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Relative measurement theory[☆]
The unification of experimental and theoretical measurements

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ABSTRACT

The discontinuous, non-causal and instantaneous changes due to a measurement that appear in quantum mechanics (QM) theory are not consistent with a classical understanding of physical reality, but are completely confirmed by experiments. Relative measurement theory explains why. This paper presents the first formal development of an experimental measurement which includes the uncertainty due to calibration and resolution. The uncertainty due to calibration and resolution, previously considered experimental artifacts, is shown to be equal to the uncertainty that appears in QM theory and experiment. When the calibration to a reference and resolution effects are considered, all the QM measurement discontinuities are consistent with classical explanations.

1. Introduction

In quantum mechanics (QM) theory "…the discontinuous, non-causal and instantaneously acting experiments or measurements" [1] create: uncertainty – when measurements of an unchanged observable change [2], disturbance – measuring one observable disturbs a different observable [3], collapse – experimental results have a lower entropy than QM theory predicts [4], and entanglement – measurement results transfer faster than the velocity of light [5]. Strangely, experiments completely support these unreasonable results [6]. And the wave function (the basis of QM theory) is a complete success at describing the probabilities of a quantum system. This agreement of extensive experiments and successful QM theory has caused many to believe that quantum mechanics is not consistent with classical mechanics, i.e., QM is not reasonable in terms of human experience.

In 1935, the EPR paper [7] proposed that the wave function must be an incomplete description of physical reality. The belief expressed in the EPR paper is that physical reality has underlying consistency and it is a fundamental task of physics to formalize this consistency [5]. Whether or not quantum and classical mechanics are consistent has been considered and tested extensively starting before 1935, without a clear resolution.

This paper develops the first formal measurement function [8] that includes calibration and resolution (Sections 2, 2.1 and 2.2), converts probabilistic QM measures to experimental measurement results (Section 2.3), explains how relative measurement theory resolves the unreasonable results (Sections 3–5), completes the QM description of physical reality (Section 5), and concludes that all mechanics are consistent (Section 6).

Euler [9] identifies that any measurement result is only relative to another measurement result. Fig. 1 presents the minimum empirical single axis *relative measurement system* [10] including three entities: observables,[1] measuring apparatus with finite intervals, and a reference. In Fig. 1, what is accepted in QM theory is the top half and unshaded. What this paper adds is the bottom half and shaded grey.

Each measuring apparatus is projected (vertical arrows) on each observable $\boldsymbol{A}$ and $\boldsymbol{B}$, establishing the A and the B vector magnitudes in intervals of a_i and b_j. The reference u is tightly correlated (relative) by *calibration* (diagonal arrows) to each experimental measuring apparatus interval (MAI). Calibration defines the interval vector magnitude of each a_i or b_j. Fig. 1 does not include resolution effects.

An experimental measurement result of an observable is the sum of each MAI magnitude (e.g., a centimeter ± uncertainty). Often measurement results are assumed to be the product of the vector magnitude (e.g., A) of the intervals times the mean ($\langle . \rangle$) interval magnitude which is $A\langle a_i\rangle$. Or measurement results may be assumed to be Au. When each a_i is not exactly equal to u, or when all a_i are not equal, or when the distribution of a_i is not symmetrical about u, i.e., $u \neq \langle a_i\rangle$, or when

☆ A precursor paper, Relational measurements and uncertainty, was published in Measurement Volume 93, November 2016, Pages 36–40. Relative measurement theory offers an expanded, clearer and more rigorous development than the precursor paper.

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[1] An observable has a magnitude of intervals, each with an interval vector magnitude. In this paper each interval vector magnitude is an independent variable.

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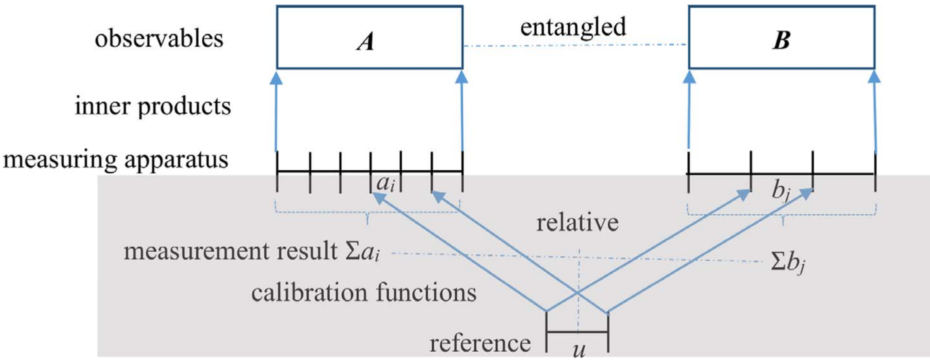

Fig. 1. Relative measurement system.

$\langle a_i \rangle \neq \langle \mathrm{MAI} \rangle$, then:

$$Au \neq A\langle a_i \rangle \neq \Sigma a_i \neq \Sigma \mathrm{MAI} \tag{1}$$

The unequalities in (1) occur when these distributions are not symmetrical to each other. None of these unequalities are recognized in existing measure theory [11] as they are assumed to be related to the experiment. Only ΣMAI in (1) describes experimental results. The uncertainty caused by these unequalities can be significant.

When two calibration functions occur (which correlate each MAI of each measuring apparatus to one reference), measurement result $\sum a_i$ and measurement result $\sum b_j$ and the reference become relative to each other (thin dash-dot line) and can be compared via a common factor of u (*common reference*).

Relative describes the now corrolated relation of the measurement result's intervals, e.g., $2a_i = u = b_j$, as well as the now correlated relation of the measurement result's relative magnitudes ($A = 6$ and $B = 3$) in u_i, i.e., $6a_i = 3b_j$. Fig. 1 does not include the uncertainty of the MAI. In QM theory the observables are termed entangled (thin dash-dot line) when the measurement results of two separated observables remain relative to each other. Entanglement is more formally developed in Section 2.1.

In metrology (the science of experimental measurement), calibration to a reference establishes the correlation between measurement results and decreases the uncertainty of distributions of measurement results. QM theory is based upon a measure theory [11] which does not consider calibration in a *reference space*.[2] A relative measurement theory (RMT) is needed.

2. Formal measurement

In an experimental measurement, the MAI and their coordinate axes are defined by the SI (International System of Units) [12]. The SI is the *experimental reference space*. This reference space must be applied in a measurement theory which describes experimental results. An MAI is correlated to the appropriate SI standard(s) using metrology. But each MAI is not exactly equal to the others from the same measuring apparatus or exactly equal to the appropriate SI standard(s).

From Fig. 1, a measurement includes the inner product function and calibration function required to establish a comparable measurement result. In this paper, the magnitudes of observables and their intervals are formalized without consideration for interactions with the measuring apparatus (i.e., observer effects) or any external effects such as noise. In carefully designed inner product and calibration experiments, these observer and external effects may be minimized or canceled and are not considered inherent.

[2] Reference space describes a vector space that also stipulates the discrete intervals applied for measures or measurements.

2.1. Inner product function

An observable (e.g., $\boldsymbol{A}$ or $\boldsymbol{B}$) exists prior to any relative relation. Therefore the observable is a norm or unity. Norms (***bold***) are self-relative and represent all the magnitude possibilities. In QM theory an observable is a superposition of complex amplitudes which represents all the magnitude possibilities.

The inner product function converts an observable's norm to a magnitude of interval norms. The measuring apparatus's intervals, before the calibration function, are norms, $\boldsymbol{u}_i$. A *measure* (observable's magnitude in $\boldsymbol{u}_i$) is calculated when each interval of the measuring apparatus ($\boldsymbol{u}_i$) which projects on the observable is counted. In Fig. 2 each projection is indicated by upward arrows.

Eq. (2) formalizes Fig. 2, as a sum of inner products $\langle . , . \rangle$ where $i \in \{1,2,...,n\}$ [13].

$$\sum_{i=1}^{i=n} \langle \frac{1}{n}, \boldsymbol{u}_i \rangle = \text{magnitude (e.g., } A \text{ of } \boldsymbol{A}\text{) in } \boldsymbol{u}_i \tag{2}$$

Eq. (2) may also be formalized in bra-ket notation [14]. Since $\boldsymbol{u}_i - \boldsymbol{u}_{i-1} = \frac{1}{n}$ and N is a vector magnitude expressing the sum of n equal intervals of both the observable and the measuring apparatus, then Fig. 2 provides a derivation of the Born Rule [15]. The Born Rule identifies that the inner product of the bra and the ket in (3) is the probability amplitude of the magnitude of a measure in $\boldsymbol{u}_i$.

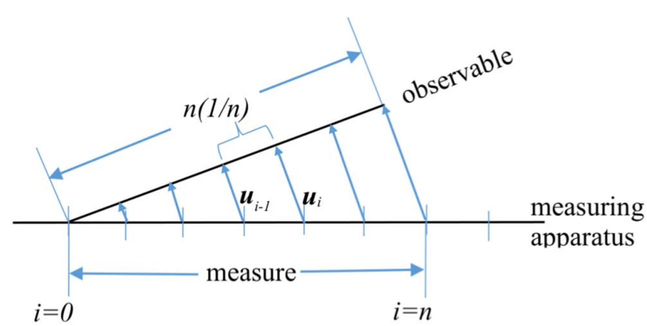

Fig. 2. Inner product function.

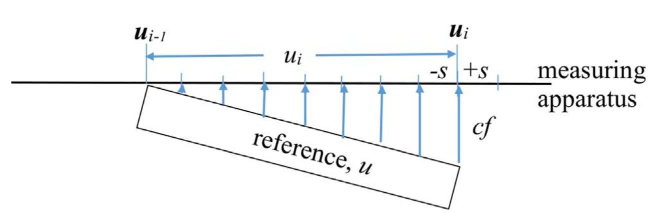

Fig. 3. Calibration function $\boldsymbol{u}_i \rightarrow u_i$.