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Investigation of polytropic corrections for the piston-in-cylinder primary standard used in dynamic calibrations of pressure sensors

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ABSTRACT

The increasing demands for more accurate dynamic measurements of pressure in different industrial and scientific applications require the use of sensors with suitable dynamic characteristics. This paper discusses a development of a piston-in-cylinder primary standard for dynamic calibration of pressure sensors. The time-varying pressure generated by a piston-in-cylinder pressure generator can be traceable to measurements of the static pressure and length at the highest and the lowest pulsation frequencies, where the process can be considered as adiabatic and isothermal, respectively. The main limitation of such a dynamic pressure calibrator to provide SI-traceable dynamic calibrations in the transition frequency range of polytropic pulsations is the fact that the value of the polytropic index depends on the degree to which the heat transfers to the surroundings during the generation of the time-varying pressure. In order to investigate the polytropic index, which defines the ratio of the amplitude of the generated relative pressure change to the amplitude of the relative volume change of the gas in the piston-in-cylinder calibrator, the analytical solution in the frequency domain is presented. The analytical solution was derived on the basis of the lumped physical-mathematical model for the gas in the cylinder chamber of the piston-in-cylinder dynamic pressure calibrator. The experimentally obtained polytropic index for the built piston-in-cylinder calibrator confirms the results obtained from the analytical solution. The paper ends with an estimation of the measurement uncertainty related to the polytropic corrections and the time-varying pressure amplitude generated by the developed piston-in-cylinder calibrator.

1. Introduction

The requirements for reliable dynamic measurements of pressure are becoming increasingly strict in many industrial and scientific applications [1,2]. The lack of metrological traceability for dynamic measurements of pressure, except for sound pressure, results in less-than-optimal measurements. Although it is well known that pressure sensors exhibit a distinctive dynamic behaviour that differs from their static sensitivity characteristic, the pressure sensors employed in such applications are often calibrated using static pressure standards to ensure metrological traceability to the International System of Units (SI).

As pressure calibration requirements vary widely in frequency and amplitude, a variety of operating principles and configurations for dynamic pressure calibrators have been developed in order to provide traceable dynamic calibrations with the required levels of uncertainty. In general, dynamic pressure generators are divided into two classes: aperiodic and periodic [3–5]. In choosing between aperiodic and periodic pressure calibrator it is recommended to choose the pressure generation that most closely resembles the actual measurement situation. The range of use for aperiodic pressure calibrators is relatively wide as they are able to generate high amplitudes and cover many areas of industrial use for pressure sensors, e.g., automotive industry, aviation, military, etc. Typical aperiodic pressure calibrators available for traceability at high-frequency time-varying pressures are shock tubes, drop-weight systems and quick-opening valve devices. The shock tubes can generate repeatable, fast-rising, step-like pressures changes, which can be calculated from the ideal gas dynamics by measuring the velocity of the shock front and the initial pressure, where relatively large uncertainties associated with determining the velocity of the shock front due to non-instantaneous opening of the diaphragm and reflections of a non-planar shock wave from the tube walls represent the main limitations of such dynamic pressure calibrators [6–11]. The drop-weight calibration systems can achieve metrological traceability by calculating the acceleration of the piston from a traceable measurement of the mass displacement, and inde-

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pendent measurements of the mass of the drop-weight and the cross-sectional area of the piston, or by measuring the time variation of the pressure-dependent refractive index of the fluid under compression. With maximum pressure values of hundreds of megapascals, drop-weight systems have much higher pressure capacities than a shock tube. However, pressure rise times for drop-weight systems are of the order of 1 ms–2 ms, leading to much reduced calibration bandwidths (less than 1 kHz) in comparison to shock tubes [8,12]. The reference time-varying pressure generated by quick-opening valve calibrators can be estimated from traceable measurements of the static pressure and the calculation of the pressure change based on the density change of the transmitting medium. The main limitations of such pressure calibrators are relatively large uncertainties related to the generated pressure profiles and much slower rise times for the generated pressure in comparison with those obtained in shock tubes [13].

In many industrial and scientific applications the ability to accurately measure small amplitudes of pressure pulsations is very important, e.g. in acoustics, in measurements of the fluid flow with differential pressure flow meters, in medicine, etc. For these applications the use of the periodic dynamic calibrators is essential as they enable determination of the dynamic response of pressure meters under constant-amplitude pulsating pressure conditions [14–16]. Typical periodic pressure calibrators available for traceability at low-amplitude time-varying pressures are rotating valves and piston-in-cylinder dynamic calibrators. A rotating valve switches the pressure supplied to the calibration object between two (or more) values, which generates approximately rectangular pressure waveform with the frequency determined by the rotational speed of the valve. The rotating-valve calibration systems can achieve metrological traceability for the measurements of the generated time-varying pressure with the use of an optical interferometric measurement technique for the pressure dependence of the refractive index of the fluid. The main limitations of such pressure calibrators are relatively large uncertainties related to the ringing of the pressure waveform, which mainly depends on the resonance frequency of the fluid inside the dynamic pressure calibrator [17]. An example of a periodic pressure calibrator capable of providing SI-traceable dynamic pressure calibrations is also the piston-in-cylinder dynamic calibrator, which is discussed in this paper. Its principle of operation is based on an oscillating piston, which causes changes in the volume of the gas in the cylinder chamber and consequently changes to the pressure inside the cylinder chamber. If we consider an ideal gas in a closed cylinder chamber, where the piston moves at a velocity much less than the speed of sound of the gas in the cylinder chamber, the relationship between the volume $V(t)$ and the pressure $p(t)$ of the gas in the cylinder chamber is given by [18,19]:

$$p(t)V^n(t) = \text{constant}, \tag{1}$$

where n is the dimensionless polytropic index. The value of n approaches the adiabatic index γ at higher frequencies of the piston oscillation, where the gas volume changes relatively quickly over time with respect to the heat transfer to the surroundings, and therefore the process can be considered as adiabatic. By decreasing the frequency of the piston oscillations, the value of n decreases. At the lowest pulsation frequencies, where the gas volume changes are relatively slow over time with respect to the heat transfer to the surroundings, the process can be considered as isothermal and the value of n approaches a value of 1.

The pressure and the volume of the gas in the cylinder chamber can be expressed as $p(t) = p_0 + p'(t)$ and $V(t) = V_0 + V'(t)$, respectively, where p_0 and V_0 represent their time-averaged components and $p'(t)$ and $V'(t)$ their time-varying components. For relatively small pressure and volume changes, the linear relationship between $p'(t)$ and $V'(t)$ can be written as:

$$\frac{p'(t)}{p_0} = -n\frac{V'(t)}{V_0}. \tag{2}$$

Considering the relationship between the volume change of the gas in the cylinder chamber and the displacement of the piston $V'(t) = Ah'(t)$, where A is the piston effective area and $h'(t)$ is the piston displacement, Eq. (2) can be written as [20]:

$$\frac{p'(t)}{p_0} = -n\frac{h'(t)}{h_0}, \tag{3}$$

where $h_0 = V_0/A$ is the effective height of the cylinder chamber. From Eq. (3) it is clear that time-varying pressure can be traceable to static pressure and length measurements at the highest and the lowest pulsation frequencies, where the process can be considered as adiabatic and isothermal, respectively. The main limitation of such a dynamic pressure calibrator for providing SI-traceable dynamic calibrations over a wide frequency range is the fact that the value of the polytropic index n in the transition frequency range of polytropic pulsations depends on the degree to which the heat transfers to the surroundings during the generation of time-varying pressure [21]. Therefore, calibrations in the transition frequency range of polytropic pulsations require the heat transfer correction to be employed to account for energy transfer that indicates a departure from adiabatic and isothermal behaviour. Many researchers have studied this general problem in the field of acoustics, where the standard governing primary reciprocity calibration of microphones provides two models for the heat transfer correction: the low frequency solution and the broadband solution [22–24]. The low frequency model derives from Gerber's treatment of heat transfer in a closed fluid-filled cylindrical chamber, where the heat flow for the finite cylinder geometry at low frequencies is determined from the equation for energy conservation in an ideal gas, under the assumption of negligible convection, viscosity, friction, and a homogenous pressure distribution in the chamber [25]. Broadband solution, on the other hand, expands on the low frequency model by also considering a high frequency viscosity effect and therefore removes the assumption of homogeneous pressure within the chamber [26,27]. Analyses revealed that even after accounting for simplification errors, the underlying models exhibit different behaviours at lower pulsation frequencies that cannot be explained by their different treatments of viscosity.

The aim of this paper is to investigate the polytropic corrections for the piston-in-cylinder calibrator with the purpose of developing the primary measurement system for dynamic calibration of pressure sensors. The obtained polytropic index variations will enable the derivation of the semi-empirical heat transfer correction factors to account for energy transfer with the surroundings that indicates a departure from adiabatic and isothermal behaviour, which will decrease the uncertainty component that covers the incompleteness of the current measurement models. In order to study the polytropic index in the frequency domain, mathematical and experimental analyses were performed. The analytical solution for the polytropic index in the frequency domain, which was derived on the basis of the lumped physical-mathematical model for the gas in the cylinder chamber of the piston-in-cylinder dynamic pressure calibrator, is presented in Section 2. Section 3 presents the measurement system for the experimental investigations of the polytropic index for the piston-in-cylinder calibrator, the mechanical implementation of which was presented by the authors of this paper in [28]. The experimentally obtained polytropic index and the contribution of its measurement uncertainty to the uncertainty of the time-varying pressure amplitude generated by the built piston-in-cylinder calibrator are discussed in Section 4.