol{M}(\boldsymbol{\sigma}_b)\boldsymbol{\omega}_b \tag{2}$$

where $\hat{\mathcal{J}} = \mathrm{diag}(J_1, J_2, J_3)$ is the nominal part of the inertia matrix $\mathcal{J} \in \mathbb{R}^{3\times3}$, and $\boldsymbol{\omega}_b \in \mathbb{R}^3$ denotes the inertial angular velocity. The superscript $(\cdot)^\times$ is the skew-symmetric matrix operator on any 3×1 vector $\boldsymbol{\alpha} = [\alpha_1, \alpha_2, \alpha_3]^{\mathrm{T}}$ such that

$$\boldsymbol{\alpha}^\times = \begin{bmatrix} 0 & -\alpha_3 & \alpha_2 \\ \alpha_3 & 0 & -\alpha_1 \\ -\alpha_2 & \alpha_1 & 0 \end{bmatrix}$$

$\boldsymbol{T}_c \in \mathbb{R}^3$ is the control torque provided by the reaction control thrusters. $\boldsymbol{T}_d \in \mathbb{R}^3$ stands for the disturbance torque, including the environmental and non-environmental torques. $\boldsymbol{T}_p \in \mathbb{R}^3$ is the torque induced by the parametric uncertainty. Let $\Delta\mathcal{J} = (\mathcal{J} - \hat{\mathcal{J}}) \in \mathbb{R}^{3\times3}$ denote the inertia matrix uncertainty, then $\boldsymbol{T}_p = -\Delta\mathcal{J}\dot{\boldsymbol{\omega}}_b - \boldsymbol{\omega}_b^\times \Delta\mathcal{J}\boldsymbol{\omega}_b$. $\boldsymbol{\sigma}_b \in \mathbb{R}^3$ denotes the Modified Rodriguez Parameters (MRP) representation for the inertial attitude of the spacecraft. $\boldsymbol{M}(\cdot) : \mathbb{R}^3 \to \mathbb{R}^{3\times3}$ is the Jacobian matrix operator such that

$$\boldsymbol{M}(\boldsymbol{\sigma}_b) = \frac{(1 - \|\boldsymbol{\sigma}_b\|^2)\mathbf{I}_3 + 2\boldsymbol{\sigma}_b^\times + 2\boldsymbol{\sigma}_b\boldsymbol{\sigma}_b^{\mathrm{T}}}{4} \tag{3}$$

where $\mathbf{I}_3$ is the 3×3 identity matrix and $\|\cdot\|$ is the vector 2-norm. Moreover, $\boldsymbol{M}^{-1}(\boldsymbol{\sigma}_b) = \boldsymbol{M}^{\mathrm{T}}(\boldsymbol{\sigma}_b)/m(\boldsymbol{\sigma}_b)$ with $m(\boldsymbol{\sigma}_b) = (1 + \|\boldsymbol{\sigma}_b\|^2)^2/16$.

Let $\boldsymbol{\sigma}_d, \boldsymbol{\omega}_d \in \mathbb{R}^3$ denote the desired attitude variables, which also satisfy the attitude kinematics in (2), i.e., $\dot{\boldsymbol{\sigma}}_d = \boldsymbol{M}(\boldsymbol{\sigma}_d)\boldsymbol{\omega}_d$. It is assumed that $\boldsymbol{\sigma}_d$ and $\boldsymbol{\omega}_d$ together with $\dot{\boldsymbol{\omega}}_d$ are all bounded. Subsequently, the attitude error variables are defined as

$$\boldsymbol{\sigma}_e = \boldsymbol{\sigma}_b \oplus \boldsymbol{\sigma}_d^* \tag{4}$$

$$\boldsymbol{\omega}_e = \boldsymbol{\omega}_b - \boldsymbol{R}(\boldsymbol{\sigma}_e)\boldsymbol{\omega}_d \tag{5}$$

where $\boldsymbol{\sigma}_e, \boldsymbol{\omega}_e \in \mathbb{R}^3$ represent the MRP error and the angular velocity error. $\oplus$ is the MRP addition operator, characterizing the successive rotations. For two MRPs, e.g., $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, it is calculated as follows:

$$\boldsymbol{\sigma}_1 \oplus \boldsymbol{\sigma}_2 = \frac{(1 - \|\boldsymbol{\sigma}_2\|^2)\boldsymbol{\sigma}_1 + (1 - \|\boldsymbol{\sigma}_1\|^2)\boldsymbol{\sigma}_2 - 2\boldsymbol{\sigma}_1^\times\boldsymbol{\sigma}_2}{1 + \|\boldsymbol{\sigma}_1\|^2\|\boldsymbol{\sigma}_2\|^2 - 2\boldsymbol{\sigma}_1^{\mathrm{T}}\boldsymbol{\sigma}_2}$$

The superscript $(\cdot)^*$ denotes the complex conjugate of MRP and $\boldsymbol{\sigma}_d^* = -\boldsymbol{\sigma}_d$. $\boldsymbol{R}(\cdot) : \mathbb{R}^3 \to \mathbb{R}^{3\times3}$ is the rotation matrix operator. For $\boldsymbol{\sigma}_e$, one has

$$\boldsymbol{R}(\boldsymbol{\sigma}_e) = \mathbf{I}_3 + \frac{8\boldsymbol{\sigma}_e^\times\boldsymbol{\sigma}_e^\times - 4(1 - \|\boldsymbol{\sigma}_e\|^2)\boldsymbol{\sigma}_e^\times}{(1 + \|\boldsymbol{\sigma}_e\|^2)^2}$$

By substituting (4) and (5) into (1) and (2), the governing equations in terms of $\boldsymbol{\omega}_e$ and $\boldsymbol{\sigma}_e$ can be described as

$$\begin{aligned} \hat{\mathcal{J}}\dot{\boldsymbol{\omega}}_e =& \hat{\mathcal{J}}\left(\boldsymbol{\omega}_e^\times\boldsymbol{R}\boldsymbol{\omega}_d - \boldsymbol{R}\dot{\boldsymbol{\omega}}_d\right) - \boldsymbol{\omega}_e^\times\hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) \\ & - (\boldsymbol{R}\boldsymbol{\omega}_d)^\times\hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right) + \boldsymbol{T}_c + \boldsymbol{T}_d + \boldsymbol{T}_p \end{aligned} \tag{6}$$

$$\dot{\boldsymbol{\sigma}}_e = \boldsymbol{M}\boldsymbol{\omega}_e \tag{7}$$

where the related arguments in $\boldsymbol{M}(\boldsymbol{\sigma}_e)$ and $\boldsymbol{R}(\boldsymbol{\sigma}_e)$ are ignored for clarity.

From a practical point of view, the disturbance torque and the inertia matrix uncertainty are both bounded. Following the same line of (Huang et al. [2008]), it is reasonable to assume that $\|\boldsymbol{T}_d + \boldsymbol{T}_p\|_\infty \le c_0 + c_1\|\boldsymbol{\sigma}_e\|_\infty + c_2\|\boldsymbol{\omega}_e\|_\infty$, where c_i $(i = 1, 2, 3)$ are known positive constants and $\|\cdot\|_\infty$ is the vector infinity norm. Thus, the control object can be summarized as follows. Find an attitude controller such that 1) $\boldsymbol{\sigma}_e$ and $\boldsymbol{\omega}_e$ can be globally stabilized in the presence of bounded disturbance and inertia matrix uncertainty; 2) transient performance of the closed-loop system is guaranteed.

3. MAIN RESULTS

In this paper, the above-mentioned control object is realized by two steps. Firstly, high-performance attitude control in the absence of disturbance and inertia matrix uncertainty is guaranteed by an enhanced Lyapunov-based control law. Then, the proposed control law is redesigned by the ISM control technique to ensure the robustness. Before moving on, current Lyapunov-based control scheme is briefly reviewed.

3.1 Current Lyapunov-based control

The basic idea of Lyapunov-based control is to design a feedback control law that renders the derivative of a specified Lyapunov function negative definite or negative semi-definite. To this end, consider the following energy-like Lyapunov function

$$V = \frac{1}{2}\boldsymbol{\omega}_e^{\mathrm{T}}\boldsymbol{\omega}_e + 2k_p\ln(1 + \boldsymbol{\sigma}_e^{\mathrm{T}}\boldsymbol{\sigma}_e) \tag{8}$$

where $k_p > 0$ is a constant scalar.

With respect to the nominal attitude control system, i.e., assuming $\boldsymbol{T}_d = \boldsymbol{T}_p = \boldsymbol{0}$, taking the derivative of (8) gives

$$\begin{aligned} \dot{V} &= \boldsymbol{\omega}_e^{\mathrm{T}}\hat{\mathcal{J}}^{-1}\hat{\mathcal{J}}\dot{\boldsymbol{\omega}}_e + 4k_p\frac{\boldsymbol{\sigma}_e^{\mathrm{T}}\dot{\boldsymbol{\sigma}}_e}{1 + \boldsymbol{\sigma}_e^{\mathrm{T}}\boldsymbol{\sigma}_e} \\ &= \boldsymbol{\omega}_e^{\mathrm{T}}\hat{\mathcal{J}}^{-1}\left[\hat{\mathcal{J}}\left(\boldsymbol{\omega}_e^\times\boldsymbol{R}\boldsymbol{\omega}_d - \boldsymbol{R}\dot{\boldsymbol{\omega}}_d\right) - \boldsymbol{\omega}_e^\times\hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right)\right] \\ &\quad + \boldsymbol{\omega}_e^{\mathrm{T}}\hat{\mathcal{J}}^{-1}\left[\boldsymbol{T}_c - (\boldsymbol{R}\boldsymbol{\omega}_d)^\times\hat{\mathcal{J}}\left(\boldsymbol{\omega}_e + \boldsymbol{R}\boldsymbol{\omega}_d\right)\right] + k_p\boldsymbol{\sigma}_e^{\mathrm{T}}\boldsymbol{\omega}_e \end{aligned}$$