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Review

A descriptor of speckle textures using box fractal dimension curve

H. Rabal [a,*], E. Grumel [a,b], N. Cap [a], L. Buffarini [a], M. Trivi [a,b]

[a] Centro de Investigaciones Opticas, CIOp (CONICET – UNLP – CIC), P.O. Box No. 3, 1897 Gonnet-La Plata, Argentina

[b] UIDET OPTIMO, Departamento de Ciencias Básicas, Facultad de Ingeniería, Universidad Nacional de La Plata, La Plata, Argentina

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ABSTRACT

We propose a simple generalization of the box fractal dimension in images by considering the curve obtained from its value as a function of the binarization threshold. This curve can be used to partially describe ordinary images, textures, static and dynamic speckle patterns. We show some examples of different applications of this approach in some cases of interest.

1. Introduction

Rough surfaces illuminated by coherent laser light show a grainy appearance called speckle [1–2]. Speckle techniques have been applied to study different experimental situations. According to the problems to be solved, different algorithms are required, for example, speckle correlation to study surface roughness [3] and digital speckle pattern interferometry (DSPI) for the study of displacements, deformations and cracks [4].

When the surface changes, its speckle pattern also changes and it is called dynamic speckle [5]. Some properties of the surface changes can be inferred from the time dynamics of its irradiance. Several applications of the measurement of dynamic speckle activity have been found in medicine, biology, industry, agriculture, etc. [see 5 and references therein]. Also, some algorithms have been developed for different applications. In general, the algorithms used in these techniques are useful to solve some situations but cannot be applied to others. For example, when we try to describe static or dynamic speckle patterns it is very difficult to find a single algorithm to analyze both situations.

In a recent work we performed an exhaustive comparative analysis of the descriptors most often used in different applications [6].

Also, we have proposed and shown the possibility to apply the box fractal dimension to characterize speckle patterns in some restricted situations [7]. In that case, we showed numerical simulations and a controlled experiment.

Fractal dimensions, introduced by B. Mandelbrot [8], have been found as useful descriptions in mathematics, in many images such as those that can be found in natural landscapes, patterns, sequences, biological tissues, simple life forms, organic systems, complex life forms, environments and many other branches of the natural sciences.

In this paper, we propose to extend the use of the box fractal algorithm to characterize both static and dynamic speckle patterns in several experimental situations, but it can also be applied to other types of images. It is possible, with a single algorithm and small adaptations, to apply it to very different problems.

We present the results on laser static speckle patterns in an example of roughness and on dynamic speckle quantitative measurements for free propagation geometry in controlled experimental conditions, in the evolution process of polymers (drying of paint) and in ultrasound speckle images.

2. Theory

2.1. Box fractal dimension

The box fractal dimension (BFD), also named the box counting dimension or similarity dimension, is a method of characterizing data, for example, curves or binary images, by decomposing the subject into boxes (usually squared) of different sizes and measuring how the data cover the plane at different scales [8]. If the image is not binary, it must binarized using some threshold U_0 so that every pixel of the image is set to 0 if its value is smaller than U_0 and to 255 in other cases.

The box fractal dimension for that threshold is obtained using the expression:

$$N(s) = B.s^{-BFD} \tag{1}$$

where s is the size of the side of each square box, B is a constant, $N(s)$ is the number of boxes with side s required to cover the image and BFD is the box fractal dimension. In the limit $s \to 0$.

* Corresponding author.
E-mail address: hrabal@ing.unlp.edu.ar (H. Rabal).

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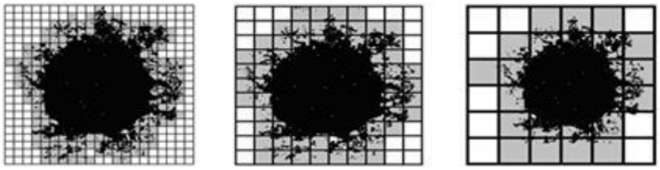

Fig. 1. A red wine drop on paper and how it is covered with three different grids.

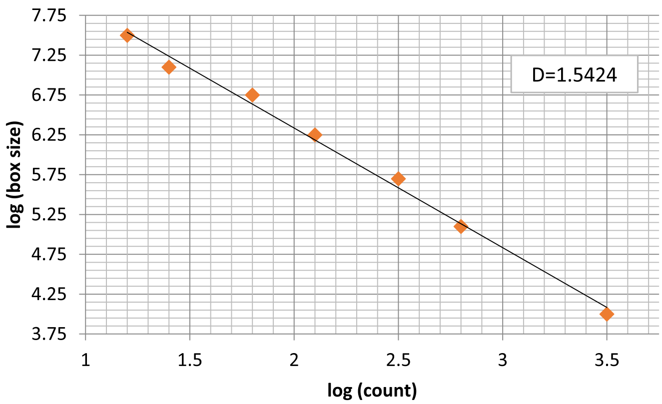

Fig. 2. log (N (s)) plotted versus log (s).

As images are only accessible as discrete integer numbers, the limits $s \rightarrow 0$ cannot be reached. Then, in practice, the BFD is estimated as the slope of the straight line best fitting (least square error) a log–log graph:

$$-(\log N(s) \text{ vs } \log s) \tag{2}$$

for a series of different values of s.

We are going to use an example to illustrate the procedure:

a) A grid is overlaid on the two-dimensional binary image of an object with grid size $= s \times s$ as shown in Fig. 1.
b) Then $N(s)$, the number of cells containing at least one white point of the object, is counted and stored. The number of cells containing at least one dark point could alternatively be used.
c) Next, the size of the grid $s \times s$ is changed and the process repeated.
d) With the obtained results, log (N (s)) is plotted versus log (s). See Fig. 2.
e) The best fitting straight line is determined by using minimum squares. Its slope with reversed sign is, by definition, the box fractal dimension (BFD) estimation for every chosen binarization threshold U.

The original image is again binarized with a new U threshold value. The BFD measurement is then repeated on the result.

Changing the value of U generally produces a different value for the BFD. There seems to be no unique criterion for the choice of the optimum threshold value in all cases. In digital image processing there are several thresholding techniques used to binarize images. Two popular thresholding criteria are to use the middle value of the dynamic range and the Otsu method [9].

It is possible that for some unfortunate choice of the threshold two or more experimental situations result in the same or close value for the fractal dimension, making it impossible to distinguish between them (see for example Figs. 12a and 13a below). For these cases, it is convenient to use more than one threshold value.

B. Chaudhuri and N. Sarkar [10] proposed an improved method for the calculation of the fractal dimension, named Differential Box Counting. In it, although there was no explicit use of a threshold, the dynamic range inside each box was used to count the number of occupied ones for the calculation of the FD. That procedure was used, for example, for segmentation of textures [11] and several improvements were proposed afterward [12–14]. The use of all possible thresholds was not contemplated because for very high values of the threshold very few points are left and the adjustment of the calculation is very poor.

Nevertheless, complex textures could include different structures with different fractal dimensions in different ranges of the image's dynamic range.

In the Chaudhuri and Sarkar method, the image is considered as a surface in a 3D space divided into boxes, with the intensity at each pixel as the third coordinate. Boxes are counted considering the maximum and the minimum value of intensity in the contained volume. This so defined dimension is then a single value between 2 and 3. No other threshold is considered.

In this work, we propose the use of all possible values of U so that information on structures with different gray levels can be preserved. We call this result the box fractal dimension curve (BFDC).

In this approach, each thresholded image is equivalent to a level cut of the surface defined in Sarkar's method and projected on the x, y plane. So, when every possible threshold is considered, a set of numbers between 0 and 2 are obtained. By continuously changing the threshold, the obtained box fractal dimension describes a curve that is characteristic of the distribution of gray levels in the image.

Theoretically, the curve should start with the value 2. In practice, nevertheless, when the curve is numerically adjusted, the result may be slightly higher or lower than the theoretical value. This is due to the error committed when the image is quantized. By increasing the number of bits this undesired effect can be alleviated.

2.2. Box fractal dimension curve (BFDC) in an image example

As a first step to illustrate the use of the BFDC algorithm, in this section we show its application to the well-known case of the mandrill image.

A digital image in incoherent light is an array of integer values, named gray levels, distributed on a bidimensional frame. Each value represents the irradiance registered on the sensitive plane of a camera. As irradiance is usually a continuous function of the position, a quantization of its values and a spatial sampling are inherent to the register [15]. The distribution of the gray levels can be visualized by its histogram.

One frequent operation in image processing is binarization. It consists in segmenting the image according to comparison of the gray levels with a threshold value. If the gray level of a pixel is higher than the threshold, the binarized image is assigned the highest value (usually 255) and the rest is assigned to be zero. The binarization operation generates a sharp, usually irregular, edge between the bright and the dark regions.

In this work we explore the use of the fractal dimension of that binarized image as a function of the chosen binarization threshold. It is evident that if the threshold is very low, most of the pixels in the binarized image are going to be bright and cover a substantial area of the image. It is to be expected that in that case the fractal dimension will be near the value 2. Conversely, when the threshold value is higher than the highest irradiance pixel value, all the binarized image will be uniformly black and the fractal dimension will be zero. Between these extreme situations, the fractal dimension as a function of the threshold will describe a curve that we name the box fractal dimension curve (BFDC).

Fig. 3a) shows a typical example image of a mandrill used in many image processing examples. Fig. 3b) shows the histogram of the image. Fig. 3c) shows the result of applying the concept of the box fractal dimension curve (BFDC) to the image of the mandrill. Notice that it starts at 2, that is, for small thresholds the binarized image covers all of the plane. This is so until the threshold reaches the smallest occupied value of the histogram (22 in Fig. 3b). Then, it decreases, in this case almost monotonically, to 0 for the highest occupied level and over. Between these two values, the particular behavior of the curve depends on the characteristics of the image.