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From Linear to Nonlinear Model Predictive Control of a Building

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Abstract: In the building climate control area, the linear model predictive control (LMPC)—nowadays considered a mature technique—benefits from the fact that the resulting optimization task is convex (thus easily and quickly solvable). On the other hand, while nonlinear model predictive control (NMPC) using a more detailed nonlinear model of a building takes advantage of its more accurate predictions and the fact that it attacks the optimization task more directly, it requires more involved ways of solving the non-convex optimization problem. In this paper, the gap between LMPC and NMPC is bridged by introducing several variants of linear time-varying model predictive controller (LTVMPC). Making use of linear time-varying model of the controlled building, LTVMPC obtains predictions which are closer to reality than those of linear time invariant model while still keeping the optimization task convex and less computationally demanding than in the case of NMPC. The concept of LTVMPC is verified on a set of numerical experiments performed using a high fidelity model created in a building simulation environment and compared to the previously mentioned alternatives (LMPC and NMPC) looking at both the control performance and the computational requirements.

Keywords: Predictive control; adaptive control; recursive identification.

1. INTRODUCTION

Energy savings in buildings and reduction of their energy consumption are some of the most emerging challenges for society today. The reason is simple and the numbers speak for themselves—up to 40 % of the total energy consumption can be owed to the building sector [1]. Out of this amount, more than half is consumed by various building heating/cooling systems. Therefore, the recent emphasis on the energy savings in this area is right on target. With the clearly evident need for savings in the area of the building climate control, improvements can be found when considering the latest control techniques.

Model Predictive Control (MPC) stands as one of the most promising candidates for the energetically efficient control strategy. This was demonstrated also within the framework of the Opticontrol project where one research team at ETH Zurich (Switzerland) showed on numerous simulations that using MPC instead of the classical control strategies, more than 16 % savings can be achieved [2,3] depending on the building type. Under real-operational conditions, these savings can be even higher than considering the simulation environment due to the software simplifications compared to the real building. Handling the real-life challenges properly, the improvement achieved by the MPC compared to the classical controller is usually more impressive. This was shown by teams from Prague [4,5] and UC Berkeley [6] where the actual cost savings were even better than the theoretical expectations.

However, MPC suffers from several drawbacks. Besides the need for a reliable mathematical model of the building which should be both simple enough (so that it can be handled effectively) and able to predict the building behavior with sufficient accuracy for several hours ahead, one very severe bottleneck is the complexity of the optimization routine. In order to be feasible and computable, simplified formulations are often considered. Moreover, linear models are usually assumed and exploited by the optimizer. Therefore, in the majority of the MPC applications, the overall task is formulated as a linear/convex optimization problem easily solvable by the commonly available solvers for quadratic or semidefinite programming [4,7]. Although being computationally favorable and able to find the global minimum in case of the convex formulation of the optimization task, their disadvantage is that they do not enable minimization of the nonlinear/nonconvex cost criteria and therefore, only certain approximation of the real cost paid for the control is optimized. Moreover, they resort to the optimization of either the setpoints or the energy delivered to the heating/cooling system while leaving all its distribution to the suboptimal low-level controllers which can lead to a significant loss of the optimality gained by the MPC.

In several recent works, the effort to introduce the nonlinearities (caused either by the dynamical behavior of the building or by the control requirements formulation) into the optimization task can be found [6,8]. In this paper, we discuss both possibilities for the zone temperature control (the linear and the nonlinear MPC) and moreover, we bridge the two banks of the gap between the nonlinear and the linear variant of the MPC by introducing linear models that change in time. Such models can describe the building dynamics in a more reliable and flexible way while they still keep the low complexity of the optimization task (since the linear model remains convex). Two ways of obtaining a time-varying model are described and the results of the modified controllers are compared with the results of the original (linear and nonlinear) MPCs.

The paper is organized as follows: Sec. 2 illustrates the problem of the building climate control on a simple example. Both the building and the heat delivery system description are provided. Furthermore, control performance criterion, comfort requirements and restrictions are introduced. In Sec. 3, the models supplying predictions to the model based controllers are described. The nonlinear model is derived based on the thermodynamics while for the linear model, the assumed simplifications are presented. For the linear time-varying models, two ways of obtaining them are explained. All four models are verified and their results are discussed. Sec. 4 brings a description of the controllers including the low level re-calculation (for the linear MPC) and the nonlinear optimization routine (for the nonlinear MPC). In Sec. 5, the control behavior of the LMPC, NMPC and LMPC with time-varying models (LTVMPC) is investigated and their results are presented and examined. Sec. 6 draws conclusion of the paper.

2. PROBLEM FORMULATION

In this section, the description of the building, constraints and the evaluative performance criterion are formulated.

2.1 Building of interest

The building under our investigation is a simple medium weight one-zone building modeled in the TRNSYS16 [9] environment, which is a high fidelity simulation software package widely accepted by the civil engineering community as a reliable tool for simulating the building behavior. The Heating, Ventilation and Air Conditioning (HVAC) system used in the building is of the so called active layer type. The pipes in the ceiling distribute supply water which then performs thermal exchange with the concrete core of the building consequently heating the air in the room.
Fig. 1 shows a sketch of the considered building. We considered four directly measured outputs: zone temperature T_Z, ceiling temperature T_C, temperature of the return water T_R and temperature of the south-oriented wall T_S. The supply water temperature T_{SW} and the mass flow rate of the supply water $\dot{m}$ are the controlled inputs while predictions of disturbances (solar radiation $\dot{Q}_S$ and outside-air temperature T_O) are considered to be available.

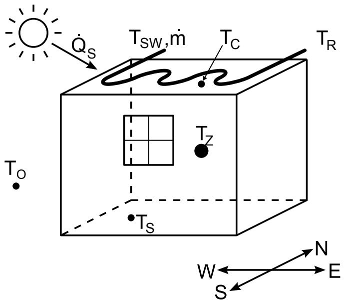

Fig. 1. A scheme of the modeled building

The last step is to describe the heat distribution system. In our application, we consider the configuration of the heating system as shown in Fig. 2. Clearly, the storage tank

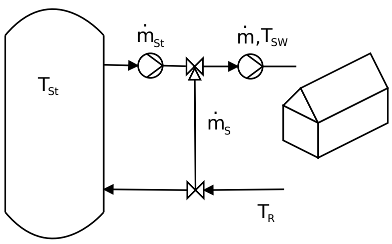

Fig. 2. A scheme of heat distribution system

plays a key role as the sole heat supplier in this system. In fact, having obtained the requirements for the supply water temperature T_{SW} and the supply water mass flow rate $\dot{m}$, these two values are "mixed" using the return water with the temperature T_R flowing into the building inlet pipe through the side-pipe at the mass flow rate $\dot{m}_s$ and the water from the storage tank which is kept at certain constant value T_{St} (in this paper, $T_{St} = 60^\circ$ C is considered) and can be withdrawn from the tank at mass flow rate $\dot{m}_{St}$. Based on this, the following set of equations can be written for the upper three-way valve:

$$\dot{m}T_{SW} = \dot{m}_{St}T_{St} + \dot{m}_S T_R$$

$$\dot{m} = \dot{m}_{St} + \dot{m}_S. \tag{1}$$

which can be further rewritten into an expression for the calculation of the storage water mass flow rate, $\dot{m}_{St} = \dot{m}(T_{SW} - T_R)/(T_{St} - T_R)$. Having the return water temperature measurement at our disposal and extracting the storage water with the temperature of T_{St} at this mass flow rate, both the supply water temperature and supply water mass flow rate related to the heating requirements can be achieved. Last of all, let us note a situation which requires a value of T_{SW} to be lower than the return water temperature T_R would mean negative storage water mass flow rate $\dot{m}_{St}$, which is practically unrealizable. On the other hand, it is also obvious that such T_{SW} requirement really can not be satisfied as only the hot water storage is considered in this configuration. With no cold water storage provided, the temperature of the supply water can not be decreased below the return water temperature which means that the active cooling mode is neither allowed nor realizable.

2.2 Control performance requirements

Besides the building description, it is important to specify the performance requirements, constraints and the criterion according to which the control strategy is evaluated. Considering the building climate control, one of the most important tasks is to ensure the required thermal comfort which is specified by a pre-defined admissible range of temperatures related to the way of use of the building (office building, factory, residential building, ...). Under the weather conditions of middle Europe with quite low average temperatures where heating is required for more than half of year, the thermal comfort satisfaction requirement can be further simplified such that the zone temperature is bounded only from below. As we consider an office building with regular time schedule, the lowest admissible zone temperature $T_Z^{min}(t)$ whose violation will be penalized is defined as a function of working hours as

$$T_Z^{min}(t) = \begin{cases} 22^\circ \text{ C} & \text{from 8 a.m. to 6 p.m.,} \\ 20^\circ \text{ C} & \text{otherwise.} \end{cases} \tag{2}$$

Then, the thermal comfort violation is expressed as

$$CV(t) = \max(0, T_Z^{min}(t) - T_Z(t)). \tag{3}$$

Besides the comfort violation $CV(t)$, the price paid for the operation of the building is penalized in the cost criterion as well. Coming out of the considered structure of the building and its energy supply system, the monetary cost includes the price for the consumed hot water and the electricity needed to operate the two water pumps. While the hot water price P_W is considered constant (see Tab. 1), the electricity price $P_E(t)$ which applies to the operation of the supply and storage water pumps is piece-wise constant and similarly to the lowest admissible zone temperature profile, it depends on the working hours as follows:

$$P_E(t) = \begin{cases} HT & \text{from 8 a.m. to 6 p.m.,} \\ LT & \text{otherwise.} \end{cases} \tag{4}$$

In order to bring our case study closer to reality, the values of high and low tariff (HT and LT) have been chosen in accordance with the real prices approved by the Regulatory Office for Network Industries of Slovak Republic [10].The exact values of HT and LT in €/kWh are listed in Tab. 1. Thus, the overall performance criterion over a time interval $\langle t_1, t_2 \rangle$ is formulated as

$$J = \int_{t_1}^{t_2} \omega CV \mathrm{d}t + \int_{t_1}^{t_2} (P_E(t)(P_C(\dot{m}) + P_C(\dot{m}_{St})) + P_W \dot{m}_{St}) \,\mathrm{d}t. \tag{5}$$

Here, ω is the virtual price for the comfort violation $CV(t)$ which is defined by (3) and $P_W \dot{m}_{St}$ represents the cost paid for the consumed hot water. Time-varying electricity price is expressed as a function of time by (4) and the power consumptions of the water pumps corresponding to $\dot{m}$ and $\dot{m}_{St}$ can be calculated as a quadratic function of the particular mass flow rate, $P_C(\dot{m}) = \alpha_0 + \alpha_1\dot{m} + \alpha_2\dot{m}^2$, $P_C(\dot{m}_{St}) = \alpha_0 + \alpha_1\dot{m}_{St} + \alpha_2\dot{m}_{St}^2$. The parameters $\alpha_{0,1,2}$ are listed in Tab. 1.