Optics and Lasers in Engineering 108 (2018) 36–40

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

Efficient tensor approach for propagation of beams of arbitrary shape and coherence through atmospheric turbulence

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ARTICLE INFO

Keywords:
Partially coherent beams
Turbulence
Coherence and statistical optics
Propagation

ABSTRACT

A tensor approach to simulate and predict the transverse intensity and distribution of a two-point correlation of beams of arbitrary shape and coherence through atmospheric turbulence has been derived on the basis of the extended Huygens–Fresnel principle. The validity of this approach is verified by comparing the reconstructed intensity and second-order correlation of symmetric points in the output plane of a Gaussian Schell-model beam with calculated results from analytic formulae. An example illustrates how this tensor approach can be applied to beams blocked by finite apertures in the atmospheric propagation. Our approach provides an efficient and universal numerical model to manage the turbulent propagation of various optical sources. This approach can be useful in long-distance imaging and optical communication.

1. Introduction

In recent years, increasing attention has been paid to the propagation of partially coherent beams (PCBs) through atmospheric turbulence [1] due to their important applications in free space communication [2], optical imaging [3–5], and remote sensing and detection [6,7]. Among these investigations, theoretically predicting the transverse intensity distribution and the correlation property of randomized optical sources is important. Through theoretical analysis, the turbulence-induced effects have been characterized and predicted [8–11]. Special beam classes and new types of partially coherent fields [12] have been analyzed to mitigate the degrading effects of atmospheric turbulence [13–16]. However, due to the complexity of turbulence and the variety of light sources, analytical solutions that present accurate formulae for the simulation of the atmospheric propagation problem are not always available and have to start again for every optical source [17]. As a result, one has to rely on numerical simulation methods. Direct integration from the extended Huygens–Fresnel integral is one option [18], but it appears to be time consuming for complicated optical sources even without turbulence [19,20]. The standard computational model for the numerical simulation utilizes a multiple-phase screen strategy in accordance with which the turbulence is described by thin random phase screens. Discrete Fourier transform (DFT) techniques are used to accomplish screen-to-screen propagation [17,21]. In these DFT-related methods, the lowest spatial frequency of the beam is approximately equal to the inverse of the width of the simulation domain, which means frequencies lower than this inverse are not included. This condition causes distortions in the reconstruction by DFT [1]. Moreover, sampling constraints have to be strictly obeyed when the DFT is carried out [21]. Any violation of the constraints would lead to errors in reconstruction. Therefore, an efficient formalism capable of managing the turbulent propagation of general beam classes and free from the DFT-related limitations is desirable but does not exist yet to the best of our knowledge.

Recently, an efficient tensor approach (ETA) has been reported for numerically reconstructing a propagated PCB in free space [22]. This approach is a direct reconstruction method, which does not carry out the Fourier transformation of the field between the space and frequency domains. Therefore, this method does not have the drawbacks of DFT. However, this approach can only manage the propagation problem in free space. Combining turbulence effects with ETA is helpful for theoretical simulations.

The atmosphere can be modeled in terms of a power spectral density of the fluctuation of the refractive index [23], and the most common models are von Karman, Kolmogorov, and non-Kolmogorov. A modified von Karman formalism, which combines the von Karman and non-Kolmogorov turbulences, has been presented [24,25]. Additionally, this formalism can reduce to the conventional Kolmogorov turbulence [26,27]. For general usage, this modified von Karman formalism seems to be a good selection to describe the atmosphere.

In this manuscript, an ETA to simulate the atmospheric propagation of partially coherent beams is derived under the extended Huygens–Fresnel principle. The generalized turbulence model, which is applicable for both Kolmogorov and non-Kolmogorov statistics, is utilized. Tensor/matrix formalisms are presented to reconstruct the spectral density (averaged intensity) and cross-spectral density (CSD) of symmetric

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https://doi.org/10.1016/j.optlaseng.2018.04.015
Received 12 February 2018; Received in revised form 18 April 2018; Accepted 21 April 2018

points in the transverse plane of arbitrary shape and coherence through atmospheric propagation.

2. Theory

2.1. Propagation of PCBs in atmospheric turbulence

In accordance with the extended Huygens–Fresnel integral, the paraxial propagation of the CSD function of a PCB in turbulence can be obtained as [1,23]

$$W(\boldsymbol{\rho_1}, \boldsymbol{\rho_2}, z) = \frac{1}{\lambda^2 z^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_0(\mathbf{r}_1, \mathbf{r_2}) \times \exp\left[-\frac{ik}{2z}(\mathbf{r_1}-\boldsymbol{\rho_1})^2 + \frac{ik}{2z}(\mathbf{r_2}-\boldsymbol{\rho_2})^2\right] \cdot \langle \exp[\psi^*(\mathbf{r}_1, \boldsymbol{\rho_1}, z) + \psi(\mathbf{r_2}, \boldsymbol{\rho_2}, z)]\rangle_m d^2\mathbf{r_1} d^2\mathbf{r_2}, \quad (1)$$

where $k = 2\pi/\lambda$ is the wave number; $\boldsymbol{\rho_1}$, $\boldsymbol{\rho_2}$ denote the two positions in the output plane; and $\boldsymbol{r}_1$,$\boldsymbol{r}_2$ represent the positions in the input plane. z refers to the propagation distance in the atmosphere. The angle bracket $< >_m$ signifies the ensemble average over the turbulence medium. To propagate a PCB in the turbulence of non-Kolmogorov statistics with the von Karman spectrum, the phase term $\langle \exp[\psi^*(\mathbf{r_1}, \boldsymbol{\rho_1}, z) + \psi(\mathbf{r_2}, \boldsymbol{\rho_2}, z)]\rangle_m$ can be written as [25–27]

$$\langle \exp[\psi^*(\mathbf{r_1}, \boldsymbol{\rho_1}, z) + \psi(\mathbf{r_2}, \boldsymbol{\rho_2}, z)]\rangle_m = \exp\left\{-\frac{\pi^2 k^2 T z}{3}\left[(\mathbf{r_1}-\mathbf{r_2})^2 + (\mathbf{r_1}-\mathbf{r_2})\cdot(\boldsymbol{\rho_1}-\boldsymbol{\rho_2}) + (\boldsymbol{\rho_1}-\boldsymbol{\rho_2})^2\right]\right\}, \quad (2)$$

where T is a turbulence parameter defined by

$$T = \frac{A(\alpha)}{2(\alpha-2)}\tilde{C}_n^2 \cdot \left[\beta(\alpha)\kappa_m^{2-\alpha}\exp\left(\kappa_0^2/\kappa_m^2\right)\Gamma_l\left(2-\alpha/2, \kappa_0^2/\kappa_m^2\right) - 4\kappa_0^{4-\alpha}\right], \quad 3 < \alpha < 4, \quad (3)$$

with $\kappa_{\mathbf{m}} = [2\pi\Gamma(5-\alpha/2)A(\alpha)/3]^{1/(\alpha-5)}/l_0$, $\kappa_{\mathbf{0}}=2\pi/L_0$. Here, L_0 and l_0 represent the outer and inner scales of the turbulence, respectively. $A(\alpha)$ and $\beta(\alpha)$ are functions of α and defined by $A(\alpha) = \Gamma(\alpha-1)\cos(\alpha\pi/2)/(4\pi^2)$ and $\beta(\alpha) = 2\kappa_0^2 - 2\kappa_m^2 + \alpha\kappa_m^2$. $\Gamma()$ denotes the gamma function, and $\Gamma_l()$ refers to the incomplete gamma function. $\tilde{C}_n^2$ signifies a generalized refractive index structure parameter with a unit of $m^{3-\alpha}$. The parameter α symbolizes the power law exponent.

Eq. (3) is applicable for the description of both Kolmogorov and non-Kolmogorov turbulences. When $\alpha = 11/3$, Eq. (3) reduces to the conventional Kolmogorov turbulence. In other cases, the equation is for the non-Kolmogorov turbulence.

2.2. Averaged intensity distribution in atmospheric turbulence

On the basis of Eq. (1), the averaged intensity distribution, i.e., $< I(\rho, z) > \equiv W(\rho, \rho, z)$ can be obtained by calculating the following integral:

$$\langle I(\rho, z)\rangle = \frac{1}{\lambda^2 z^2}\iint\iint W_t(\mathbf{r}_1, \mathbf{r}_2) \times \exp\left[\frac{ik}{2z}(\mathbf{r}_1-\rho)^2 + \frac{ik}{2z}(\mathbf{r}_2-\rho)^2\right]d^2\mathbf{r}_1 d^2\mathbf{r}_2, \quad (4)$$

where

$$W_t(\mathbf{r}_1, \mathbf{r}_2) = W_0(x_1, x_2, y_1, y_2)\exp\left\{-\frac{\pi^2 k^2 T z}{3}\left[(x_1-x_2)^2 + (y_1-y_2)^2\right]\right\}. \quad (5)$$

To indicate the spatial vector points in a transverse plane, we have used $\rho \equiv (u, v)$ and $\mathbf{r} \equiv (x, y)$. To indicate discrete coordinates, we have set $x_{j_1} = j_1\Delta_1$, $y_{k_1} = k_1\Delta_1$, $u_m = m\Delta_2$, and $v_n = n\Delta_2$, where the grid (sampling) separations in the input and output planes are Δ_1 and Δ_2, respectively. The discrete form of Eq. (4) is as follows:

$$\langle I(u_m, v_n, z)\rangle \equiv [\boldsymbol{I}]_{mn} = \sum_{j_1}^{N_1}\sum_{j_2}^{N_1}\sum_{k_1}^{N_1}\sum_{k_2}^{N_1}\left[\boldsymbol{H}_y{}^T\right]_{nk_1} \times \left[\boldsymbol{H}_x{}^T\right]_{mj_1}\left[\boldsymbol{W}_t\right]_{j_1 j_2 k_1 k_2}\left[\boldsymbol{H}_x\right]_{j_2 m}\left[\boldsymbol{H}_y\right]_{k_2 n}, \quad (6)$$

where the average intensity $\boldsymbol{I} \equiv \{I(u_m, v_n)\}$ with $m, n = 1 \cdots N_2$ is a matrix of $N_2 \times N_2$. $\boldsymbol{H}_x \equiv \{H_x(x_j, u_m)\}$ and $\boldsymbol{H}_y \equiv \{H_y(y_k, v_n)\}$ with $j, k = 1 \cdots N_1$ represent the impulse response functions of a free propagation system in x and y directions, respectively, and both are $N_1 \times N_2$ matrices. N_1 and N_2 denote integers, representing the numbers of sampling points in the input and output planes, respectively. Within the paraxial approximation, i.e., $\Delta_1 \ll \sqrt{\lambda z}$, the response matrix is as follows [22,28]:

$$[\mathbf{H}_x]_{jm} \equiv H_x(x_j, u_m) = \Delta_1\frac{\exp[i\pi z/\lambda]}{\sqrt{i\lambda z}}\mathrm{sinc}\left[\frac{(u_m - x_j)\Delta_1}{\lambda z}\right]\exp\left[i\frac{\pi}{\lambda z}\left(x_j^2 - 2x_j u_m + u_m^2\right)\right]. \quad (7)$$

$\boldsymbol{H}_y$ can be represented in the same way. The superscript "T" indicates the matrix transpose and complex conjugate. $\mathbf{W}_t \equiv \{W_t(x_{j_1}, x_{j_2}, y_{k_1}, y_{k_2})\}$ with $j_{1,2}, k_{1,2} = 1 \cdots N_1$ signifies the discrete form of the input CSD function, which is a tensor (multidimensional array) of $N_1 \times N_1 \times N_1 \times N_1$. The discrete form of Eq. (5) is as follows: $[\boldsymbol{W}_t]_{j_1 j_2 k_1 k_2} \equiv [\mathbf{W}_0]_{j_1 j_2 k_1 k_2} \cdot \exp\{-\pi^2 k^2 T z[(x_{j_1} - x_{j_2})^2 + (y_{k_1} - y_{k_2})^2]/3\}$.

In some cases, the tensor calculation of Eq. (6) can be further simplified. For example, when the CSD function of the optical source is x-y separable, Eq. (5) can be separated, i.e., $W_t(\mathbf{r}_1, \mathbf{r_2}) = W_{tx}(x_1, x_2)W_{ty}(y_1, y_2)$, where

$$W_{tx}(x_1, x_2) = W_0(x_1, x_2)\exp[-\pi^2 k^2 T z(x_1 - x_2)^2/3], \quad (8)$$

and $W_{ty}(y_1, y_2)$ can be obtained in a similar way. Eq. (6) can be simplified into a matrix form

$$[\boldsymbol{I}]_{mn} = \left[\boldsymbol{H}_x{}^T\boldsymbol{W}_{tx}\boldsymbol{H}_x\right]_{mm} \cdot \left[\boldsymbol{H}_y{}^T\boldsymbol{W}_{ty}\boldsymbol{H}_y\right]_{nn}, \quad (9)$$

where $\boldsymbol{W}_{\mathbf{tx}}$ and $\boldsymbol{W}_{\mathbf{ty}}$ are $N_1 \times N_1$ matrices: $\boldsymbol{W}_{tx} \equiv \{W_{tx}(x_{j_1}, x_{j_2})\}$, $\boldsymbol{W}_{ty} \equiv \{W_{ty}(y_{k_1}, y_{k_2})\}$. The dot $(\cdot)$ indicates a scalar multiplication.

Through Eqs. (6) and (9), the evolution of the intensity of various PCBs in both Kolmogorov and non-Kolmogorov turbulences can be quantitatively analyzed.

2.3. Second-order correlation of symmetric points in turbulence

In this section, a concrete formula for the CSD function between two symmetric points in the output plane, i.e., $(\boldsymbol{\rho},-\boldsymbol{\rho})$, is derived. Setting $\boldsymbol{\rho}_1 = -\boldsymbol{\rho}_2 = \boldsymbol{\rho}$ in Eqs. (1) and (2), one obtains

$$W(\boldsymbol{\rho}, -\boldsymbol{\rho}, z) = \Gamma(\boldsymbol{\rho}, z) = \iint\iint W_0(\mathbf{r}_1, \mathbf{r_2})\exp[-\boldsymbol{\delta}(\mathbf{z})\cdot(\mathbf{r}_1 - \mathbf{r_2})^2]\exp[-8\delta(z)\cdot\boldsymbol{\rho}^2] \cdot \exp\left[-\frac{ik}{2z}(\mathbf{r_1} - \tilde{\boldsymbol{\rho}})^2 + \frac{ik}{2z}\left(\mathbf{r_2} + \tilde{\boldsymbol{\rho}}^*\right)^2\right]d^2\mathbf{r_1}d^2\mathbf{r_2}, \quad (10)$$

where $\tilde{\boldsymbol{\rho}} = (1 + i2z\delta(z)/k)\boldsymbol{\rho}$ with $\boldsymbol{\delta}(\mathbf{z}) = \boldsymbol{\pi^2 k^2 Tz/3}$. Thus, the output CSD can be represented as

$$[\boldsymbol{\Gamma}]_{mn} \equiv \Gamma(u_m, v_n, z) = \exp\left[-8\delta(z)\left(u_m{}^2 + v_m{}^2\right)\right] \cdot \left[\tilde{\boldsymbol{W}}\right]_{m,(N_2+1-m),n,(N_2+1-n)}, \quad (11)$$

where $\tilde{\boldsymbol{W}} = \tilde{\boldsymbol{H}}_y^T\tilde{\boldsymbol{H}}_x^T\tilde{\boldsymbol{W}}_t\tilde{\boldsymbol{H}}_x\tilde{\boldsymbol{H}}_y$with $\tilde{\boldsymbol{W}}_t = \boldsymbol{W}_0 \cdot \exp[-\delta(z)\cdot(\mathbf{r}_1 - \mathbf{r_2})^2]$, $[\tilde{\mathbf{H}}_x]_{jm} = H_x(x_j, \tilde{u}_m)$, and $[\tilde{\mathbf{H}}_y]_{kn} = H_y(y_k, \tilde{v}_n)$. $\tilde{u}_m$ and $\tilde{v}_n$ are complex coordinates and defined by $\tilde{u}_m = (1 + i2z\delta(z)/k)u_m$ and $\tilde{v}_n = (1 + i2z\delta(z)/k)v_n$.

When the CSD function of the input signal $W_0(\mathbf{r}_1, \mathbf{r_2})$ is mathematically separable in x and y, Eq. (11) can be simplified as follows:

$$[\boldsymbol{\Gamma}]_{mn} = \exp\left[-8\delta(z)(u_m{}^2 + v_m{}^2)\right] \cdot [\tilde{\boldsymbol{H}}_x^T\tilde{\boldsymbol{W}}_{tx}\tilde{\boldsymbol{H}}_x]_{m,(N_2+1-m)} \cdot [\tilde{\boldsymbol{H}}_y^T\tilde{\boldsymbol{W}}_{ty}\tilde{\boldsymbol{H}}_y]_{n,(N_2+1-n)}, \quad (12)$$

where $\tilde{W}_{tx}(x_{j_1}, x_{j_2}) = W_0(x_{j_1}, x_{j_2})\exp[-\delta(z)(x_1 - x_2)^2]$. $\tilde{W}_{ty}$ can be obtained in a similar way.