Sensors and Actuators A 279 (2018) 537–542

Contents lists available at ScienceDirect

Sensors and Actuators A: Physical

journal homepage: www.elsevier.com/locate/sna

Impedance calculation of arbitrary-shaped thin-walled coils for eddy-current testing of planar media

Wu Dehui, Yang Fan, Wang Xiaohong*, He Tianfu

Department of Electronic Mechanical Engineering, School of Aerospace Engineering, Xiamen University, Fujian Xiamen, 361005, China

ARTICLE INFO

Article history:
Received 3 December 2017
Received in revised form 2 April 2018
Accepted 7 May 2018
Available online 9 May 2018

Keywords:
Arbitrary-shaped coils
Impedance
Coil function
Eddy-current testing
Planar media

ABSTRACT

Eddy-current testing is used in a variety of fields, so it is important to analyze the coil impedance. Previous studies on impedance calculation have mainly focused on circular and rectangular coils. This paper presents a general method to evaluate the impedance of an arbitrary-shaped thin-walled coil facing with a planar media. The impedance formulas are deduced by using second order vector potential (SOVP) method. In the formulas, the coil function, which is in double-integral form, is defined. The impedance calculation for arbitrary-shaped coils can be represented by the corresponding coil function. The coil functions for common coils such as circular, triangular, rectangular and trapezoidal coils are also derived in this paper. Finally, the impedance and impedance change of various shaped coils above an aluminum plate are measured, and compared with calculated values. The results indicated that they have a good agreement.

1. Introduction

The eddy-current testing (ECT) is a common method for non-destructive testing (NDT) of conductive materials. It is being increasingly used for some industrial fields, for instance, device fabrication, aeronautics and astronautics, petrochemical and power system, etc. [1]. ECT uses an excitation coil to produce eddy-current over the conductive specimen. In turn, the induced eddy-current also affects the excitation magnetic field, causing a change in coil impedance when abnormalities such as cracks occur in the specimen. By calculating the coil impedance, the object's electromagnetic properties, geometry, and testing condition can be measured. In practical use of ECT, it is important to master the coil impedance.

The coil impedance over conductive materials has subsequently been analyzed by many researchers [1–8]. However, these analyses were for coils with some specific shapes, such as circular and rectangular, not available for coils of other shapes. A closed-form expression has been proposed by Lv C. et al. for the impedance of a rectangular coil moving across a conductor with right-angled wedge [2]. Fava J.O. et al. presented expressions for calculating the self-inductance and impedance produced by a planar rectangular spiral coil of arbitrary number of turns and a finite rectangular cross-section placed on a conductive material [6]. In addition, Burke S.K. et al. derived the formulations for the inductance of a circular spiral coil in free space and the impedance change when the coil is wound on the surface of a magnetically conductive cylinder [8]. But for impedance of coils with more complex shapes, to the best knowledge of the authors, there are currently no reports. For coils with arbitrary shape, there are only analysis about their magnetic field and self-inductance [9,10].

In this paper, based on the previous studies, we deduced the formula for calculating the impedance of an arbitrary-shaped coil facing with a planar media by using second order vector potential(SOVP). And we also defined a coil function for arbitrary-shaped coil which is a double-integral form expression derived from the impedance calculating formula. Therefore, the impedance calculation for arbitrary-shaped coils can be represented by the coil function. Finally, we experimentally measured different shape coils' self-inductance and scattering field impedance under different lift-off conditions. And the measured and calculated values were compared and verified.

2. Impedance problem of eddy-current coils

2.1. Problem definition

A sketch of the problem studied in this paper as shown in Fig. 1. An arbitrary-shaped thin-walled coil (solenoid) with N -turns and a thickness of δ, lies above a conductive half-space. The interface between the air region R_0 and the conductive region R_1 is the plane $z = 0$. Assume that Region R_1 is symmetrical and isotropic with a constant conductivity σ and a magnetic permeability $\mu = \mu_r\mu_0$,

* Corresponding author.
E-mail addresses: wudehui@xmu.edu.cn (D. Wu), yangfan2017xmu@163.com (F. Yang), wxh@xmu.edu.cn (X. Wang), 19920151153691@stu.xmu.edu.cn (T. He).

https://doi.org/10.1016/j.sna.2018.05.014
0924-4247/

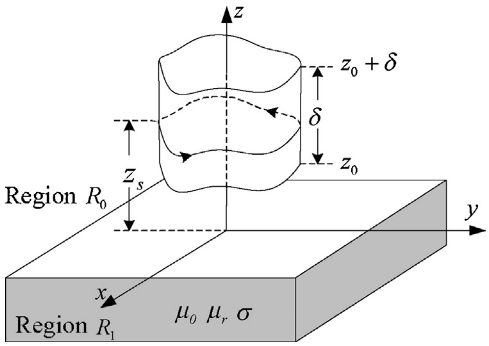

Fig. 1. An arbitrary-shaped coil above a conductive half-space.

where μ_r is the relative per*l*meability and μ_0 is the vacuum permeability. The coil is parallel to the conductor surface with a lift-off z_0.

When the coil is energized by alternating current q with $\omega = 2\pi f$, an electromagnetic field (EMF) $\mathbf{B_s}$ will be generated, which is called incident field. The $\mathbf{B_s}$ is only related to the excitation source and has nothing to do with the presence or absence of the conductor. According to Faraday-law, the eddy-current will be induced over the conductor under the influence of $\mathbf{B_s}$. The eddy-current also produces another EMF $\mathbf{B}_{ec}$, which is called scattering field.

We first consider the EMF of an arbitrary-shaped filament coil which parallels to the conductor surface. For this task, the SOVP method is used. Following [6], the EMF problem for a filament coil has been solved. The EMF $\mathbf{B}$ in Region R_0 is the gradient of scalar potential W, namely [6]

$$\mathbf{B} = \nabla(\frac{\partial W}{\partial z}). \tag{1}$$

The scalar potential W in (1) consists of two parts: the primary potential W_s induced by the excitation current I in Region R_0 and the secondary potential W_{ec} due to the eddy-current over Region R_1, that are

$$\begin{cases} \mathbf{B} = \mathbf{B}_s + \mathbf{B}_{ec} \\ W = W_s + W_{ec} \end{cases}. \tag{2}$$

The expressions of above potentials have been given by Kriezis and Xypteras [11] as

$$W_s = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_s e^{kz} e^{j\xi x} e^{j\eta y} d\xi d\eta,\ \ Z \in R_0, \tag{3}$$

$$W_{ec} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} D_s \frac{k\mu_r - \lambda}{k\mu_r + \lambda} e^{-kz} e^{j\xi x} e^{j\eta y} d\xi d\eta,\ \ Z \in R_0, \tag{4}$$

where ξ and η are the integral variables, D_s is a coefficient related to the coil, $k = \sqrt{\xi^2 + \eta^2}$ and $\lambda = \sqrt{k^2 + j\omega\mu_0\mu_r\sigma}$.

2.2. Electromagnetic field analysis

Based on Biot–Savart law, the primary potential W_s in (2) is written as

$$W_s = \frac{\mu_0 I}{4\pi}\iint_S \frac{1}{R} dx_s dy_s. \tag{5}$$

where S denotes the area surrounded by the filament coil, $R = \sqrt{(x-x_s)^2 + (y-y_s)^2 + (z-z_s)^2}$ denotes the distance between field point (x, y, z) and source point (x_s, x_s, z_s) on the coil.

The generalized double integral form of R^{-1} is expressed as [2]

$$\frac{1}{R} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{k} e^{-k|z-z_s|} e^{j\xi(x-x_s)} e^{j\eta(y-y_s)} d\xi d\eta. \tag{6}$$

Substituting (6) into (5), the primary potential W_s can be calculated by

$$W_s = \frac{\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{\iint_S e^{-j(\xi x_s + \eta y_s)} dx_s dy_s\right\} \frac{e^{-k|z-z_s|}}{k} e^{j(\xi x + \eta y)} d\xi d\eta, \tag{7}$$

We denote the contents in {} as the coil function C, which is only related to the shape and position of coil,

$$C(\xi, \eta) = \iint_S e^{-j(\xi x_s + \eta y_s)} dx_s dy_s. \tag{8}$$

Substituting (7) into (1), we can obtain x -component, y -component and z -component of incident field $\mathbf{B_s}$ as follows

$$B_{s,x} = sgn(z_s - z)\frac{j\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \xi e^{-k|z-z_s|} e^{j(\xi x + \eta y)} C(\xi, \eta) d\xi d\eta, \tag{9}$$

$$B_{s,y} = sgn(z_s - z)\frac{j\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \eta e^{-k|z-z_s|} e^{j(\xi x + \eta y)} C(\xi, \eta) d\xi d\eta, \tag{10}$$

$$B_{s,z} = \frac{\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k e^{-k|z-z_s|} e^{j(\xi x + \eta y)} C(\xi, \eta) d\xi d\eta. \tag{11}$$

By using the formula above, we can calculate the EMF produced by arbitrary-shaped filament coils, and the $C(\xi, \eta)$ varies with the shape and position of coil.

Comparing (7) with (3), we could get the expression of secondary potential from (4),

$$W_{ec} = \frac{\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{e^{-k(z+z_s)}}{k}\frac{k\mu_r - \lambda}{k\mu_r + \lambda} e^{j(\xi x + \eta y)} C(\xi, \eta) d\xi d\eta. \tag{12}$$

Similarly, substituting (12) into (1), we can also obtain x -component, y -component and z -component of scattering field $\mathbf{B}_{ec}$ as follows

$$B_{ec,x} = \frac{-j\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \xi e^{-k(z+z_s)}\frac{k\mu_r - \lambda}{k\mu_r + \lambda} e^{j(\xi x + \eta y)} C(\xi, \eta) d\xi d\eta, \tag{13}$$

$$B_{ec,y} = \frac{-j\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \eta e^{-k(z+z_s)}\frac{k\mu_r - \lambda}{k\mu_r + \lambda} e^{j(\xi x + \eta y)} C(\xi, \eta) d\xi d\eta, \tag{14}$$

$$B_{ec,z} = \frac{\mu_0 I}{8\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} k e^{-k(z+z_s)}\frac{k\mu_r - \lambda}{k\mu_r + \lambda} e^{j(\xi x + \eta y)} C(\xi, \eta) d\xi d\eta. \tag{15}$$

2.3. Impedance calculation of a thin-walled coil

A thin-walled coil can be recognized as the result of superimposing the filament coils in z-direction, so the EMF $\mathbf{B}^{thin-walled}$ of a thin-walled coil can be determined as following

$$\mathbf{B}^{thin-walled} = \frac{1}{\delta}\int_\delta \mathbf{B} dz = \frac{1}{\delta}\left(\int_\delta \mathbf{B}_s dz_s + \int_\delta \mathbf{B}_{ec} dz_s\right). \tag{16}$$

Due to the influence of eddy-current, the impedance Z of a coil is given by

$$Z = Z_0 + \Delta Z, \tag{17}$$

where Z_0 represents incident field impedance and ΔZ represents scattered field impedance.

The impedance Z of a thin-walled coil with N -turns can be determined by the magnetic flux Φ that passes throughout the coil, namely

$$Z = \frac{Nd\Phi}{Idt} = \frac{Nj\omega}{I\delta}\int_\delta\int_S \mathbf{B}^{thin-walled} d\vec{S} dz. \tag{18}$$