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Uncertainty measure for interval-valued belief structures

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ABSTRACT

Interval-valued belief structures, as an extension of belief structures in classical evidence theory, are developed for better exploitation of uncertain and imprecise information. From the point of view of information theory, uncertainty measure of an interval-valued belief structure is critically important for information processing. However, it is still an open issue to measure its uncertainty. Besides discord and non-specificity, which hide in a precise belief structure, we claim that fuzziness is also associated with an interval-valued belief structure. In this paper, axiomatic requirements for uncertainty measure of interval-valued belief structure are defined. Then an uncertainty measure is proposed to measure the information conveyed by interval-valued belief structures. Its properties are mathematically proved. Finally, numerical experiments are employed to illustrate the performance of the proposed uncertainty measure. It is illustrated that the proposed uncertainty measure is sensitive to the change of belief structures, which might have beneficial effects on decision making.

1. Introduction

Since it was firstly presented by Dempster [1], and was later extended and refined by Shafer [2], the Dempster-Shafer evidence theory, or evidence theory for short, has generated considerable interest. It has been successfully applied in many areas such as expert systems [3], pattern classification [4–7], knowledge reduction [8], and data mining [9]. However, counter-intuitive results obtained in some special cases are roadblocks in its development. Many works have been done to prevent so-called counter-intuitive combination results [10–17]. Researchers holding two major viewpoints have debated for decades. The focus of the disputation lies on the cause of counter-intuitive results. The first viewpoint is that the counter-intuitive results are caused by Dempster's rule of combination, especially its normalization step. Thus, a number of researchers have proposed alternative combination rules that use various strategies to redistribute the conflict and provide a fusion tool that produces results that match expectations [10–13]. The second viewpoint is that the counter-intuitive results come from unreliable evidences to be combined. According to this viewpoint, there are no counter-intuitive behavior results from the use of Dempster's rule of combination, and the mass functions should be modified before combination [14–17]. Evidence weighted averaging [14,15] and evidence discounting [16,17] are two methods for evidence modification.

The original D-S theory requires precise belief degrees and belief structures, whereas in practice some or all probability masses may be uncertain or imprecise. Such uncertainty or imprecision may be caused by the lack of information, linguistic ambiguity or vagueness. For example, in the problem of pattern recognition, a classifier may be unable to give a precise decision about the unknown pattern, and even worse, its belief degree on each class may be imprecise. In this case, an interval-valued belief degree rather than a precise one should be adopted. In the group

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decision making, different experts may give different degrees of belief. Of course these degrees can be averaged to get a real valued estimate, but this leads inevitably to the loss of some important information. Therefore, the use of interval-valued belief degree seems to be a sensible option. This would preserve different belief degrees, thereby facilitating further discussion. So we can say that interval-valued belief structures represent the belief information which is known but not precise. Hence the problem of extending evidence theory to interval-values belief structure arises.

There have been several attempts to extend evidence theory for handling interval-valued belief structure [18–24]. Based on the generalized summation and multiplication operations, Lee and Zhu [18] defined the combination of two pieces of interval evidence. Denoeux [19,20] constructed a pair of quadratic programming models to calculate the combined probability masses of two pieces of interval evidence. Yager [21] explored the issues of the combination and normalization of interval evidence based on the application of interval arithmetic operations. Wang et al. [22,23] investigated the issues of combination and normalization of interval-valued belief structures within the framework of evidence theory. They presented a new logically correct optimality approach, where the combination and the normalization were optimized together rather than separately. As the latest work on the combination and normalization of interval-values belief structures, Sevastianov et al. [24] developed a new framework for rule-based evidential reasoning in the interval setting. We have also proposed a combination rule of interval-valued belief structures based on intuitionistic fuzzy set [37]. However, the issues of combination and normalization of interval-valued belief structures have not been fully resolved. Combination of interval-valued belief structures is still attracting much attention.

Another issue involving in interval-valued belief structure is how to measure its uncertainty, which can supply new viewpoints for analyzing interval-valued data. The concept of uncertainty also plays a fundamental role in computational intelligence. It provides a measure of the amount of information contained by belief structures or probability distribution. Its role in belief structure theory is analogous to the role that entropy plays in probability theory. The uncertainty measure evaluates the degree to which a belief structure points to one and only one element of the discernment frame. This is significant for the processing and accessing of belief structure.

To the best of our knowledge, there are few studies on uncertainty measure of interval-valued belief structures except for Denoeux's [19] work. Denoeux [19] first extended uncertainty measures of precise belief structures into imprecise belief structures. Then he proposed several uncertainty measures for interval-valued belief structures in the form of interval value. Their bounds are the minimum and maximum values of uncertainty measure defined for precise belief structures, obtained by a nonlinear programming model, which is similar to his proposed combination approach [19].

Intuitively, there are at least two drawbacks in Denoeux's uncertainty measure. Firstly, the manifestation of uncertainty measure may be a straightforward one. Interval-valued measure cannot genuinely reflect the uncertainty in the belief structures. We argue that the uncertainty measure of interval-valued belief structures should be a precise value, analogous to the ambiguity measure of precise belief structures [25]. Since there is much uncertainty hidden by interval values, it cannot be applied to depict the uncertainty of interval-valued belief structures. Moreover, it is also an open issue to compare interval values. Although Denoeux's uncertainty measure can capture all types of the uncertainty identified by Klir and Yuan [26], it is not a sensible choice to quantify uncertainty by an uncertain measurement. Secondly, in the calculation of Denoeux's uncertainty measure, optimization algorithm is required when solving the nonlinear programming problem. Although many algorithms can be applied, it will bring extra computation burden, which cannot satisfy the requirement of real time information system.

To process and access interval-valued information better, we reinvestigate the uncertainty measure of interval-valued belief structures in this paper. Based on uncertainty measures in the evidence theory, axiomatic requirements for uncertainty measure of an interval-valued belief structures are presented. By taking the span of the interval value into account, we proposed a new uncertainty measure for interval-valued belief structures. Mathematical proofs and numerical simulations are presented to illustrate properties of our proposed uncertainty measure. It is demonstrated that the uncertainty measure can capture all kinds of uncertainty contained by interval belief structures, and it is sensitive to the change of belief structures. Hence the proposed uncertainty measure offers an alternative for measuring uncertainty or information conveyed by interval-valued belief structures.

The rest of this paper is organized as follows. Section 2 gives a brief recall of evidence theory, together with introductions of interval-valued belief structures. In Section 3, we briefly review existing uncertainty measures for precise belief structures. A new uncertainty measure for interval-valued belief structures and its properties are proposed in Section 4. Illustrative examples are given to show the performance of the proposed measure in Section 5. This paper is concluded in Section 6.

2. Preliminaries

The background material presented in this section deals with the following three main points: (1) the interpretation of evidence theory and terminologies which will be used in this paper to ease the exposition, (2) axiomatic definitions on interval-valued belief structure, and (3) the presentation of interval-valued Bayesian belief structure.

2.1. Dempster-Shafer evidence theory

Dempster-Shafer theory of evidence is modeled based on a finite set of mutually exclusive elements, called the frame of discernment denoted by Θ [1]. The power set of Θ, denoted by 2^{Θ}, contains all possible unions of the elements in Θ including Θ itself and empty set $\emptyset$. Singleton