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Resolver angle estimation using parameter and state estimation

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ABSTRACT

In this paper, a new type of a resolver angle estimator that utilizes a combined parameter and state estimation scheme is proposed. A state-space model of a resolver is first developed with unknown parameters. Least square estimation is employed to obtain some unknown model parameters by using the measurements up to the current time. Based on the state-space model with estimated parameters, a constrained state estimator with finite memory is constructed to estimate the resolver angle. It is shown through simulation that the proposed scheme is very effective in suppressing noise and overcoming amplitude and phase imbalances compared with common angle tracking observers.

1. Introduction

Resolvers have been used extensively as transducers in all kinds of position and speed control applications such as motors, since they are very simple and reliable. These devices employ two sinusoidal signals with a phase difference of $\pi/2$ to represent the angular position. From the values of two sinusoidal signals, the angular position can be easily computed.

The recent main research areas focussing on development of resolvers can be grouped into the following three fields [1].

- model development [1–7]
- ADCs and DSP [8–11]
- applications [12]

Specially, advanced design schemes related to ADCs and DSP using models have been proposed for high performance. Several methods for implementation have been proposed to more accurately compute the angular position from two sinusoidal signals with a phase difference of $\pi/2$. Most work is based on classical approaches based on deterministic and nominal models. H/W and S/W implementation issues of algorithms based on the nominal models without noise and model uncertainties are considered [2,13,14]. Since noises and model uncertainties such as amplitude imbalance may happen, the deterministic and fixed models are not practical. For better robust performance, a feedback loop with integrators was proposed to track down the angle position and it has been commonly used in industry applications [15,16]. However, this scheme has limited performance since it is not based on mathematical models. The least square technique for parameter estimation with a mathematical model was employed without considering noise [3]. This work only considers a static model, not a dynamic time-series model, so the transient performance and the tracking ability are not guaranteed. Until now, heuristic, non-dynamic model-based, deterministic, and static approaches have been tried without consideration of both noise and model uncertainties. To the best of the authors' knowledge, a modern signal processing approach based on state-space models and optimal design has not been developed for resolver angle estimation.

In this paper, a state space model of a resolver is first suggested and its efficiency is illustrated by applying it to the design of the Kalman filter. State space models are more efficient and more elaborate than I/O models such as transfer functions in some respects. In state space models, system noises and measurement noises can be considered separately. If noise covariances are available or estimated from experiments, the better performance can be expected by using optimal Kalman or FIR filters. Additionally, parameters in state space models can be easily obtained by employing on-line parameter estimation schemes. In this regard, the proposed scheme could be said to have good tracking ability, noise-suppressing effect, and adaptive properties.

In this paper, we develop a stochastic state-space model for a resolver and employ parameter and state estimation schemes for computing the angular position. Since the proposed state-space model includes some unknown parameters, the well-known least

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square curve fitting method is first applied to estimate them [17,18]. Then, the state including information on the resolver angle is recovered from the quadratic constrained least square technique with a finite horizon. Since the proposed method is based on a state-space model or a time-series model, better tracking performance can be expected in comparison with existing non-dynamic model-based approaches. Furthermore, noise-suppressing and adaptive properties could be provided since state estimation has a noise filtering effect and parameter estimation is applied on line. The finite horizon state estimation employed in this paper has been known to be more robust than conventional growing horizon methods [19–21]. It is shown through simulation that the proposed scheme is very effective in suppressing noise and overcoming amplitude and phase imbalances compared with common angle tracking observers.

In Section 2, a state-space model is set up for resolver angle estimation, and parameter and state estimation are applied. The simulation is carried out in Section 3 and conclusions are drawn in Section 4.

2. Resolver angle estimation

As depicted in Fig. 1, the reference voltage for excitation U_{ref} is applied and two induced voltages with a phase difference of $\pi/2$ are obtained from two coils. These resolver outputs can be represented as follows:

$$U_{\sin}(t) = a\sin(\theta(t)) + v_s(t), \tag{1}$$
$$U_{\cos}(t) = b\cos(\theta(t)+\phi) + v_c(t), \tag{2}$$

where a and b are the amplitudes, ϕ is the phase shift due to the imperfect placement of the stator wirings of a motor, and $v_s(\cdot)$ and $v_c(\cdot)$ are measurement noise. Since resolver parameters gradually vary with wear and aging, a and b are generally different from each other, so its imbalance should be compensated. a, b, and ϕ in (1) and (2) are regarded as unknown variables that will be estimated later on. It is assumed that the angular velocity, $\dot{\theta}(t)$, is constant and available. The objective is to estimate $\theta(t)$ from two measurements $U_{\sin}(t)$ and $U_{\cos}(t)$.

If the state variable $x(t)$ is defined by

$$x(t) \triangleq \begin{bmatrix} \sin(\theta(t)) \\ \cos(\theta(t)) \end{bmatrix}, \tag{3}$$

the following state space model can be obtained:

$$\dot{x}(t) = \begin{bmatrix} 0 & 2\pi f \\ -2\pi f & 0 \end{bmatrix} x(t) \triangleq Ax(t), \tag{4}$$

where $d\theta/dt = 2\pi f$. Using the trigonometric identity, $\cos(\theta(t)+\alpha) = \cos(\theta(t))\cos(\alpha) - \sin(\theta(t))\sin(\alpha)$, the measurement equation can be written as

$$\begin{aligned} y(t) &= \begin{bmatrix} a & 0 \\ d & e \end{bmatrix} x(t) + \begin{bmatrix} v_s(t) \\ v_c(t) \end{bmatrix}, \\ &\triangleq Cx(t) + v(t), \end{aligned} \tag{5}$$

where $d = -b\sin(\phi)$, $e = b\cos(\phi)$, and $v(t)$ is assumed to be zero mean white Gaussian noise with the variance R. It is noted that a, d, and e in (5) are unknown variables since a, b, and ϕ in (1) and (2) are considered to be unknown.

All we have to do is to estimate the state $x(t)$ in (5). To do so, the necessary parameters, a, d, and e of C in (5) must first be obtained. Once a, d, e in C are known, the state estimator can be applied and finally $x(t)$ in (3) can be recovered. The angle position $\theta(t)$ can be computed from $\theta(t) = \mathrm{atan2}(x_1(t), x_2(t))$, where $x(t) = [x_1(t), x_2(t)]$ and atan2 is the arctangent function with two arguments in order to return the appropriate quadrant of the computed angle. The overall system is constructed as in Fig. 2.

2.1. Parameter estimation

If there is no noise, or $v_x(t) = v_y(t) = 0$, $y(t) = [y_1(t), y_2(t)]^T$ in (5) should be on the following ellipsoidal trajectory:

$$\beta_1 y_1^2(t) + \beta_2 y_1(t) y_2(t) + \beta_3 y_2^2(t) = 1, \tag{6}$$

where $\beta_1 > 0$, $\beta_3 > 0$, and $\beta_2^2 - 4\beta_1\beta_3 < 0$ since $x(t)$ in (3) is on the unit circle, or $x^T(t)x(t) = 1$. Actually, measurements $y(t)$ filtered by the state estimator may be used, so the noise-free assumption for parameter estimation can be considered to be reasonable.

It can be easily shown through algebraic calculation that, from β_1, β_2 and β_3, a, d, and e in (5) can be computed as

$$a = \frac{1}{\sqrt{\beta_1}}, \tag{7}$$
$$d = -\frac{\beta_2}{2\sqrt{\beta_1\beta_3}}, \tag{8}$$
$$e = \frac{1}{\sqrt{\beta_3}}. \tag{9}$$

β_1, β_2 and β_3 in (6) are obtained to minimize the following cost function:

$$\sum_{i=0}^{k} \left(\beta_1 y_1^2(t_i) + \beta_2 y_1(t_i) y_2(t_i) + \beta_3 y_2^2(t_i) - 1\right)^2, \tag{10}$$

where $y_1(t_i)$ and $y_2(t_i)$ are measurements at sampling times t_i. A parameter set $\bar{\beta} = [\beta_1, \beta_2, \beta_3]^T$ minimizing the cost function (10)

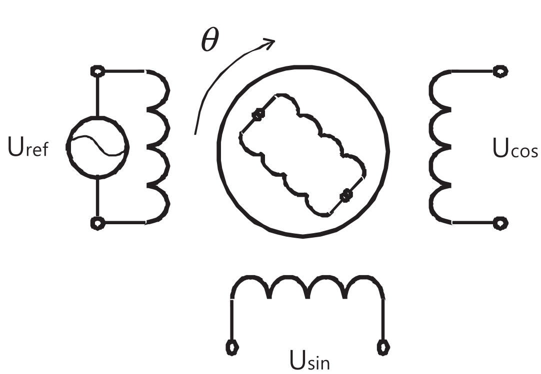

Fig. 1. The structure of a resolver.

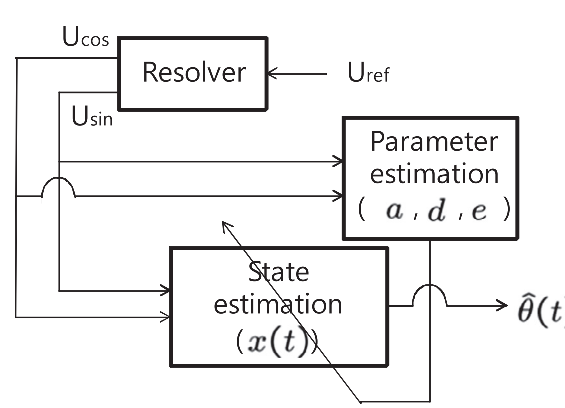

Fig. 2. The overall structure of a resolver angle estimator.