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A systematic analysis for the quantitative comparison of phase retrieval methods based on alternating projections

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A R T I C L E I N F O

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A B S T R A C T

We present a numerical and experimental scheme for the systematic analysis and comparison of phase retrieval techniques based on alternating projection numerical methods. This comparison allows us to evaluate the most common and recently introduced phase retrieval methods. The proposed scheme gives a quantitative comparison that helps to elucidate the differences between them and develop proper technical implementations of phase retrieval. The comparison is made by means of a numerical and experimental scheme that allows us to evaluate phase retrieval experiments. In this work, the drawbacks of using arbitrary random initial seeds to support the phase retrieval numerical algorithms are also analyzed and discussed. Moreover, we show the convenience of using a rough object phase estimation, which is obtained by means of a simple holographic technique, as the initial seed. This seed dramatically reduces the computational load of the algorithms by decreasing the successive iterations from hundreds to less than twenty. The experimental object under study is a random phase object within a micro-channel. As a proof of concept, this micro-channel combined with a millimeter size semicircular hole, which provides a reference wave, conforms a primitive sensor. The performance of the algorithms is not only measured by the usual convergence error, but also by means of a quality index that requires a direct comparison against the generally unknown original phase object. Thus, in order to evaluate the experimental performance of the phase retrieval techniques, we implement an interferometric optical setup that allows us to compare the results obtained by both techniques. The experiment proposed is a valuable tool for quantitative experimental evaluation of phase retrieval techniques in the optical domain.

1. Introduction

Phase retrieval techniques are particularly used in different fields such as electron microscopy, X–ray crystallography and astronomy. Moreover, there are a variety of phase retrieval techniques. Some of them introduce changes in the recording scheme such as changing wavelengths, changing the distance between the sample and the sensor or the distance between the illumination source and the sample, or moving the illumination laterally across the sensor (ptychography) and illuminating the sample from different directions (Fourier-ptychography). As is very well known, this problem plays a central role in various fields of science and engineering when it is investigated from a more general point of view. In this work, we avoid the use of interferometric setups or recently introduced multi-image approaches [1,2]. A contemporary overview of the phase retrieval problem with application to optical imaging should be consulted in Ref. [3] with an exhaustive list of references therein. Interested readers can find in this review links between relevant optical physics and signal processing methods and algorithms.

We focus on phase retrieval techniques based on alternating projection numerical methods. This approach only requires knowing the Fourier magnitude and the support of the tested object. Therefore, it is very attractive for the development of refractive sensors since the complexity level of the optical setup is reduced. However, these recursive numerical methods either fail to work or show partial results that are difficult to interpret. A reason to explain this difficulty is that there is no guarantee that a solution can be found algorithmically. This problem is not convex, and the solution depends on the initialization and the complex object signal. Therefore, it is convenient to carry out theoretical and experimental comparisons between the well-known recursive algorithms based on a proper object signal as a standard for analysis. To our knowledge, this kind of comparisons cannot be found in the relevant lit-

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erature. Moreover, we could not find a quantitative comparison of the retrieved phase against the object optical phase.

With the name of *Optical Phase Retrieval* (OPR), we refer to the classic problem that can be shortly described as the reconstruction of an object signal $g_o \in \mathbb{C}$ from the magnitude of its Fourier transform $F = \mathcal{F}(g_o)$. OPR is formulated as the empirical risk minimization expressed by the following equation:

$$\hat{g} = min_{\{g\}} \sum_{k,l=1}^{2M} [|F_{k,l}|^2 - | < a_{k,l}, g > |^2]^2, \quad (1)$$

where $\hat{g} \in \mathbb{C}^{2N \times 2N}$ is the complex object function to be recovered given the intensity measurements $|F| \in \mathbb{R}_+^{2M \times 2M}$. $< a_{k,\,l}, \cdot >$ denotes the decomposition in vectors $a_{k,\,l}$ of the Fourier basis being $M > 2N - 1$ and adopting a frame of work based on the oversampled discrete Fourier transform (zero padding for the object function and oversampling by 2 or more). From the analysis of Eq. (1), it is not clear how to find a global minimum, even if one exists. In addition, it should be noted that all of the trivial ambiguities for $\hat{g}$: a)- global phase shift, b)- conjugate inversion, c)- spatial shift, have the same Fourier modulus.

It is known that prior information increases the probability of convergence to the true solution [3]. Then, to initialize the algorithms, we obtain a rough seed by means of a holographic technique [4] and test its consequent benefits. In order to experimentally evaluate the OPR techniques, we propose a simple two beam interferometric setup to recover the object phase and compare it against the phase retrieved by the recursive OPR algorithms. Since the object phase is also interferometrically determined, this gave us the opportunity to introduce the structural similarity index measure (SSIM) to the OPR study framework.

This paper is organized as follows: in Section 2, we briefly describe the compared algorithms based on alternating projections and the criteria for comparison. Section 3 presents the theoretical object to be tested and introduces the framework of the exact complex–wave reconstruction to obtain a rough estimate object used as an initial guess. In Section 4, we show the performances obtained in the phase retrieval problem by using numerical simulations. Section 5 describes the experimental setup and analyzes the results and the different sources of uncertainty when the object phase map is embedded in a micro-channel. In Section 6, a summary and conclusions are offered.

2. Algorithms based on alternating projections

The most popular kind of phase retrieval methods are based on alternating projections. These methods are of low algorithmic complexity and easy application. Thus, they can be used by non-specialized operators. In 1982, Fienup proposed a family of iterative algorithms that are related to different interpretations of the Gerchberg and Saxton method [5,6]. The general framework is the Error-Reduction iterative algorithm (ER), which consists of the following four steps shown in the block diagram of Fig. 1 for iteration *n*: (1) Fourier transform the object complex signal g_n; (2) make minimum changes in $|G_n|$ to satisfy the Fourier domain constraints and form G'_n; (3) inverse Fourier transform of G'_n; and (4) make minimum changes in g'_n to satisfy the object domain constraints to form a new estimate of the object signal g_{n+1}. An initial guess g_i is commonly given to the iterative process by assigning to each object coordinate location $\mathbf{x} = (x, y) \in \mathbb{R}^2$ a phase composed of uniformly distributed values between $-\pi$ and π. The Fourier constraints are satisfied by replacing $|G_n| = |F|$, where $|F| = \sqrt{I}$ with I the measured intensity in the Fourier domain of the object signal. The object constraints are described as

$$g_{n+1} = \begin{cases} 0 & \{x, y\} \in \gamma, \\ g'_n & \text{otherwise}, \end{cases} \quad (2)$$

where γ includes all points at which the n^{th} estimate of the object function g'_n violates the object extent constraints. We employ the nomenclature proposed in the literature for the reviewed algorithms. In this case, γ is the region in the input object plane where the field values are all zero. One of the most commonly used variant to this ER iterative algorithm is referred to as the Hybrid Input-Output (HIO) method

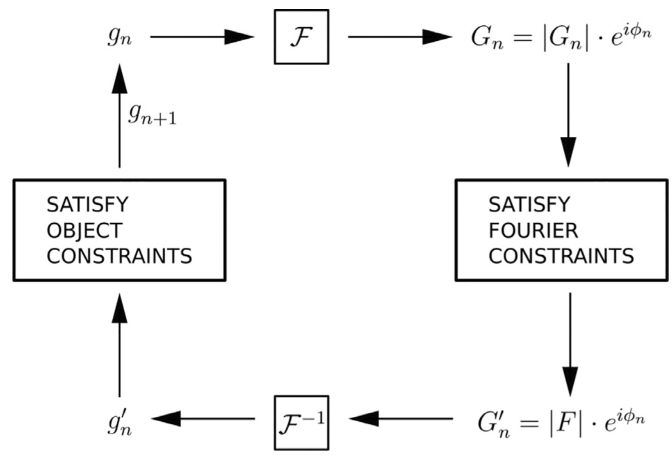

Fig. 1. Block diagram of the ER iterative phase retrieval algorithm.

$$g_{n+1} = \begin{cases} g_n - \beta g'_n & \{x, y\} \in \gamma, \\ g'_n & \text{otherwise}, \end{cases} \quad (3)$$

where β is a constant feedback parameter with values in [0.5, 1]. The HIO algorithm is currently the most widely used algorithm in comparison to the other variants known as the Input-Output (IO) algorithm

$$g_{n+1} = \begin{cases} g_n - \beta g'_n & \{x, y\} \in \gamma, \\ g_n & \text{otherwise}, \end{cases} \quad (4)$$

and the Output-Output (OO) algorithm

$$g_{n+1} = \begin{cases} g'_n - \beta g'_n & \{x, y\} \in \gamma, \\ g'_n & \text{otherwise}. \end{cases} \quad (5)$$

Only the amplitude of the Fourier image and γ are necessary for the object phase recovery. As is known in the specialized literature, some precautions must be taken when applying iterative methods to avoid stagnation, slow convergence and the twin image problem [7]. The combination of the ER and HIO iterative algorithms can perform a better phase retrieval process than separated realizations [8]. To avoid confusion, we name the combination of both a distinct standard iterative method ER/HIO.

As shown in Ref. [9] the combination of the HIO and ER algorithms is significantly outperformed by an extension of this combination based on randomized overrelaxation. The authors show that this extension can enhance the success rate of reconstructions for a fixed number of iterations as compared to reconstructions solely based on the traditional algorithm. We briefly review this algorithm for completeness and name it HIO/O/ER. Therefore, it is convenient to define projection operators P_S and P_A from the operations shown in Fig. 1. It is direct to observe that $g_{n+1} = P_S P_A g_n$ for the ER algorithm. The operator P_A performs the Fourier transformation and conserves the measured amplitude and P_S inverse Fourier transform by fixing the block of zeros corresponding to γ. We encourage the readers to consult Ref. [9] for a proper review. In these terms, HIO is rewritten as $g_{n+1} = [1 - P_S - \beta P_A + (1 + \beta) P_S P_A] g_n$.

The extension of the ER/HIO to the HIO/O/ER is based on overrelaxation and randomization. The authors replace the projection operator P_A by the relaxed expression $L = 1 + \lambda_A (P_A - 1)$ obtaining a new expression for the HIO with overrelaxation $g_{n+1} = [1 - P_S - \beta L + (1 + \beta) P_S L] g_n$, where λ_A is a real constant called relaxation parameter. To include the randomization, in each iteration λ_A is randomly selected with a uniform distribution within a given range of specific values. Formally, the authors present a framework for studying randomization of any iterative projection algorithm and limit its use to parameter values whose deterministic contribution coincide with the HIO algorithm. In this framework, a projection polynomial operator is considered, and by means of