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Guidance Law Design via Variable Structure Control with Finite Time Sliding Sector

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Abstract: A new guidance law utilizing variable structure control with finite time sliding sector is proposed. First, a finite time sliding sector is defined. The finite time sliding sector is a subset of state space in which the Lyapunov function candidate satisfies the finite time stability condition, in contrast to the commonly used notion of asymptotic stability in conventional sliding sector. Then, based on the finite time sliding sector, a sliding sector control law is designed to move the system state in to the sector in finite time. The target acceleration is considered as an uncertainty. The proposed sliding sector guidance law is derived by supposing the target acceleration upper bound can be estimated as a priori. Simulation results show that the new guidance law is highly effective.

Keywords: Guidance law, Variable structure control, Lyapunov function, Finite time stability, Sliding sector.

1. INTRODUCTION

A large number of design methods have been applied to missile guidance problems, ranging from proportional navigation (PN) to robust control algorithms. The PN guidance law has been widely used due to its advantages such as simple form and easy implementation (Guelman, 1971; Zarchan, 2012). If the target does not maneuver, the PN guidance law can achieve high precision. When the targets acceleration information can be obtained, the augmented proportional navigation (APN) and the predictive guidance law (PGL) are proposed to intercept maneuvering targets (Ha et al., 1990; Talole & Ravi, 1998). However, the target acceleration is hard to be estimated precisely in practical applications. Therefore, some robust control algorithms have been applied to guidance problems such as the H_∞ guidance law (Yang & Chen, 1998; Chen et al., 2002; Shieh, 2007), the L_2 gain guidance law (Zhou et al., 2001), the Lyapunov nonlinear guidance law (Lechevin & Rabbath, 2004) and the variable structure guidance law (Zhou et al., 1999; Moon et al., 2001; Zhou et al., 2009; Babu et al., 1994; Zhou et al., 2013).

The variable structure control is well known for its robustness properties, but it is suffering the chatting phenomena. To deal with the chatting phenomena existing in a VSC system, a sliding sector (Furuta & Pan, 2000) for a linear time invariant (LTI) system has been proposed instead of sliding mode. The sector is designed by the algebraic Riccati equation (ARE). For the nonlinear time varying (NTV) system with a matched uncertainty, the forward integration of state dependent differential Riccati equation (SDDRE) is used to design an NTV sliding sector (Pan et al., 2009). It has been show that the Lyapunov function decreases with a VSC law inside the sector. And outside the sector, a sliding sector control (SSC) law is designed to move the system state into the sector in finite time. But it can't ensure the finite time stability while inside the sector.

In recent years, the finite time stability (Hong, 2002; Huang et al., 2002) for feedback control systems has gained increased attention. It was demonstrated that finite time stable systems might enjoy not only faster convergence but also better robustness and disturbance rejection properties (Huang et al., 2002; Bhat & Bernstein, 1997; Bhat & Bernstein, 1998; Bhat & Bernstein, 2000). In this paper, the relative motion equations between the missile and the target are represented in a state dependent linear time variant (SDLTV) form. The finite time sliding sector is designed in which the Lyapunov function candidate satisfies the finite time stability condition. A SSC guidance law using the finite time sliding sector is proposed based on the solution of SDDRE.

The paper is organized as follows. In Sec. 2, the missile-target engagement problem is formulated. In Sec. 3, the finite time sliding sector is defined and the new guidance law is proposed utilizing the VSC with finite time sliding sector. Numerical simulation results are shown in Sec. 4, and conclusions are reported in Sec. 5.

2. PROBLEM FORMULATION

Considering the spherical line of sight (LOS) coordinates (r, θ, ϕ) with origin fixed at the missile's gravity center. Let e_r , e_θ and e_ϕ be the unit vectors along the coordinate axes. Fig.1 is the three-dimensional pursuit-evasion geometry between the missile and the target.

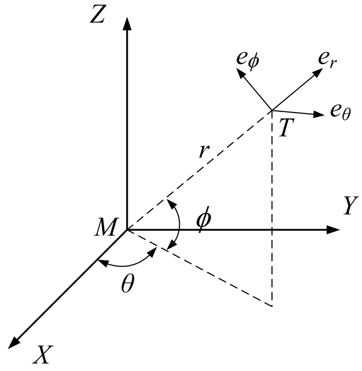

Fig.1. Three-dimensional interception geometry.

In Fig.1, the missile *M* is attempting to intercept a target *T*. In the guidance process, the missile and the target are assumed as two point masses. By virtue of the principles of kinematics, the relative motion can be expressed by the following set of second-order nonlinear differential equations (Chen et al., 2002; Shieh, 2007):

$$\ddot{r} - r\dot{\phi}^2 - r\dot{\theta}^2\cos^2\phi = a_{Tr} - a_{Mr} \tag{1}$$

$$r\ddot{\theta}\cos\phi + 2\dot{r}\dot{\theta}\cos\phi - 2r\dot{\phi}\dot{\theta}\sin\phi = a_{T\theta} - a_{M\theta} \tag{2}$$

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} + r\dot{\theta}^2\sin\phi\cos\phi = a_{T\phi} - a_{M\phi} \tag{3}$$

where r is the relative distance between missile and target, ϕ and θ are LOS angles in elevation loop and the azimuth loop, respectively.

Let $V_r = \dot{r}$, $V_\theta = r\dot{\theta}\cos\phi$, $V_\phi = r\dot{\phi}$, the relative velocity in the LOS coordinates can be expressed as

$$\boldsymbol{V}_{mt} = \begin{bmatrix} V_r & V_\theta & V_\phi \end{bmatrix}^{\mathrm{T}} \tag{4}$$

In the terminal guidance phase, the relative speed and distance satisfy the following condition:

$$V_r < 0,\ \ 0 < r < r(0) \tag{5}$$

The purpose of design a guidance law is to make sure that the tangential relative velocities V_θ and V_ϕ converge to zero. It means that the missile and target are in head-on condition. To design such a guidance law, (2)-(3) can be rewritten as

$$\dot{V}_\theta = -\frac{V_r V_\theta}{r} + \frac{V_\theta V_\phi \tan\phi}{r} - a_{M\theta} + a_{T\theta} \tag{6}$$

$$\dot{V}_\phi = -\frac{V_r V_\phi}{r} - \frac{V_\theta^2 \tan\phi}{r} - a_{M\phi} + a_{T\phi} \tag{7}$$

The missile's acceleration $a_{M\theta}$ and $a_{M\phi}$ are chosen as a form of an extension PN guidance law, that is

$$a_{M\theta} = -\frac{N V_r V_\theta}{r} + u_{M\theta} \tag{8}$$

$$a_{M\phi} = -\frac{N V_r V_\phi}{r} + u_{M\phi} \tag{9}$$

where N is a navigation constant, $N > 1$. $u_{M\theta}$ and $u_{M\phi}$ will be designed in the sequel. Substituting (8)-(9) into (6)-(7), the nonlinear system can be represent in the SDLTV form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}(\boldsymbol{x},t)\boldsymbol{x} + \boldsymbol{B}(\boldsymbol{x},t)(\boldsymbol{u} + \boldsymbol{w}) \tag{10}$$

where,

$$\boldsymbol{x} = \begin{bmatrix} V_\theta \\ V_\phi \end{bmatrix},\ \boldsymbol{A}(\boldsymbol{x},t) = \begin{bmatrix} \frac{(N-1)V_r}{r} & \frac{V_\theta \tan\phi}{r} \\ -\frac{V_\theta \tan\phi}{r} & \frac{(N-1)V_r}{r} \end{bmatrix},$$

$$\boldsymbol{B}(\boldsymbol{x},t) = -\boldsymbol{I}\ ,\ \boldsymbol{u} = \begin{bmatrix} u_{M\theta} \\ u_{M\phi} \end{bmatrix},\ \boldsymbol{w} = \begin{bmatrix} -a_{T\theta} \\ -a_{T\phi} \end{bmatrix}$$

In practical applications, the target acceleration $\boldsymbol{w}$ is unknown and is usually difficult to estimate, but its upper bound can be estimated as a priori. Suppose the target acceleration is bounded as

$$\|\boldsymbol{w}\|_\infty \le f \tag{11}$$

with a positive constant f , where $\|\boldsymbol{w}\|_\infty$ denotes infinity norm of $\boldsymbol{w}$.

3. GUIDANCE LAW DESIGN

3.1 Finite time sliding sector

An LTI sliding sector (Furuta & Pan, 2000) and an NTV sliding sector (Pan et al., 2009) are defined for the LTI system and the SDLTV system respectively. Considering the parameter uncertainties or external disturbances, a VSC law is implemented to ensure the decrease of the Lyapunov function candidate inside the sliding sector (Pan et al., 2009). In recent years, finite time stability of nonlinear systems has gained increased attention. The finite time stability theory for time-invariant nonlinear systems (Hong, 2002) is extended to time-varying nonlinear systems (Zhou, 2009) in the following lemma.

Lemma 1 (Zhou, 2009): Consider the nonlinear system described as

$$\dot{x} = f(x,t),\ f(0,t) = 0\ ,\ x \in R^n$$

Suppose that there is a continuously differentiable function $V(x,t)$ defined in the neighborhood $\hat{U} \subset R^n$ of the origin, and that there are real numbers $\alpha > 0$ and $0 < \lambda < 1$, such that $V(x,t)$ is positive definite on $\hat{U}$ and that $\dot{V}(x,t) + \alpha V^\lambda(x,t) \le 0$ on $\hat{U}$. Then, the zero solution of the nonlinear system is finite time stable. The settling time, depending on initial state x_0 , is given by

$$T_r \le \frac{V^{1-\lambda}(x_0,0)}{\alpha(1-\lambda)}.$$

In this paper, based on the above lemma, a finite time sliding sector is defined as follows.

Definition 1: A finite time sliding sector for the SDLTV system (10) is defined as

$$\mathrm{S}(\boldsymbol{x},t) = \left\{\boldsymbol{x}\ \middle|\ \|\boldsymbol{\sigma}(\boldsymbol{x},t)\| - \|\boldsymbol{\delta}(\boldsymbol{x},t)\| \le 0, t \in R^+\right\} \tag{12}$$

inside which a Lyapunov function candidate satisfies the finite time stability condition

(1) $V(\boldsymbol{x},t)$ is positive define;